BEHAVIOR OF DOMAIN STRUCTURE UNDER THE INFLUENCE OF ELASTIC STRESSES

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The problem of the behavior of the simplest domain structure under the influence of homogenous elastic stresses is solved by the thermodynamic method developed for ferromagnetic materials by Landau and Lifshitz. A crystallite of the iron type, a surface of which coincides with a (001) plane, is considered. The good agreement of the calculation with experimental data permits use of the formulas derived for estimation of the density of magnetic poles on the boundaries between crystallites.

As is shown by experiment, in a plane crystallite of iron, whose surface coincides with the crystallographic plane (001), there occurs a plane-parallel domain structure, with domains oriented along one of the easy axes. At the edges of the crystallite, at the place of emergence of the vector I_S, there may be a formation of closure domains; sometimes this is not advantageous. In general we may assume the presence of partial-closure domains (Fig. 1), whose width d may take a value from zero to D, the width of the basic domains. We shall calculate the free energy of such a structure. We take the coordinate axes along the tetragonal axes of the crystallite; we denote by x_0, y_0, z_0 the dimensions of the crystallite along the respective coordinate axes.

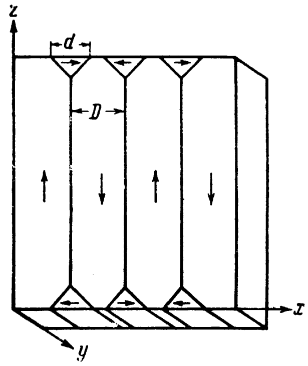

FIG. 1

1. THE FREE ENERGY

Kittel[2] gave a general method of calculating the energy of magnetic poles. He showed that for a structure periodic along one axis, the surface density of pole energy is given by the expression

$$\gamma_M = \pi \sum_{-\infty}^{+\infty} C_m C_{-m} P_m^{-1}, \qquad P_m = |m|\pi/D,$$

where the C_m's are the Fourier coefficients of the pole density. On applying this method to our case, we get

$$\gamma_M = \frac{4\omega^2 D}{\pi^2}\left[1.05 + \sum_{m=1}^{\infty} \frac{\cos(2m-1)\pi k}{(2m-1)^3}\right], \tag{1.1}$$

where ω is the surface density of magnetic poles, and where $k = d/D$.

For poles on an iron-air boundary, in the absence of wedge-shaped domains, ω is equal to the saturation magnetization I_S. On the boundary between two crystallites, for a number of reasons, ω may be appreciably less than I_S. For example, if the domains in the adjacent crystallite are prolongations of the domains under consideration (Fig. 2), then $\omega = I_S(1 - \cos\varphi)$; a finer domain structure in the adjacent crystallite also leads to a diminution of ω; there may occur a formation of a sort of interdomain partition on small sections of the boundary between crystallites; the effective value of ω is sharply decreased by wedge-shaped domains, which may remain unnoticed if they are located in the body of the material and do not extend to the surface of observation. Therefore ω for the crystallite must be considered an unknown quantity, lying in the interval 0 to I_S. For each specific case, the value of ω can be calculated

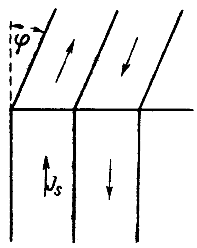

FIG. 2

from observation of the domain structure, as will be shown below. Assuming that the effective densities ω are the same on both faces of the crystallite, we write the magnetic pole energy, referred to unit volume, in the form

$$F_M = \frac{8\omega^2}{\pi^2 z_0} Df(k). \quad (1.2)$$

Here and below, all energies are referred to unit volume of the crystallite. By f(k) we denote the expression enclosed in square brackets in formula (1.1). Its value can be calculated with tables compiled by Kitover.[11] For rough estimates we approximated f(k) with the cubic polynomials

$$f(k) = \begin{cases} 5.28\,k^3 - 6.84\,k^2 + 2.1, & k \leqslant 0.5 \\ 5.28\,k^3 - 9\,k^2 + 2.16\,k - 1.56, & k \geqslant 0.5 \end{cases}.$$

If a surface of the crystallite does not coincide exactly with the direction [001], then the energy of the magnetic poles on this surface is

$$F_M^* = 1.7 I_s^2 \sin^2\theta \cdot 2D/(1+\mu^*)\,y_0 = aD/y_0,$$

$$\mu^* = 1 + 2\pi I_s/K, \quad (1.3)$$

where θ is the angle between the crystallite surface and the direction [001], and where K is the magnetic anisotropy constant.

The problem of calculating the magnetostrictive energy connected with the formation of closure domains (a problem in the theory of elasticity) has not been solved in its general form. Use is usually made of crude approximations, which give (for complete-closure domains) satisfactory agreement with experiment.[2,3] To calculate the dependence of the energy on $k = d/D$, we shall also make use of simplifying assumptions.

Because of the inequality of the magnetostrictive deformations along and across the magnetization, the closure domains are, in effect, squeezed between the basic ones. The latter prevent them from deforming in accordance with the requirements of magnetostriction, and this produces internal stress both in the closure domains and in the basic domains. The diagram (Fig. 3) may clarify this statement. We suppose at first that magnetostrictive deformation is absent; then a

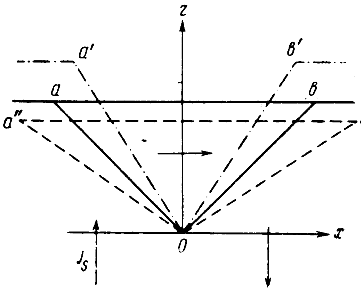

FIG. 3

closure domain occupies the region aOb. Now we take account of magnetostriction, taking the point O fixed. The basic domain elongates along the z axis, with relative elongation λ_{100}, and contracts along the x axis, with relative contraction $\lambda_{100}/2$. As a result it tries to take the position a′Ob′; for the same reasons, the closure domain tries to take the position a″Ob″. In actuality, one may reason, the points a′ and a″, and likewise the points b′ and b″, will coincide, and there will occur some equilibrium state with a complicated distribution of the stress tensor, both in the basic and in the closure domains.

We suppose for simplicity that deformation occurs only along the x axis, and we calculate the density of energy in the closure domains. Each half of such a domain (Fig. 4) undergoes at its top part an absolute deformation $\lambda_{100}d/2$. Consequently

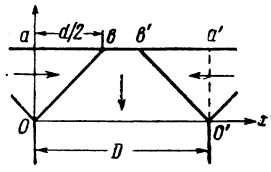

FIG. 4

the relative deformation of the segment aa′ will be $\bar{u}_{11} = -\lambda_{100}d/D$. By assuming that the deformation decreases linearly with z and taking $u_{11} = 0$ at $z = 0$, we get for the mean deformation $u_{11} = -\lambda_{100}k/2$. We take all the other components of the strain tensor equal to zero. Since the energy density of magnetostriction is connected with the components of the strain tensor by the relation[3]

$$f_{ms} = -3c_2\lambda_{100}\left(\sum_i \alpha_i^2 u_{ii} - \frac{1}{3}u_{ii}\right) - 3c_3\lambda_{111}\sum_{i\neq k}\alpha_i\alpha_k u_{ik}, \quad (1.4)$$

we get for the magnetostriction energy the expression

$$F_{ms} = c_2\lambda_{100}^2 Dk^3/2z_0. \quad (1.5)$$

The wall energy is determined with sufficient accuracy by the formula

$$F_w = \gamma/D, \quad (1.6)$$

where γ is the surface density of wall energy.

The energy density of elastic stresses, in the case in which the magnetization is parallel to an easy axis,[4] is

$$f_\sigma = -\frac{3}{2}\lambda_{100}\sigma\left(\cos^2\alpha - \frac{1}{3}\right), \quad (1.7)$$

where α is the angle between the direction of the applied stress and the vector $\mathbf{I_S}$. Calculation of the energy leads, both for the basic and for the closure domains, to the expression

$$F_\sigma = -\frac{3}{2}\lambda_{100}\sigma\left(\cos^2\varphi - \frac{1}{3}\right) + \frac{3}{4}\lambda_{100}\sigma\cos 2\varphi\,\frac{Dk^2}{z_0}, \quad (1.8)$$

where φ is the angle between the stress direction and the z axis (the direction [001]).

The complete free energy of the structure,

$$F = F_M + F_M^* + F_{ms} + F_w + F_\sigma \quad (1.9)$$

is in our case a function of the two parameters D and k.

2. BEHAVIOR OF THE STRUCTURE UNDER THE INFLUENCE OF STRESS

For an arbitrary value of σ, the equilibrium state of the structure is determined by the conditions

$$\partial F(D,\,k)/\partial D = 0, \qquad \partial F(D,\,k)/\partial k = 0, \quad (2.1)$$

which lead to the equations

$$D = \left[\frac{\gamma z_0}{a z_0/y_0 + 8\omega^2\pi^{-2} f(k) + c_2\lambda_{100}^2 k^3/2 + {}^3/_4\lambda_{100}\sigma k^2\cos 2\varphi}\right]^{1/2}, \quad (2.2)$$

$$(16\omega^2/3\pi^2) f'(k) + \lambda_{100}^2 c_2 k^2 + \lambda_{100} k\sigma\cos 2\varphi = 0. \quad (2.3)$$

The type of behavior of the structure depends on the value of the angle φ at which the stress acts. For $\varphi = \pi/4$, the stress has no effect on the structure. For $\varphi < \pi/4$, the value of k decreases with increase of σ and becomes zero when

$$\sigma\cos 2\varphi \approx 7.4\omega^2/\lambda_{100}. \quad (2.4)$$

Things are more complicated as regards the parameter of the basic structure, D. On increase of σ, one term in the denominator of (2.2) increases, the others decrease. According to the value of ω, either decrease or increase of D upon increase of σ may occur in the same material. When k becomes zero, D ceases to depend directly on the stress:

$$D = \left[\frac{\gamma z_0}{a z_0/y_0 + 8\omega^2\pi^{-2}\cdot 2.1}\right]^{1/2}.$$

However, even for k = 0 a change of structure is possible, because of a change of the value of ω; for it is determined by conditions at the boundary of the crystallites, i.e., depends on the structure in adjoining crystallites.

For $\varphi > \pi/4$, the value of k increases with the value of σ, though complete closure does not occur. The direction of change of D now depends on ω. For k = 1

$$D = \left[\frac{\gamma z_0}{a z_0/y_0 + \lambda_{100}c_2/2 + {}^3/_4\lambda_{100}\sigma\cos 2\varphi}\right]^{1/2}, \quad (2.5)$$

i.e., from the moment of complete closure, D increases with increase of σ. However, this formula does not completely describe the behavior of the structure for k = 1. Physically, the tendency toward increase of D is a consequence of the tendency toward increase of the volume of the closure domains, in which the vector $\mathbf{I}_S$ is more favorably oriented with respect to the applied stress. Increase of the volume of the closure domains can occur not only by increase of D, but also by growth of some of the closure domains and shrinking of their neighbors, without violation of the closure condition (Fig. 5). Consequently, the structure has an additional degree of freedom which must be taken into account. The energy of elastic stresses for such a structure is

$$F_\sigma = -\frac{3}{2}\lambda_{100}\sigma\left(\cos^2\varphi - \frac{1}{3}\right) + \frac{3}{4}\lambda_{100}\sigma\cos 2\varphi\cdot\frac{D}{z_0}(\eta^2 - 2\eta + 2), \quad (2.6)$$

where $\eta = d_1/D$, $d_1 + d_2 = 2D$, and d_1 is the width of the larger of the adjoining closure domains.

The magnetostriction energy we compute by the same approximation as before. The mean deformation along the x axis (Fig. 6) is, for the volumes Oaa′O′ and abb′a′ respectively,

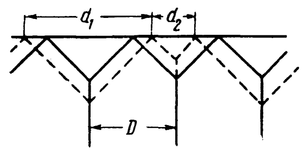

FIG. 5

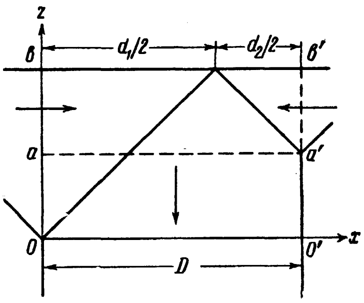

FIG. 6

$$\overline{u}_{11}^{(1)} = -\lambda_{100}d_1/4D \quad\text{and}\quad \overline{u}_{11}^{(2)} = -\lambda_{100}(d_1 + d_2)/4D.$$

Hence

$$F_{ms} = \frac{1}{2}\lambda_{100}^2 c_2 D z_0^{-1}(\eta^3 - 3\eta^2 + 3\eta). \quad (2.7)$$

A systematic consideration of the case $\varphi > \pi/4$ requires introduction of the variable η from the beginning, and solution of the problem of a minimum of a free energy dependent on three parameters: D, k, and η. For simplicity we have assumed that the influence of η begins to be felt only after closure has occurred. The free energy F_η

of such a structure is the sum of the energies determined by formulas (1.3), (1.6), (2.6), and (2.7). The equilibrium conditions

$$\partial F_{\eta}(D, \eta)/\partial D = 0, \qquad \partial F_{\eta}(D, \eta)/\partial \eta = 0 \tag{2.8}$$

lead to the equations

$$D = \left[\frac{\gamma z_0}{az_0/y_0 + \lambda_{100}^2 c_2(\eta^3 - 3\eta^2 + 3\eta)/2 + {}^3/_4\lambda_{100}\sigma\cos 2\varphi(\eta^2 - 2\eta + 2)}\right]^{1/2}, \tag{2.9}$$

$$\eta = 1 - \sigma\cos 2\varphi/\lambda_{100}c_2. \tag{2.10}$$

With increase of σ, η increases; as $\sigma \cos 2\varphi \to -\lambda_{100}c_2$, η approaches the value 2, corresponding to the usual structure with uniform complete closure domains (Fig. 7, c). Under these conditions, D also increases. Upon increase of σ, sooner or later there comes a moment when F_X becomes less than F_Z (here F_Z is the free energy of a structure in which the basic domains are parallel to the z axis, and F_X is the energy of a structure whose basic domains are parallel to the x axis). Then there occurs a complete reorganization, and a new structure is formed, with domains parallel to the x axis.

We summarize the results obtained. In the absence of stress in the crystallite (Fig. 7) one of the three structures a, b, c is established, according to the value of ω. Suppose structure a has been established. Upon application of a homogeneous tensile stress at an angle greater than $\pi/4$ to the z axis, closure appears at some value of σ (b); it increases, until complete closure occurs (c); meanwhile D may either increase or decrease. There further occurs a growth of some closure domains and a shrinking of other (adjoining) ones, until structure e is formed; in this stage, D increases. When F_X becomes smaller than F_Z, there occurs a changeover to basic domains along the x axis; in the new direction one of the three structures f, g, h may form, according to the value of ω on the new boundary and to the value of σ at which the changeover occurred. On increase of σ, the closure will diminish (g), until state h occurs. Further increase of σ will have no effect on the nature of the domain structure (unless it produces a change of ω).

3. COMPARISON WITH EXPERIMENT

Various aspects of the process pictured in Fig. 7 have been observed by many authors. Structure b has been produced[5,6] with $k \sim 0.5$ at $\sigma = 0$; calculation by formula (2.3) gives for this case $\omega \approx 5$ gauss. A process similar to the transition

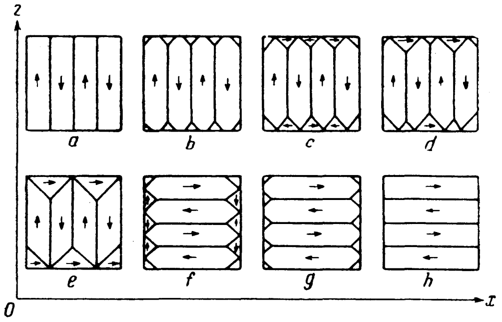

FIG. 7. Successive stages of change of the domain structure with extension [(100) plane; extension in a direction at angle greater than $\pi/4$ to the z axis].

from c to d has been described;[5] the stress was applied at an angle slightly exceeding $\pi/4$ (in the paper it was erroneously implied that $\varphi = \pi/2$). The transition occurred by a jump; this is presumably attributable to the usual departure of the system from a state of macroscopic equilibrium, because the crystallite is not "ideal." In these same researches, transitions from state b to c and from state f to h were observed. In the latter case the value of ω was about 70 gauss. A transition from f to g was observed by Shur and Zaĭkova.[7] In other work of the same authors[8] a transition from b to g was observed; in this case the intermediate stages turned into a complication of the structure inside the crystallite, a fact explained by peculiar circumstances (in the crystal, of inaccurate shape, there were always two domains in the initial state).

Thus the calculation carried out agrees well with known experiments and forecasts the possibility of producing structure e, which has not yet been observed under the influence of elastic stresses. The formulas obtained make it possible to estimate, from observation of the partly closed structure of type b, the effective density ω of magnetic poles on the boundaries of the crystallites; this is of interest in relation to several questions in the theory of the technical magnetization curve. Such an estimate, carried out by us in two cases, gave values of ω appreciably less than the value I_S. It is possible that this sheds light on the disagreements with experiment of the theoretical predictions of the magnetic influence of the crystallites in the law of approach to saturation.[9,10]

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