

MAGNETIC RELAXATION IN FERROMAGNETIC METALS

L. I. BUIHVILLI, G. R. KHUTSISHVILI, and O. D. CHEISHVILI

Institute of Physics, Academy of Sciences, Georgian S. S. R.

Submitted to JETP editor April 7, 1960

J. Exptl. Theoret. Phys. (U.S.S.R.) **39**, 726-736 (September, 1960)

A microscopic calculation of magnetic relaxation in ferromagnets, due to sd exchange interaction, is carried out. Expressions are derived for the corresponding kinetic coefficients. A general phenomenological analysis of the problem is made with allowance for the so-called indirect relaxations.

1. A theoretical analysis of magnetic relaxation in ferroelectrics has been the subject of many papers.¹⁻⁵ An additional relaxation, due to the sd interaction, should be observed in the case of a ferromagnetic metal.

The purpose of the present investigation was an analysis of magnetic relaxation in ferromagnetic metals. We use the simplified model of a ferromagnetic metal, which starts out with the existence of two groups of electrons (cf., e.g., Akhiezer and Pomeranchuk⁶), the conduction (s) and ferromagnetic (d) electrons, and proceed to calculate the relaxation terms due to the sd exchange interaction, in the spin-wave approximation.

We note that Turov⁷ and Mitchell⁸ carried out microscopic calculations of the relaxation due to sd interaction. They have assumed, however, that the conduction electrons are also in equilibrium relative to the spin direction.

2. The model we use for the ferromagnetic metal is quite crude. We therefore confine ourselves to a consideration of the simplest case of a cubic crystal (iron, nickel). We assume furthermore that the external field H (which is aligned with the z axis) is so large that the sample consists of a single domain.

We denote by S and S_d the respective spin operators of the conduction electron and of the electron shell of the ferromagnetic ion, by g_s and g_d the absolute values of their g factors, and by N the concentration of the ferromagnetic atoms. We have

$$Na^3 = \nu, \quad (1)$$

where a is the edge of the elementary cube and ν is the number of atoms per elementary cube.

We denote furthermore by M the magnetic moment (due to the d electrons) per unit volume of the ferromagnet, and by M_G the value of M in absolute saturation (i.e., at $0^\circ K$). We have

$$M_0 = -Ng_d\beta S_d = -\nu a^{-3}g_d\beta S_d, \quad (2)$$

where β is the Bohr magneton.

The energy operator of the exchange-interaction of the conduction electron with the ferromagnetic electrons will be written in the following form:⁶

$$V(r) = (A/Ng_d\beta)SM(r), \quad (3)$$

where A is the constant of the sd exchange interaction.

In the expression

$$SM = \frac{1}{2}(S_+M_- + S_-M_+) + S_zM_z$$

$$(S_\pm = S_x \pm iS_y, \quad M_\pm = M_x \pm iM_y)$$

the last term near the ground state is much greater than the others. Therefore, if we confine ourselves to states close to the ground state (i.e., if we consider the case of temperatures which are much lower than the Curie temperature Θ), the operator S_z will commute with the exchange energy and its eigenvalue will be a good quantum number.

It is well known that for states close to the ground state, the operators M_x and M_y are small quantities of first order of smallness, and M_z differs from M_G by a quantity of second order of smallness. Therefore, when finding the energy levels (states close to the ground state), we can replace M in (3) by M_G . This yields (see reference 9)

$$V = -ASS_d. \quad (4)$$

We denote by H_s the effective magnetic field due to the sd exchange interaction, which acts on the spin of the conduction electron. Analogously, let H_d represent the effective field due to the sd exchange and acting on the spin of the ferromagnetic ion. Using formula (4), we find

$$H_s = -A\langle S_{dz} \rangle / g_s\beta, \quad H_d = -A\langle S_z \rangle \rho / g_d\beta, \quad (5)$$

where the brackets $\langle \rangle$ denote averaging, and ρ

is the number of conduction electrons per atom. Considering that in states close to the ground state almost all the d spins are directed opposite to the field, we have

$$H_s = AS_d / g_s \beta. \quad (6)$$

The spin of the conduction electron is in a field $H + H_s$. Recalling the expression for the Pauli paramagnetic susceptibility, we obtain

$$\langle S_z \rangle = -3g_s \beta (H + H_s) / 8\mu_0,$$

where μ_0 is the chemical potential of the conduction-electron gas. Consequently

$$H_d = (3g_s A \rho / 8g_d \mu_0) H + 3A^2 S_d \rho / 8g_d \beta \mu_0. \quad (7)$$

However, the question of the second term in H_d calls for a detailed analysis. The point is that when we substitute the value of A , taken for the free ion (~ 0.3 eV), the resultant second term is of the order of 100 koe, which sharply contradicts the experiment (this would cause a very strong shift of the resonant frequency). In this connection, Kittel and Mitchell believe that in a ferromagnet the constant A is reduced by screening by a factor of several times. However, so strong a screening appears little likely to us.

We adhere to a second explanation. The second term (7) is due to the part of the conduction-electron polarization caused by the field H_s . Therefore the second term in (7) coincides in direction with the magnetization due to the d electrons. It follows therefore that the second term in H_d does not give a rotational moment that acts on \mathbf{M} , and consequently does not change the resonant frequency (in this connection, see the papers by Yosida and Hasegawa¹⁰). In quantum language, we can say that the Zeeman energy of the vector \mathbf{M} in the additional field is independent of the orientation of \mathbf{M} , and therefore drops out in the energy difference. Thus, instead of (7) it would be more correct to write

$$H_d = (3g_s A \rho / 8g_d \mu_0) H. \quad (8)$$

3. The spin Hamiltonian of the conduction electron can be written in the following form

$$\mathcal{H}(r) = g_s \beta H S_z + (A a^3 / \nu g \beta) S M(r). \quad (9)$$

In the ground state, all the ferromagnetic spins are directed opposite to the z direction. The operator M_+ increases the overall d -electron spin projection by unity, while M_- decreases it. In other words, the operator M_+ causes the production of a ferromagnon, while the operator M_- causes its annihilation. According to the commuta-

tion rule (see, for example, reference 2)

$$[M_+(r), M_-(r')] = -2g_d \beta M_s \delta(r - r'). \quad (10)$$

Let the volume of the sample be unity. Then

$$M_+(r) = -\sqrt{2g_d \beta M_s} \sum_{\mathbf{f}} a_{\mathbf{f}}^{\dagger} e^{-i\mathbf{f}r},$$

$$M_-(r) = -\sqrt{2g_d \beta M_s} \sum_{\mathbf{f}} a_{\mathbf{f}} e^{i\mathbf{f}r}, \quad (11)$$

where $a_{\mathbf{f}}^{\dagger}$ and $a_{\mathbf{f}}$ are the operators of production and annihilation of a ferromagnon with wave vector \mathbf{f} , respectively. Substitution in (9) yields

$$\mathcal{H}(r) = (g_s \beta H + AS_d) S_z$$

$$- AS_d \sqrt{g_d \beta / 2M_s} \sum_{\mathbf{f}} [a_{\mathbf{f}}^{\dagger} S_- e^{-i\mathbf{f}r} + a_{\mathbf{f}} S_+ e^{i\mathbf{f}r}]. \quad (12)$$

In the derivation of (12), we substitute M_{σ} for M_z in the term $S_z M_z$.

Let us carry out second quantization over the conduction electrons. We readily obtain for the second-quantized Hamiltonian

$$\mathcal{H} = \frac{1}{2} (g_s \beta H + AS_d) \sum_{\mathbf{k}} (b_{\mathbf{k}}^{\dagger} b_{\mathbf{k}} - d_{\mathbf{k}}^{\dagger} d_{\mathbf{k}})$$

$$- AS_d \sqrt{g_d \beta / 2M_s} \sum_{\mathbf{k}\mathbf{f}} [a_{\mathbf{f}}^{\dagger} d_{\mathbf{k}-\mathbf{f}}^{\dagger} b_{\mathbf{k}} + a_{\mathbf{f}} b_{\mathbf{k}-\mathbf{f}}^{\dagger} d_{\mathbf{k}}], \quad (13)$$

where $b_{\mathbf{k}}^{\dagger}$ and $b_{\mathbf{k}}$ are respectively the operators of production and annihilation of a conduction electron with a wave vector \mathbf{k} and a spin projection $1/2$ on the z axis. The operators $d_{\mathbf{k}}^{\dagger}$ and $d_{\mathbf{k}}$ are analogously defined for an electron with a spin projection $-1/2$.

It follows from (12) that the energy spectrum of the conduction electron in the ferromagnet has the form

$$\varepsilon(\mathbf{k}, m_s) = \varepsilon_0(\mathbf{k}) + (g_s \beta H + AS_d) m_s, \quad (14)$$

where m_s is the eigenvalue of the operator S_z and $\varepsilon_0(\mathbf{k})$ is the energy of the conduction electron in the absence of an external field and of sd exchange interaction. In further calculations we assume that

$$\varepsilon_0(k) = \hbar^2 k^2 / 2m \quad (15)$$

(m is the effective mass of the conduction electron).

For the energy of the ferromagnon we use the expression¹¹

$$\varepsilon(\mathbf{f}) = 2JS_d (a_{\mathbf{f}})^2 + g_d \beta H_{\text{eff}}, \quad (16)$$

where J is the dd exchange integral, and H_{eff} is the total field acting on the spin of the ferromagnetic ion. It is given by the formula

$$H_{\text{eff}} = H + H_a + H_d,$$

where H_a is the anisotropy field. Considering that A is considerably smaller than μ_0 , we can neglect H_d compared with H [see Eq. (8)]. Next, since we consider the case of large H (compared with H_a), we can put $H_{\text{eff}} = H$ (we note that in the final result H_{eff} is contained only under the logarithm sign).

The dispersion law, given by (16), is valid only for values of f that are not too small. Accordingly, on the one hand, as indicated in references 3 and 12, formula (16) can be used for temperatures higher than about 2 or 3°K; on the other hand, the spin-wave picture can be used for temperatures lower than approximately one-tenth the Curie temperature.

4. The second term in (13) causes transitions between the stationary states of the system. For example, the term $a_{\mathbf{f}}^{\dagger} d_{\mathbf{k}-\mathbf{f}}^{\dagger} b_{\mathbf{k}}$ causes a process in which an electron with wave vector \mathbf{k} and a spin projection $1/2$ is annihilated, and an electron with a wave vector $\mathbf{k} - \mathbf{f}$ and a spin projection $-1/2$ appears together with a ferromagnon with wave vector \mathbf{f} . The overall projection of the spin is conserved here (the projection of the ferromagnon spin is equal to $+1$) since this an exchange interaction.

Using standard perturbation theory, we readily obtain

$$\begin{aligned} dn(\mathbf{f})/dt = C \sum_{\mathbf{k}} \delta[\varepsilon(\mathbf{k}, +1/2) - \varepsilon(\mathbf{k} - \mathbf{f}, -1/2) - \varepsilon(\mathbf{f})] \\ \times \{ [n(\mathbf{f}) + 1][1 - g(\mathbf{k} - \mathbf{f}, \mu_-)] g(\mathbf{k}, \mu_+) \\ - n(\mathbf{f}) [1 - g(\mathbf{k}, \mu_+)] g(\mathbf{k} - \mathbf{f}, \mu_-) \}. \end{aligned} \quad (17)$$

In this formula $n(\mathbf{f})$ denotes the distribution function of the ferromagnons; $g(\mathbf{k}, \mu_+)$ and $g(\mathbf{k}, \mu_-)$ are the distribution functions of the conduction electrons with spin projections $1/2$ and $-1/2$, respectively; μ_+ and μ_- are the corresponding chemical potentials; C is a constant given by the formula

$$C = (AS_d)^2 \pi g_d \beta / \hbar) M_a. \quad (18)$$

We assume that the totality of conduction electrons with given spin direction is both in internal equilibrium and in equilibrium with the lattice (therefore each of these totalities is described by a Fermi distribution with a temperature equal to the lattice temperature). This assumption is correct, since the corresponding relaxation time is exceedingly small (the process is not connected with spin flip). However, these two electron gases are generally speaking not in equilibrium with each other, and therefore $\mu_+ \neq \mu_-$. On the other hand, the ferromagnon gas is not in equilibrium with the lattice.

Our problem is to calculate the relaxation terms due to the sd exchange interaction. In addition, there are other relaxation mechanisms present, namely: a) the mechanism connected with the direct interaction between the conduction electrons and the lattice,^{13,14} without account of the sd exchange, b) the mechanism connected with the interaction between the ferromagnetic spins and the lattice, c) internal relaxation in the ferromagnon gas. Neither the lattice nor the conduction electron participates in the latter case. In this relaxation, therefore, the total ferromagnon-gas energy and the z component of the magnetization remain constant. A Planck distribution is established with a certain temperature which, in general, is different from the lattice temperature.

Processes b) and c) are not connected with the conduction electrons, and must therefore proceed in the same manner in a ferromagnetic metal as in a ferroelectric.¹⁻⁵

In accordance with the foregoing, we represent $n(\mathbf{f})$ in the following form:

$$n(\mathbf{f}) = n^0(\mathbf{f}) + \Delta n(\mathbf{f}), \quad n^0(\mathbf{f}) = (e^{\varepsilon(\mathbf{f})/T} - 1)^{-1}; \quad (19)$$

$\Delta n(\mathbf{f})$ is a small addition. The temperature is measured everywhere in energy units.

Taking into consideration the form of the functions $n^0(\mathbf{f})$ and $g(\mathbf{k}, \mu)$, and also the conservation of energy (which is taken into account by the δ function), we can readily bring (17) to the form

$$\begin{aligned} dn(\mathbf{f})/dt = C \sum_{\mathbf{k}} \delta[\varepsilon(\mathbf{k}, 1/2) - \varepsilon(\mathbf{k} - \mathbf{f}, -1/2) - \varepsilon(\mathbf{f})] \\ \times \{ n^0(\mathbf{f}) [1 - g(\mathbf{k}, \mu_+)] g(\mathbf{k} - \mathbf{f}, \mu_-) \\ \times [\exp\{(\mu_+ - \mu_-)/T\} - 1] \\ - [g(\mathbf{k} - \mathbf{f}, \mu_-) - g(\mathbf{k}, \mu_+)] \Delta n(\mathbf{f}) \}. \end{aligned} \quad (20)$$

5. We denote by $L/2$ the absolute value of the overall projection of all the d spins, and by L_0 the equilibrium value of L . We have

$$\begin{aligned} L = 2[NS_d - \sum_{\mathbf{f}} n(\mathbf{f})], \quad L_0 = 2[NS_d - \sum_{\mathbf{f}} n^0(\mathbf{f})], \\ L - L_0 = -2 \sum_{\mathbf{f}} \Delta n(\mathbf{f}). \end{aligned} \quad (21)$$

When $S_d = 1/2$, L represents the excess of the d spins directed against the field.

We denote further by D the excess of the conduction electrons with spins directed opposite the field, and by D_0 the equilibrium value of D . It is easy to see that the z components of the magnetic moments of the sample, caused by the d and s electrons, are to $(1/2)g_d\beta L$ and $(1/2)g_s\beta D$ respectively. According to Overhauser,¹⁵ we have

$$\mu_+ - \mu_- = 4\mu_0(D_0 - D)/3N\beta. \quad (22)$$

We confine ourselves henceforth to the case of linear relaxation. For this purpose it is necessary to satisfy the rather stringent condition

$$|\mu_+ - \mu_-|/T = 4\mu_0 |D_0 - D|/3N\rho T \ll 1. \quad (23)$$

Considering that

$$D_0 = 3N\rho (AS_d + g_s\beta H)/4\mu_0,$$

the last condition becomes

$$\frac{|D_0 - D|}{D_0} \frac{AS_d + g_s\beta H}{T} \ll 1. \quad (24)$$

In addition, in all the foregoing derivations we have assumed that the condition

$$|L_0 - L|/L_0 \ll 1 \quad (25)$$

is satisfied. Actually, the usual spin-wave picture is not applicable otherwise.

Taking all the foregoing into account, we now readily obtain

$$\begin{aligned} dL/dt = & -2C \sum_{\mathbf{k}\mathbf{f}} \delta[\varepsilon(\mathbf{k}, +1/2) - \varepsilon(\mathbf{k} - \mathbf{f}, -1/2) - \varepsilon(\mathbf{f})] \\ & \times \{(\mu_+ - \mu_-) T^{-1} n^0(\mathbf{f}) [1 - g(\mathbf{k}, \mu_0)] g(\mathbf{k} - \mathbf{f}, \mu_0) \\ & - [g(\mathbf{k} - \mathbf{f}, \mu_0) - g(\mathbf{k}, \mu_0)] \Delta n(\mathbf{f})\}. \end{aligned} \quad (26)$$

The calculation of the second term in the case of arbitrary $\Delta n(\mathbf{f})$ is impossible. We shall assume that the ferromagnon gas is in internal equilibrium, i.e., $n(\mathbf{f})$ is in the form of a Planck function with a temperature different from the lattice temperature. Then, taking (21) into account, we obtain

$$\Delta n(\mathbf{f}) = \frac{\partial n^0(\mathbf{f})}{\partial T} \Delta T = \left[2 \sum_{\mathbf{f}} \frac{\partial n^0(\mathbf{f})}{\partial T} \right]^{-1} \frac{\partial n^0(\mathbf{f})}{\partial T} (L_0 - L),$$

which can be reduced to the form

$$\Delta n(\mathbf{f}) = \frac{(8\pi J S_d)^{3/2} a^3}{3\zeta(3/2) T^{3/2}} [n^0(\mathbf{f})]^2 \varepsilon(\mathbf{f}) e^{\varepsilon(\mathbf{f})/T} (L_0 - L), \quad (27)$$

where ζ is the Riemann zeta function. Taking (22) and (27) into account, (26) becomes

$$dL/dt = -(D_0 - D)/T_{sd} + (L_0 - L)/T_{ds}, \quad (28)$$

and after rather lengthy calculations we obtain

$$\frac{1}{T_{sd}} = \frac{maA^2T}{8 \cdot 3^{1/2} \pi^{3/2} \hbar^3 (N\rho)^{1/2} J} I_{sd}, \quad \frac{1}{T_{ds}} = \frac{2^{1/2} m^2 J^{1/2} S_d^{3/2} a A^2 T^{1/2}}{3\pi^{3/2} \zeta(3/2) \hbar^3 N} I_{ds}, \quad (29)$$

where

$$\begin{aligned} I_{sd} &= \frac{ze^z}{e^z - 1} - \ln(e^z - 1), & I_{ds} &= 2 \int_z^\infty \frac{zdz}{e^z - 1} + \frac{z^2}{e^z - 1}, \\ z &= \frac{1}{T} \left[\frac{(AS_d)^2}{B\mu_0} + g_d\beta H \right], & B &= \hbar^2/ma^2JS_d. \end{aligned} \quad (30)$$

In the extreme cases we have

$$I_{sd} = \ln z^{-1}, \quad I_{ds} = \pi^2/3 \text{ for } z \ll 1; \quad (30a)$$

$$I_{sd} = ze^{-z}, \quad I_{ds} = z^2e^{-z} \text{ for } z \gg 1. \quad (30b)$$

We have calculated the relaxation terms due to the sd exchange interaction. Since the quantity $L + D$ is conserved in exchange interaction, dD/dt is given by (28) but with the opposite sign.

6. Let us now take into account the fact that the spins of the conduction electrons, and also the ferromagnetic spins, interact not only with each other, but also with the lattice directly. The relaxation times of the direct interaction with the lattice will be denoted by T_{sl} and T_{dl} respectively. Then the relaxation equations assume the form

$$\dot{D} = \frac{D_0 - D}{T_s} - \frac{L_0 - L}{T_{ds}}, \quad \dot{L} = \frac{L_0 - L}{T_d} - \frac{D_0 - D}{T_{sd}}, \quad (31)$$

where

$$1/T_s = 1/T_{sl} + 1/T_{sd}, \quad 1/T_d = 1/T_{dl} + 1/T_{ds}. \quad (32)$$

The solution of the system (31) has the form

$$\begin{aligned} D(t) &= D_0 + d_+ \exp(-\lambda_+ t) + d_- \exp(-\lambda_- t), \\ L(t) &= L_0 + l_+ \exp(-\lambda_+ t) + l_- \exp(-\lambda_- t), \end{aligned} \quad (33)$$

with

$$\lambda_{\pm} = \frac{1}{2} \left(\frac{1}{T_s} + \frac{1}{T_d} \right) \pm \frac{1}{2} \left[\left(\frac{1}{T_s} - \frac{1}{T_d} \right)^2 + \frac{4}{T_{sd}T_{ds}} \right]^{1/2}. \quad (34)$$

Further, the quantities l_+ and l_- are expressed in terms of d_+ and d_- by formulas such as

$$\frac{d}{l} = \frac{1/T_{ds}}{1/T_s - \lambda} = \frac{1/T_d - \lambda}{1/T_{sd}}.$$

The coefficients d_+ and d_- are determined from the initial conditions.

Thus, D and L in general are sums of two exponentials.

7. Let us consider two particular cases.

Case 1 (see reference 9). Assume that the following conditions are satisfied

$$T_{sl} \ll T_{sd}, T_d. \quad (35)$$

The condition $T_{sl} \ll T_{sd}$ denotes that the direct coupling between the conduction electrons and the lattice is considerably stronger than the couplings between these electrons and the d spins. We shall have here $T_s \approx T_{sl}$. In the equation for \dot{D} we can neglect the term $(L_0 - L)/T_{ds}$. We then obtain $\dot{D} = (D_0 - D)/T_s$. It follows therefore that D approaches an equilibrium value with a relaxation time T_s . For times much longer than T_s we can neglect the term $(D_0 - D)/T_{sd}$ in the equation for \dot{L} , and therefore $L = (L_0 - L)/T_d$.

As a result we obtain the solutions

$$D(t) = D_0 + [D(0) - D_0] e^{-t/T_s},$$

$$L(t) = L_0 + [L(0) - L_0] e^{-t/T_d}. \quad (36)$$

We thus have in this case independent relaxation of D and L .

Case 2. Assume that the conditions

$$T_{sd} \ll T_{st}, \quad T_{ds} \ll T_{dt}, \quad (37)$$

are satisfied; in other words, the *s* and *d* spins are coupled more strongly to each other than to the lattice. Here we have

$$T_s \approx T_{sd}, \quad T_d \approx T_{ds}.$$

The sum $D + L$ is conserved in the *sd* exchange interaction. Therefore in the case of total absence of direct coupling with the lattice, D and L cannot relax to the values D_0 and L_0 [provided the condition $D(0) + L(0) = D_0 + L_0$ is not satisfied]. It is easy to see that D and L will relax with a relaxation time equal to $T_s T_d / (T_s + T_d)$ to values D_s and L_s , which are determined from the condition that $D + L$ is conserved and from the condition that they are stationary with respect to *sd* interactions.

$$(D_0 - D_s) / T_s = (L_0 - L_s) / T_d. \quad (38)$$

It is easy to get

$$\begin{aligned} D_s &= (T_d + T_s)^{-1} \{T_d D_0 - T_s L_0 + T_s [D(0) + L(0)]\}, \\ L_s &= (T_d + T_s)^{-1} \{T_s L_0 - T_d D_0 + T_d [D(0) + L(0)]\}. \end{aligned} \quad (39)$$

For complete relaxation we have

$$\begin{aligned} D(t) &= D_0 + (D_s - D_0) \exp(-\lambda_- t) + [D(0) \\ &\quad - D_s] \exp(-\lambda_+ t), \\ L(t) &= L_0 + (L_s - L_0) \exp(-\lambda_- t) + [L(0) \\ &\quad - L_s] \exp(-\lambda_+ t), \end{aligned} \quad (40)$$

where

$$\lambda_+ = \frac{1}{T_s} + \frac{1}{T_d}, \quad \lambda_- = \frac{1}{T_s + T_d} \left(\frac{T_s}{T_{st}} + \frac{T_d}{T_{dt}} \right) \ll \lambda_+. \quad (41)$$

Adding D and L and taking into consideration the fact that $D_s + L_s = D(0) + L(0)$, we obtain from (40)

$$\begin{aligned} D(t) + L(t) &= (D_0 + L_0) + \{[D(0) \\ &\quad - D_0] + [L(0) - L_0]\} \exp(-\lambda_- t). \end{aligned}$$

In other words, λ_-^{-1} is the relaxation time of the quantity $D + L$.

We introduce the quantities

$$\begin{aligned} D_s(t) &= D_0 + (D_s - D_0) \exp(-\lambda_- t), \\ L_s(t) &= L_0 + (L_s - L_0) \exp(-\lambda_- t). \end{aligned}$$

Then (40) assumes the form

$$\begin{aligned} D(t) &= D_s(t) + [D(0) - D_s] \exp(-\lambda_+ t), \\ L(t) &= L_s(t) + [L(0) - L_s] \exp(-\lambda_+ t). \end{aligned} \quad (42)$$

It is easy to see that $D_s(t)$ and $L_s(t)$ satisfy the condition of stationarity with respect to *sd* ex-

change interaction, (38), and their sum is $D(t) + L(t)$, i.e., $D_s(t)$ and $L_s(t)$ are the quasi-equilibrium values of $D(t)$ and $L(t)$, corresponding to equilibrium under *sd* exchange interaction for specified $D(t) + L(t)$.

Thus, in our case we have the following final result: an equilibrium with respect to *sd* exchange interaction is established with a relaxation time λ_+^{-1} ; this is followed by a slower establishment of complete equilibrium, with a relaxation time λ_-^{-1} , and the equilibrium relative to exchange interaction is maintained all the time during this process.*

Case 2a. Assume that the following condition is satisfied in addition to conditions (37):

$$T_s \ll T_d \quad (\text{i. e. } T_{sd} \ll T_{ds}), \quad (43)$$

Now Eqs. (41) and (39) yield

$$\lambda_+ = \frac{1}{T_s}, \quad \lambda_- = \frac{T_s}{T_d} \frac{1}{T_{st}} + \frac{1}{T_{dt}}; \quad (44)$$

$$\begin{aligned} D_s &= D_0 - [L_0 - L(0)] T_s / T_d - [D_0 - D(0)] T_s / T_d, \\ L_s &= L(0) + [L_0 - L(0)] T_s / T_d - (1 - T_s / T_d) [D_0 - D(0)]. \end{aligned} \quad (45)$$

Let us consider relaxation in ferromagnetic resonance after turning off the alternating field. Under ordinary ferromagnetic resonance, a reduction in L takes place (saturation). Since our analysis is suitable only for relatively small deviations of L from L_0 (i.e., for small values of the resonance saturation parameter), we confine ourselves to the case $L(0) \approx L_0$.

Let us consider furthermore that the magnetization due to the *d* spins is considerably higher than the magnetization due to the *s* spins [i.e., L_0 and $L(0)$ are much greater than T_0 and $D(0)$]. Then (45) yields

$$D_s = D_0 - [L_0 - L(0)] T_s / T_d, \quad L_s = L(0). \quad (46)$$

According to (40) we obtain

$$\begin{aligned} D(t) &= D_0 + [D(0) - D_0] \exp(-\lambda_+ t) \\ &\quad + [L(0) - L_0] \{ \exp(-\lambda_- t) - \exp(-\lambda_+ t) \} T_s / T_d, \\ L(t) &= L_0 + [L(0) - L_0] \exp(-\lambda_- t). \end{aligned} \quad (47)$$

Thus, in Case 2a, the fast relaxation drops out from L and only the slow relaxation remains. As regards D , it first deviates rapidly from its initial value (and at the same time an equilibrium is established relative to the exchange interaction†), after which it slowly relaxes to its equilibrium value.

*This means that when $\lambda_+ t \gg 1$ we have $D(t) = D_s(t)$ and $L(t) = L_s(t)$.

†The corresponding term in L can be neglected, in view of the large value of $L(0)$.

8. Let us estimate the quantities T_{sd} and T_{ds} , given by (29), in the case of iron.

From a measurement of the magnetization of the absolute saturation it follows that the number of magnetons per atom of iron is 2.22 (reference 16). This leads to an average configuration* $3d^{7.78} 4s^{0.22}$ for the iron atom, which yields $S_d = 1.1$ and $\rho = 0.22$. For the dd exchange integral we take the value† $J = 175k = 2.4 \times 10^{-14}$ erg. We substitute furthermore the value $\nu = 2$ (since the iron lattice is body-centered cubic), $a = 2.85 \times 10^{-8}$ cm, $N = 0.86 \times 10^{23}$ cm $^{-3}$, and $g_d = 2$. The effective mass of the conduction electron in iron is unknown, and we shall use for it a value equal to 0.3 of the true mass of the electron, obtaining $\mu_0 = 1.4 \times 10^{-11}$ erg. At the present time, there are no data on the numerical value of the sd interaction constant A in ferromagnetic metals. As was indicated in Sec. 3, it appears little likely to us that screening would reduce this constant by a factor several times ten compared with its value in the free atom; we put in (29) a value $A = 4.8 \times 10^{-13}$ erg. For the constant B we obtain from (30) $B = 190$. It then follows from (30) that

$$z = (0.8 + 1.3 \cdot 10^{-4}H)/T,$$

where T is given in degrees Kelvin.

Since our analysis holds only for temperatures greater than 2 or 3° K, we confine ourselves to the case $z < 1$. Equations (29) and (30a) yield

$$\frac{1}{T_{sd}} = 2 \cdot 10^9 T \ln \frac{T}{0.8 + 1.3 \cdot 10^{-4}H}, \quad \frac{1}{T_{ds}} = 10^9 \sqrt{T}. \quad (48)$$

These equations can be used from 2 or 3° K approximately to 100° K. The condition $T_{ds} \gg T_{sd}$ is satisfied over this entire temperature interval.

The expression for T_{dl} can be taken from the paper by Akhiezer, Bar'yakhtar, and Peletminskiĭ.⁴ The condition $T_{dl} \gg T_{ds}$ is satisfied over the entire temperature interval of interest to us. For T_{sl} we use the expression obtained by Andreev and Gerasimenko.¹⁴ Substituting the values of the constants for iron in the suitable formula, we obtain

$$T_{sl} \sim 10^{-11}/(\Delta g)^2 T, \quad (49)$$

where Δg is the deviation of the conduction-electron g factor from its value for the free electron (2.0023). If Δg is of the same order (10^{-4} to 10^{-3})

*The question of the average configuration is not completely resolved. See the paper by Mott and Stevens¹⁷ regarding another possibility.

†This value of J follows from the work by Fallot,¹⁸ in which the coefficient preceding $T^{3/2}$ in the temperature dependence of the magnetization is determined. Here we took account of the fact that $S_d = 1.1$.

for iron as for alkaline metals, then $T_{sl} \gg T_{sd}$, and Case 2a will take place (see Sec. 7).

9. Kittel and Mitchell believe⁹ that in metals of the transition group of iron (in particular, in ferromagnetic metals), Δg will be considerably greater than in the case of alkaline metals, in view of the overlap of the 3d and 4s bands. However, comparing (48) with (49) we see that in order for T_{sl} to become of the same order as T_{sd} , it is necessary that Δg in iron be on the order of 0.2.

The values of Δg for metals of the transition groups are still unknown. In addition, in the derivation of (48) we used the unscreened value of the sd interaction constant. We cannot therefore draw any final conclusions concerning the mechanism of magnetic relaxation in ferromagnetic metals.

We consider that $T_{sl} \gg T_{sd}$, and therefore Case 2a, considered above, takes place. The fact that the resultant value of T_{sd} is so small should not cause any surprise: actually, the sd exchange interaction is strong (provided the constant A is of the same order as in the free atom).

One might ask why the relaxation time T_{sd} does not cause a broadening of ferromagnetic resonance (then the line width would be greater than the width measured in experiment). The point is, as was shown in Sec. 7, that in Case 2a the rapid relaxation drops out from the magnetization relaxation due to the d spins, and only the slow relaxation with relaxation time λ_+^{-1} remains.

We assume that the width of the ferromagnetic resonance (corresponding to a relaxation time 10^{-10} – 10^{-9} sec) is due to the establishment of internal equilibrium in the d-spin system. This is very probable also because, according to experiment, the width of ferromagnetic resonance is of the same order in ferromagnetic metals and in ferroelectrics.

However, in this case our calculation of T_{ds} is inconsistent. Actually, in calculating the terms that contain $\Delta n(\mathbf{f})$ in expression (26), we assumed that internal equilibrium is established in the ferromagnon gas. Strictly speaking, such a consideration would be valid only if the time of internal relaxation in the ferromagnon gas is smaller than λ_+^{-1} , which, in our opinion, does not take place. However, we believe that our calculation still yields an approximately correct value* of T_{ds} .

*We have also calculated the second term in the expression (26), using the method employed in references 1–5. In other words, we have used the formula

$$\sum_{\mathbf{f}} W(\mathbf{f}) \Delta n(\mathbf{f}) = \left[\sum_{\mathbf{f}} n^0(\mathbf{f}) \right]^{-1} \sum_{\mathbf{f}} W(\mathbf{f}) n^0(\mathbf{f}) \sum_{\mathbf{f}} \Delta n(\mathbf{f}).$$

In the case $z < 1$ we obtain here for $1/T_{ds}$ an expression that differs from (29) only by a factor equal to 3/4. We note also

It would be quite interesting to carry out resonance experiments on the conduction electrons in a ferromagnet. However, if the constant A is not greatly reduced by screening, the resonance will be in the infrared region (the value $A = 0.3$ eV corresponds to a frequency 0.8×10^{14} cps). In particular, it would be quite interesting to investigate the effect of saturation of ferromagnetic resonance on the resonance of the conduction electrons and vice versa. It would be also interesting to investigate the Overhauser effect in ferromagnetic metals, i.e., to measure the polarization of the nuclei in saturation of ferromagnetic resonance or resonance of conduction electrons.

At high temperatures, near the Curie point, the field due to the ds spins acting on the conduction electrons will be considerably less than the value H_S given by (6). The conduction-electron resonance should therefore shift towards the centimeter waves. It must be considered, however, that in this case the eigenvalue of the conduction-electron spin projection on the direction of the field will not be a good quantum number, since S_Z does not commute with the sd exchange interaction Hamiltonian at high temperatures.

The authors are grateful to M. I. Kaganov and to V. G. Bar'yakhtar for useful discussions.

¹A. I. Akhiezer, *J. of Phys. U.S.S.R.* **10**, 217 (1946).

²C. Kittel and E. Abrahams, *Phys. Rev.* **88**, 1200 (1953); *Revs. Modern Phys.* **25**, 233 (1953).

that if the Case 2a considered above takes place, then only the slow relaxation, whose time is considerably greater than the time for establishment of internal equilibrium in the spin system, remains in ordinary ferromagnetic resonance.

³M. I. Kaganov and V. M. Tsukernik, *JETP* **34**, 1610 (1958) and **36**, 224 (1959), *Soviet Phys. JETP* **7**, 1107 (1958) and **9**, 151 (1959).

⁴Akhiezer, Bar'yakhtar, and Peletminskiĭ, *JETP* **36**, 216 (1959), *Soviet Phys. JETP* **9**, 146 (1959).

⁵V. G. Bar'yakhtar, *JETP* **37**, 690 (1959), *Soviet Phys. JETP* **10**, 493 (1960).

⁶A. I. Akhiezer and I. Ya. Pomeranchuk, *JETP* **36**, 859 (1959), *Soviet Phys. JETP* **9**, 605 (1959).

⁷E. A. Turov, *Izv. Akad. Nauk SSSR, Ser. Fiz.* **19**, 462 (1955), *Columbia Tech. Transl.* p. 414.

⁸A. H. Mitchell, *Phys. Rev.* **105**, 1439 (1957).

⁹C. Kittel and A. H. Mitchell, *Phys. Rev.* **101**, 1611 (1956).

¹⁰K. Yosida, *Phys. Rev.* **106**, 893 (1957); H. Hasegawa, *Progr. Theoret. Phys.* **21**, 483 (1959).

¹¹J. Van Kranendonk and J. H. Van Vleck, *Revs. Modern Phys.* **30**, 1 (1958).

¹²C. Herring and C. Kittel, *Phys. Rev.* **81**, 869 (1950).

¹³R. J. Elliot, *Phys. Rev.* **96**, 266 (1954).

¹⁴V. V. Andreev and V. I. Gerasimenko, *JETP* **35**, 1209 (1958), *Soviet Phys. JETP* **8**, 846 (1959).

¹⁵A. Overhauser, *Phys. Rev.* **92**, 311 (1953).

¹⁶S. V. Vonsovskiĭ, *Современное учение о магнетизме (Modern Theories of Magnetism)*, Gostekhizdat, 1952.

¹⁷N. F. Mott and W. H. Stevens, *Phil. Mag.* **2**, 1364 (1957).

¹⁸M. Fallot, *Ann. phys.* **6**, 305 (1938).