CALCULATION OF AVERAGE PARAMETERS OF EXTENSIVE COSMIC-RAY AIR SHOWERS

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Results of a calculation of nuclear-cascade showers containing a given number of electrons at the observation level are presented. The distribution of the production levels is determined for extensive air showers containing 10^4, 10^5, and 10^6 particles at sea level and at 3860 m altitude. Various shower parameters, averaged over different production levels, are compared with experimental data.

THE experiments[1,2] carried out in Pamir (3860 m altitude) in the course of many years have yielded very detailed information on the average parameters of extensive air showers (EAS) with a total number of particles from 10^4 to 10^6 at mountain altitudes. The question arises as to what extent the experimental data correspond with present views concerning the elementary act of nuclear interaction over a wide energy range of the interacting particles.

In experimental studies of the different parameters of EAS, the total number of charged particles (i.e., shower size) at the observation level is determined, rather than the energy of the primary particle producing the shower. Because of fluctuations in the shower production level,[3] in the nuclear interactions of the "leading" particle along the depth of the atmosphere,[4] and in the characteristics of the elementary act of nuclear interaction,* the total number of particles at the observation level reflects the primary-particle energy only on an average basis. In our calculations, we have taken the fluctuations in the shower-production level into account. Other fluctuations are not essential for the assumed model of the elementary act of nuclear interaction upon which the calculations are based. The presence of nuclei with Z > 1 in the primary cosmic radiation has also not been taken into account, which leads to a slight overestimate of the fluctuations in the shower-production level.

*The hypothesis that there is a strong influence of such fluctuations on the development of EAS has been discussed by Grigorov and Shestoperov.[5] In quantitative estimates, however, the fluctuations in the characteristics of the elementary act of interaction have been reduced to fluctuations in the shower production level.

1. NUCLEAR-CASCADE SHOWERS PRODUCED BY PARTICLES OF A GIVEN ENERGY

The EAS that are within the size range of interest at the observation level are nuclear-cascade showers produced by primary particles of 10^{12} to 10^{16} ev.

The calculation of nuclear-cascade showers was carried out under the following assumptions:

1. The cascade of nuclear-active particles consists of nucleons and $\pi^\pm$ mesons which, in nuclear interactions, lose energy for the production of π^0 mesons. The number of $\pi^\pm$ mesons decreases also as a result of the $\pi \to \mu$ decay. The possibility of the production of other nuclear-active particles and their influence on the cascade process was not taken into account.

2. To describe the elementary act of nuclear interactions of nucleons and $\pi^\pm$ mesons with air nuclei at $E_0 > 10^{13}$ ev, the hydrodynamical model was used,[6] taking the traveling wave[7] into account. The particle energy corresponding to the traveling wave was assumed to be $0.8\, E_0\, (E_0/\mu c^2)^{-1/15}$, where μ is the rest mass of the meson. The production probability of nucleons or π mesons of such an energy corresponds to the fraction of nucleons or π mesons among all secondary particles, assumed to be 0.27* and 0.73 respectively.

*Such a fraction of nucleons among all secondary particles corresponds to a disintegration temperature of the hydrodynamical system $T_K \sim 1.5\, \mu c^2$. Preliminary calculations based on the fraction of nucleons corresponding to a disintegration temperature $T_K \sim \mu c^2$ have yielded a lower number of nuclear-active particles in EAS at the observation level than that obtained in experiments. It is possible that, by taking hyperons into account within the framework of the hydrodynamical theory, we shall obtain a sufficient number of particles different from π mesons even for $T_K \sim \mu c^2$. Incidentally, the value of 0.27 as the fraction of π mesons does not contradict certain available emulsion data.[8]

3. The relations used for the description of the nuclear interaction of nucleons with energy $E_0 < 10^{13}$ ev differ from the above-mentioned hydrodynamical model only in one additional assumption, namely that the particle corresponding to the traveling wave is a nucleon. For $\pi^{\pm}$ mesons with energy $E_0 < 10^{13}$ ev, the same hydrodynamical model was used, but without taking the traveling wave into account. Although this formal extrapolation into the low-energy region (down to 10^{10} ev) of the hydrodynamical-model relations does not have sufficient physical justification, the required characteristics of nuclear interactions obtained in this manner nevertheless do not contradict the experimental data in the corresponding energy range.[9]

4. The nuclear-interaction mean free path in air was assumed to be $\lambda_0 = 75$ g/cm^2. In the following discussion, length is measured in units of λ_0.

5. The minimum energy of the particles taking part in the nuclear-cascade process was assumed to be 3.7×10^9 ev.

In carrying out the calculations, the method of successive generations[10] was used. The number of nucleons of the i-th generation of energy E found in a shower produced by a primary particle of energy E_0 was assumed to be

$$N_i(E, E_0) = \int_E^{E_0} N_{i-1}(\varepsilon, E_0)\frac{dn^n}{d\ln\varepsilon}\,d\ln\varepsilon + \int_E^{E_0} \pi_{i-1}^{\pm}(\varepsilon, E_0)\frac{dn^{\pi^-}}{d\ln\varepsilon}\,d\ln\varepsilon \quad (1)$$

and, correspondingly, the number of $\pi^{\pm}$ mesons is

$$\pi_i^{\pm}(E, E_0) = A_i(E)\int_E^{E_0} N_{i-1}(\varepsilon, E_0)\frac{d\pi^n}{d\ln\varepsilon}\,d\ln\varepsilon + A_i(E)\int_E^{E_0} \pi_{i-1}^{\pm}(\varepsilon, E_0)\frac{d\pi^{\pi}}{d\ln\varepsilon}\,d\ln\varepsilon, \quad (2)$$

where $A_i(E) = [1 + k(E)/i]^{-1}$ is a coefficient accounting for the decay of charged π mesons of energy E, and k(E) is given by the expression[10]

$$k(E) = l_0\lambda_0/\rho_0(E/\mu c)\tau_0\cos\theta.$$

Here ρ_0 is the mean air density in g/cm^3 at the depth l_0 corresponding to sea level, μ the $\pi^{\pm}$ meson mass, and τ_0 the lifetime of the $\pi^{\pm}$ meson. An isothermic atmosphere was assumed. In Eqs. (1) and (2), $dn^n/d\ln\epsilon$ and $dn^{\pi}/d\ln\epsilon$ are the spectra of secondary nucleons produced in the interaction between a nucleon or $\pi^{\pm}$ meson respectively and an air nucleus, and $d\pi^n/d\ln\epsilon$ and $d\pi^{\pi}/d\ln\epsilon$ are the spectra of secondary $\pi^{\pm}$ mesons

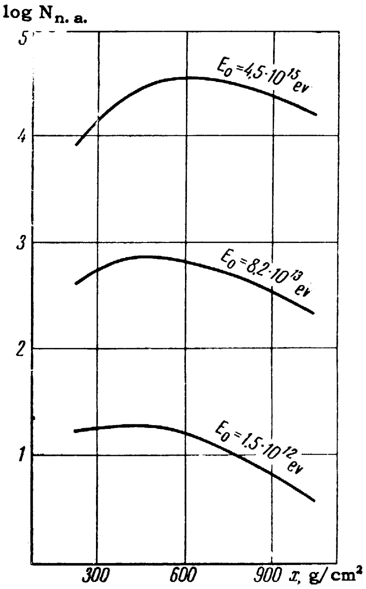

FIG. 1

produced in the interaction between a nucleon or $\pi^{\pm}$ meson respectively and an air nucleus. The total number of nucleons and $\pi^{\pm}$ mesons of energy $\geq E$ at a depth x is given by the equations[10]

$$N(\geqslant E, x) = \sum_i^{\infty}\int_E^{E_0}\frac{e^{-x}x^i}{i!}N_i(E', E_0)\,dE',$$
$$\pi^{\pm}(\geqslant E, x) = \sum_i^{\infty}\int_E^{E_0}\frac{e^{-x}x^i}{i!}\pi_i^{\pm}(E', E_0)\,dE'. \quad (3)$$

The variation of $N_{na} = N(\geq E, x) + \pi^{\pm}(\geq E, x)$ with depth x for $E \geq 3.7 \times 10^9$ ev is shown in Fig. 1 for three values of E_0. In the summation, we cut the series off at that generation whose contribution to the nuclear-active component of the shower amounts to < 1% of the total number of nuclear-active particles at the given depth.*

In order to obtain the variation of the number of electrons with the depth x, it is necessary to add together the cascade showers from the photons produced in the decay of the π^0 mesons. The number of γ rays of energy E of the i-th generation, produced by the π^0 mesons in a shower with total energy E_0, can be assumed to be

$$\gamma_i(E/2, E_0) = \pi_i^{\pm}(E, E_0)/A_i(E).$$

The quantity $\pi_i^{\pm}(E, E_0)$ is determined by Eq. (2). The total number of electrons at a depth x, produced by photons of energy $\geq E$ produced directly in the decay of the π^0 mesons, is given by the equation[10]

*The calculation was carried out by numerical approximation. The error of the results is less than 10%.

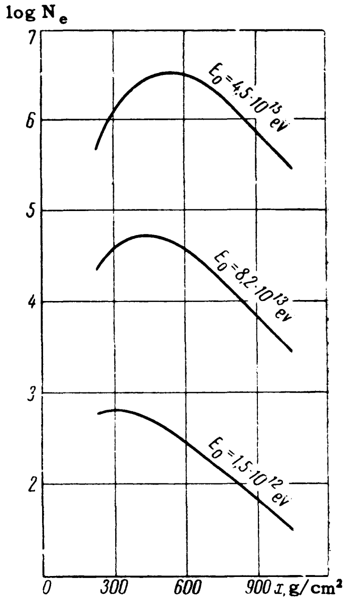

FIG. 2

$$N_e(x) = \sum_i \int_E \gamma_i(E, E_0) \int_0^x e^{-(x-t)} \frac{(x-t)^{i-1}}{(i-1)!} P(E, t)\, dt\, dE, \quad (4)$$

where $P(E, t)$ is the total number of cascade electrons produced by a photon of energy E if the cascade has traversed the depth t.[11] This expression represents a good approximation for $E \gg \beta$, where $\beta = 7.2 \times 10^7$ ev is the critical energy for air. For $E < 10^9$ ev, cascade curves calculated by the moment method were used. The variation of $N_e(x)$ with the depth x for three values of E_0 is shown in Fig. 2.

2. INFLUENCE OF FLUCTUATIONS IN THE PRODUCTION LEVEL OF EAS

By measuring any parameter of an EAS with a given number of particles at the observation level, we obtain an average over a certain energy range of the primary particles that have interacted at different altitudes above the observation level. The results of the calculations of the preceding section refer to showers produced by primary particles of a given energy. The distribution of the production levels and of the energies of the primary particles can be found if we know the energy spectrum of the cosmic radiation at the upper edge of the atmosphere $f(E_0)\,dE_0 = A\,dE_0/E^{\gamma+1}$, the depth of the observation level x_0, the mean free path of nuclear interaction λ_0, and the relation between the primary-particle energy and the total number of charged particles in the shower at various depths in the atmosphere. In the range of production

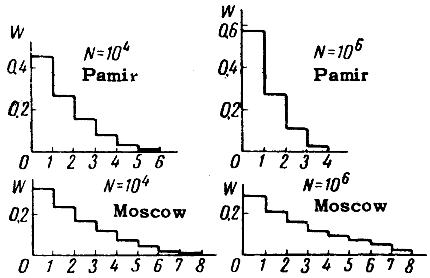

FIG. 3. Production probability W at a given atmospheric depth of a shower having a given size at the observation level (Moscow or Pamir). (The depth is measured in nuclear-interaction mean free paths.)

levels and shower sizes of interest we can, with sufficient accuracy, substitute the number of electrons $N(E_0, x) \approx N_e(E_0, x_0 - \bar{x})$ (where $\bar{x}$ is the shower production depth) for the total number of charged particles in the shower.

The number of showers observed at the depth x_0 with a number of particles N and produced at depth $\bar{x}$, is given by the expression

$$C(N, \bar{x})\, dN d\bar{x} = A e^{-\bar{x}} E_0^{-\gamma} \frac{\partial E_0}{\partial N}\, dN d\bar{x}, \quad (5)$$

where $E_0 = E_0(N, x_0 - \bar{x})$. The probability of shower production at various depths in the atmosphere, given by Eq. (5), is shown by the histograms in Fig. 3 for two given values of N at mountain altitudes and at sea level. It can be seen that the maximum contribution is due to the showers produced along the first nuclear-interaction mean free path. However, while the only appreciable contribution at mountain altitudes is made by the showers produced along the first three interaction mean free paths, at sea level it is also necessary to take the showers produced at greater depths into account.

From a known distribution of the shower-production levels, we can calculate the energy distribution of the particles producing showers of a given size at the observation level. Such distributions are represented by the histograms in Fig. 4 for showers with a total number of particles $N = 10^5$. The y axis denotes the shower-production probability W for a particle with a given energy. The dotted and solid lines show the distribution for sea level and for the mountain altitude (depth 675

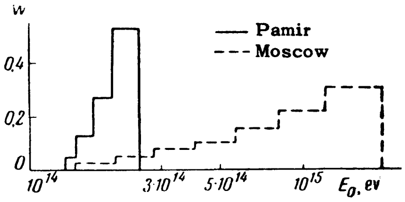

FIG. 4.

TABLE I

Observation level, m	N		
	10^4	10^5	10^6
3860	$2.6 \cdot 10^{13}$	$2 \cdot 10^{14}$	$1.6 \cdot 10^{15}$
0	$1.3 \cdot 10^{14}$	$0.9 \cdot 10^{15}$	$0.7 \cdot 10^{16}$

g/cm^2) respectively. The mean energy (in ev) of the primary particles which produce showers of a given size N at sea level, obtained from analogous distributions, is shown in Table I.

It can be seen from the table that the relation between the mean energy of the primary particles and the total number of charged particles at the observation level differs little from the usual factor relating the number of particles with the energy of the shower-producing primary particle.[12]

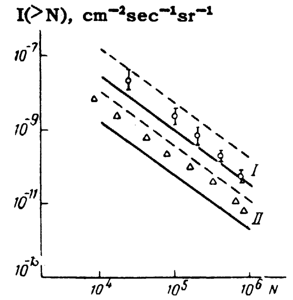

FIG. 5. Intensity of EAS with size greater than N. I – calculation for mountain altitudes, II – calculation for sea level, o – experimental data[15] for mountain altitudes, Δ – experimental data[15] for sea level.

The number of showers of a given size at the observation level (x_0) can be obtained from Eq. (5) by integrating over $\bar{x}$. The frequency of showers of size $>N$ is shown in Fig. 5.

$$J(>N) = \int_N^\infty dN \int_0^{x_0} C(N, \bar{x})\, dx.$$

The absolute intensity of the primary cosmic radiation was taken as $f(E_0)\,dE_0 = AE_0^{-2.7}\,dE_0$, where the coefficient A was normalized to experimental data[13] for 10 Bev. The results of this calculation are shown by the solid line in Fig. 5. A calculation was also carried out based on the data on the spectrum of primary cosmic radiation from underground measurements.[14] The result is shown by the dotted line in Fig. 5. Experimental data on the intensity of EAS are taken from reference 15. It can be seen from the figure that the experimental data on the absolute frequency of EAS can be made to agree well with the calculation if, for the intensity of primary cosmic radiation, we assume an intermediate value between the results of references 13 and 14.

Table II. Mean free path for particle absorption in showers, g/cm^2

Observation level, m	Number of particles in the shower		
	10^4	10^5	10^6
3860	230	280	340
0	170	175	180
From the calculated altitude dependence between the levels 3860 and 0 m	210	210	—

The altitude variation of the number of EAS calculated by us is in agreement with experimental data, within the statistical spread of the latter.

Showers with a given number of particles at the observation level are produced at a different altitude, and consequently have a different absorption coefficient than at the observation level. The average calculated values of the absorption mean free path of the particles in showers observed at sea level and at mountain altitudes are shown in Table II. It can be seen from the table that the altitude variation of the showers does not depend on the average parameters of individual showers (see also reference 5). Using the altitude variation of EAS obtained before (Fig. 5) and the size spectrum of showers, one can determine the absorption coefficients of the shower particles[12] neglecting the fluctuations in shower development (lower row of Table II).

3. ENERGY FLUX CARRIED BY SHOWER PARTICLES AT THE OBSERVATION LEVEL

The total energy of the nuclear-active component in showers of a given size at the observation level x_0 was calculated from the energy spectra of nuclear-active particles at different depths in the atmosphere in a shower produced by a primary particle with energy E_0. Nucleons and charged π mesons, calculated according to Eq. (3), were regarded as nuclear-active particles. If the differential energy spectrum of nuclear-active particles in the shower produced by a primary particle with energy E_0 at a depth $x = x_0 - \bar{x}$ is $f(E, x_0 - \bar{x}, E_0)\,dE$, then the total energy carried by nuclear-active particles at the observation level in showers of size N is

$$\varepsilon(N, x_0) = \frac{1}{C(N)} \int_0^{x_0} C(N, \bar{x})\, d\bar{x} \int_{E_{min}}^{E_{max}} Ef(E, x_0 - \bar{x}, E_0)\, dE,$$

where

$$C(N) = \int_0^{x_0} C(N, \bar{x})\, d\bar{x}.$$

TABLE III

N	Altitude, m	Energy of nuclear-active shower component, ev	
		Calculation	Experiment
10^4	3860	$6\cdot10^{12}$	$\sim4.4\cdot10^{12}$
	0	$8.8\cdot10^{12}$	$>1.7\cdot10^{12}$
10^5	3860	$4.6\cdot10^{13}$	$\gtrsim1.1\cdot10^{13}$
	0	$7.7\cdot10^{13}$	$>2\cdot10^{13}$
10^6	3860	$4\cdot10^{14}$	$\sim2.3\cdot10^{14}$
	0	$8.2\cdot10^{14}$	$\gtrsim1.2\cdot10^{14}$

Moreover, the distribution of shower-production levels $\bar{x}$ and the relation between $\bar{x}$ and E_0 is given by Eq. (5). The limits of integration $E_{min} = 3.7 \times 10^9$ ev and $E_{max} < E_0$ represent the minimum energy of the nuclear-active particles that were taken into account and the maximum energy of the nuclear-active particles encountered in the calculated spectrum. The values of $\epsilon(x_0, N)$ obtained in the calculation are shown in Table III. The same table also shows the experimental data on the energy flux carried by the nuclear-active component of EAS.[18,19] A comparison of the experimental data with the calculation reveals an agreement only for showers with a total number of particles $N = 10^4$ at an altitude of 3860 m above sea level. For the remaining cases, the calculated energy fluxes of the nuclear-active component agree with experimental data only in their order of magnitude.

TABLE IV

N	Altitude, m	Energy of electron-photon shower component, ev		$\bar{\varepsilon}(N, x_0)$, ev
		Calculation	Experiment	
10^4	3860	$2.8\cdot10^{12}$	$2.3\cdot10^{12}$	$4.1\cdot10^{12}$
	0	$2.3\cdot10^{12}$	$2\cdot10^{12}$	$3.5\cdot10^{12}$
10^5	3860	$3.2\cdot10^{13}$	$2.3\cdot10^{13}$	$3.8\cdot10^{13}$
	0	$2.4\cdot10^{13}$	$2\cdot10^{13}$	$3.5\cdot10^{13}$
10^6	3860	$3.4\cdot10^{14}$	$2.2\cdot10^{14}$	$4.1\cdot10^{14}$
	0	$2.5\cdot10^{14}$	$2\cdot10^{14}$	$3.5\cdot10^{14}$

The energy of the electron-photon component in showers of a given size at the observation level cannot be obtained directly from the calculations, since the energy spectrum of electrons and photons was not calculated by us. The energy flux carried by the electron-photon shower component can be estimated, assuming that the shower-particle absorption coefficient corresponds to the average cascade parameter $\bar{s}$ at the observation level. The results of such an estimate and the experimental data on the energy of the electron-photon component are shown in Table IV. The agreement of the calculated values with the experiment can be regarded as sufficient, especially since the method used can lead to an underestimate of the parameter $\bar{s}$ and, consequently, to an overestimate of the energy.

We can calculate more exactly the energy lost below the observation level by the electron-photon component of showers of a given size. This energy $\bar{\epsilon}(N, x_0)$, in addition to the energy of the electron-photon component at the observation level, includes the energy transferred to π^0 mesons below this level. Using the notation introduced above, we can express the calculation procedure by the relation

$$\bar{\varepsilon}(N, x_0) = \frac{1}{C(N)}\int_0^{x_0} C(N, \bar{x})d\bar{x} \int_{x_0-\bar{x}}^{\infty} N_e(x')\,dx',$$

where $N_e(x') = N$ for $x' = x_0 - \bar{x}$. The results are shown in the last column of Table IV.

The corresponding quantity cannot be measured experimentally, and is usually estimated from the absorption of showers in the atmosphere as $4.4 \times 10^8 N_e$, where N_e is the number of electrons in the shower at observation level. A comparison with the values in the table shows that, by taking the fluctuations in the observation level into account, we cause a small decrease in the calculated energy lost by the electron-photon component in the atmosphere.

4. CONCLUSIONS

1. The above calculations of various parameters of EAS of a given size at the observation level show that the study of the shower structure, for the purpose of obtaining information on the variation of the elementary nuclear act of interaction with the energy of the primary particle, can be carried out more efficiently at mountain altitudes, near the maximum of the nuclear-cascade shower development. At mountain altitudes, the number of particles (shower size) at the observation level represents sufficiently well the energy of the primary particle which produced the shower. In the interpretation of experimental data obtained at sea level, the distribution of shower-production levels may already change the energy of the primary particle producing a shower of a given size by a factor of ten.

2. The assumptions on which the calculations are based lead to a satisfactory description of the composition of EAS at mountain altitudes, and agree with the experimentally observed altitude dependence of the showers. However, a number of parameters referring to the number of nuclear-active particles and the energy carried by them at sea level are markedly different from experimental data. The main reasons for this discrepancy

are the assumption that the inelasticity coefficient of nucleon interactions decreases in the $10^{11} - 10^{9}$ ev energy range, and the identification of all those secondary nuclear-active particles which are not π mesons as nucleons.

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