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LIMITS OF STABILITY AND PROTON AND TWO-PROTON RADIOACTIVITY OF NEUTRON-DEFICIENT ISOTOPES OF LIGHT NUCLEI

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Application of isotopic invariance principles to light nuclei yields a very simple relation between the neutron and proton binding energies in distant mirror nuclei. This relation enables us to establish the limits of stability of neutron-deficient isotopes of light nuclei with respect to proton emission and to predict the existence and properties of approximately 90 such isotopes. Nuclei are indicated for which proton radioactivity or the very unique phenomenon of two-proton radioactivity can be observed. The chief properties of this interesting phenomenon are analyzed.

By using the principles of isotopic invariance we can readily show that the difference ΔE_{np} between the binding energy E_n of the Z-th neutron in a nucleus ${}_NM_Z^A$ and the binding energy E_p of the Z-th proton in the mirror nucleus ${}_ZM_N^A$ is determined by the relation

$$\Delta E_{np} = E_n({}_NM_Z^A) - E_p({}_ZM_N^A)$$
$$= [E_{\text{Coul}}({}_ZM_N^A) - E_{\text{Coul}}({}_{Z-1}M_N^{A-1})]$$
$$- [E_{\text{Coul}}({}_NM_Z^A) - E_{\text{Coul}}({}_NM_{Z-1}^{A-1})], \qquad (1)$$

in which the first two terms describe the change in the Coulomb energy when one proton is removed from the nucleus, while the last two terms take into account the corresponding change when one neutron is removed from the nucleus (owing to the reduction in the nuclear dimensions).

Accurate to about 1%, the value of δE_{np} should in general be independent of N and determined only by the value of Z, so that instead of the relation $\delta E_{np} \approx 1.2(Z-1)(Z+N-1)^{-1/3}$ which is expected at first glance, we obtain

$$\Delta E_{np} \approx \Delta E_0 = E_n({}_ZM_Z^{2Z})$$
$$- E_p({}_ZM_Z^{2Z}) \approx 1.2(Z-1)(2Z-1)^{-1/3}. \qquad (2)$$

It can be readily shown that another simple expression of the consequences of the principles of isotopic invariance is a relation that characterizes the difference of masses of the remote mirror nuclei

$${}_ZM_N^A - {}_NM_Z^A \approx (Z-N)\Delta M_0, \qquad (3)$$

where

$$\Delta M_0 = {}_{A/2+1/2}M_{A/2-1/2}^A - {}_{A/2-1/2}M_{A/2+1/2}^A, \quad \text{for odd A,}$$
$$\Delta M_0 = \tfrac{1}{2}\left\{{}_{A/2+1}M_{A/2-1}^A - {}_{A/2-1}M_{A/2+1}^A\right\} \quad \text{for even A.}$$

In many cases this formula may be useful and even more convenient than relation (2), which we use in this paper. As can be seen from Table I, relation (2) is confirmed by all the experimental data available for nuclei up to scandium (Z = 21).

Making use of (2), i.e., comparing δE_{np} with ΔE_0, or (if ΔE_1 is unknown) using the calculated value of the Coulomb energy, we can predict the properties of a rather large number of neutron-deficient isotopes of light nuclei (thereby adding to the similar isotopes discussed in the papers of Baz'[1] and Zel'dovich[2]) from the known properties of the corresponding mirror neutron-rich nuclei. Among the particular properties are the binding energies of the neutrons and the protons, the mass defect, the half lives of β^+ decay and its mechanism, and the possibility of observing proton and two-proton radioactivity.

A summary of all these properties, for almost 100 presently unknown isotopes, but which are stable to the emission of protons, is contained in a detailed communication which is now in press.[3] We give here only a general illustration (Table II) of the stability limits of the neutron-deficient isotopes of light nuclei. The conclusions regarding the position of these limits do not change if corrections similar to those considered by Swamy and Green[4] are introduced. These corrections are due to the Coulomb exchange interaction of the protons.

Proton and two-proton radioactivity should be

TABLE I. Difference between the binding energy of the Z-th neutron in the ${}_{N}M_{Z}^{A}$ nucleus and the binding energy of the Z-th proton of the nucleus ${}_{Z}M_{N}^{A}$ (ΔE_{np}).

Z \ N	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	$1{,}2\frac{Z-1}{(2Z-1)^{1/3}}$
H 1																						
He 2	0,76	0,76																				
L 3		0,84	0,84	0,83																		1,42
Be 4		1,63	1,64	1,64	1,70	1,78																1,91
B 5			1,90	1,86	1,86	2,02																2,33
C 6			3,01	2,94	2,76	2,76	2,64															2,73
N 7					2,88	3,01	3,01	3,02	3,2													3,10
O 8						3,55	3,54	3,54	3,25													3,45
F 9						3,50	3,26	3,55	3,55	3,43												3,78
Ne 10								3,91	4,05	4,05	(5,55)											4,10
Na 11									(5,81)	4,30	4,30											4,40
Mg 12											4,86	4,86	5,00									4,70
Al 13											5,21	5,08	5,08									4,98
Si 14													5,61	5,61	(4,95)							5,28
P 15													(5,09)	5,76	5,75							5,54
S 16															6,25	6,25						5,80
Cl 17																5,98	5,88					6,05
Ar 18																	6,76	6,76				6,32
K 19																		6,88	6,88			6,56
Ca 20																			7,43	7,43	7,31	6,80
Sc 21																			6,63	6,74	6,74	7,04

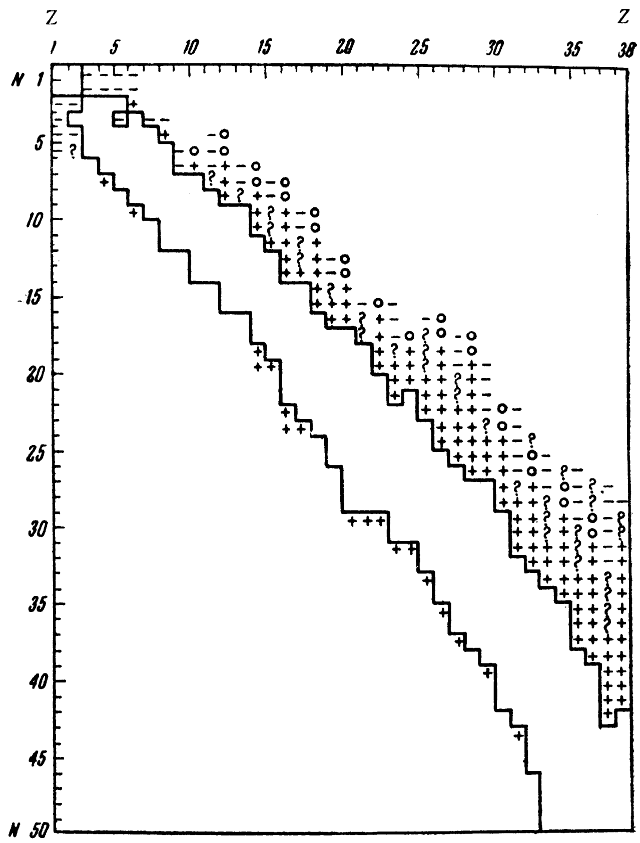


TABLE II. The continuous line outlines the area of the isotopes already known; + – predicted isotopes, stable to the emission of p and n, ? – doubtful p-stability, o – possible 2p-activity, – – isotopes known to be unstable to the emission of p or n.

observed near the stability limits of the neutron-deficient isotopes shown in Table II.

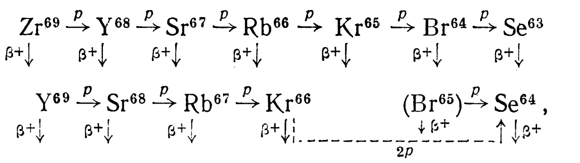
The probability of observation of the generally-trivial proton radioactivity is relatively small, for great difficulties are encountered in its experimental observation if the lifetimes of the p-decay are excessively short, while in the case of long p-decay time, this effect will be strongly screened by the β^+ decay. In the interval of observed p-decay times, from 10^{-12} sec (emulsion method) to 10 sec, the corresponding energies of the emitted protons range up to 0.04 Mev for Z = 10, 0.1 — 0.35 Mev for Z = 20, 0.2 — 0.7 Mev for Z = 30, and 0.35 — 1.1 Mev for Z = 40.

A much more interesting consequence of the considered properties of the neutron-deficient isotopes of light nuclei is the feasibility of two-proton radioactivity. The point is that for isotopes with even Z, instability to simultaneous emission of two protons can occur even when the binding energy of one proton is still positive; this takes place, for example, for Be^6 (reference 5). However, the presence of the Coulomb barrier, can cause such an instability to lead to two-proton radioactivity of many isotopes which are stable both to proton decay and to α decay.

Inasmuch as the width of the ground state of the Li^5 nucleus, which is formed in the decay $Be^6 \rightarrow Li^5 + p$, exceeds the instability of Be^6, we can consider in this case the emission of each of the two protons as independent.

On the other hand, if the energy that must be expended to detach one proton exceeds greatly the half width (reduced by the action of the Coulomb barrier) of the level from which the emission of the second proton takes place, we should have a very unique phenomenon of two-proton radioactivity — detachment of one proton is impossible, and correlated proton pairs are emitted.

One must not confuse this phenomenon with the usual chains of successive β^+ and (or) p decays of the type



although even in chains of p-decay the possibility of two-protons emission can reduce substantially the lifetime of p-radioactive isotopes with even Z = 2m + 2, in those rare occasions when the energy of the 2p decay is

$$E_{pp}(2m+2\rightarrow 2m) > 8E_p(2m+2\rightarrow 2m+1).$$

Two-proton radioactivity, which is also conveniently observed with the aid of nuclear emulsions, can take place for Ne^{16}, $Mg^{17(18?)}$, $Si^{21(22?)}$, $S^{25(24?)}$, $Ar^{29(28?)}$, $Ca^{33(34?)}$, Ti^{38}, Cr^{42}, $Fe^{44(43?)}$, $Ni^{46(47?)}$, $Zn^{53(54?)}$, $Ge^{59(58?)}$, $Se^{63(62?)}$, $Kr^{67(66?)}$. The probability of the sub-barrier emission of two protons at the same time contains the product of two coefficients of penetrability of the barrier for protons, or else the product of the coefficient of penetrability for a doubly-charged particle and a small factor in front of the exponent, characterizing the probability of two-proton correlation in the nucleus. Consequently the lifetime of the isotopes relative to two-proton radioactive decay falls within limits that are convenient for registration in an energy interval which is considerably greater than that for proton radioactivity.

The possibility of emission of two protons in the case when the nucleus is stable to single-proton decay is a direct consequence of the excess of the binding energy of the even proton over the binding energy of the preceding odd proton, i.e., it is a consequence of the pairing effects. It is therefore clear that two-proton radioactive decay should exhibit the properties of a diproton that is stable within the confines of the nucleus, and that the angle and energy distributions and correlations of the emitted protons are connected with the character of the paired interaction of the protons and the original nuclei. On the other hand, inasmuch as the diproton breaks up in a "tunnel" under the barrier, this break-up determines the effective height of the barrier and its penetrability.

The simplest approach to the development of a theory of two-proton decay is to introduce the product of two ordinary proton barrier factors, i.e., an exponential multiplier of the form

$$w(E) = \exp\left\{-\frac{2\pi(Z-2)e^2\sqrt{M}}{\hbar\sqrt{2E_{pp}}}\left[\frac{1}{\sqrt{x}}+\frac{1}{\sqrt{1-x}}\right]\right\},$$

where E_{pp} is the sum of the energies of the two protons (energy of the emitted diproton), x and (1 – x) are the fractions of the energy belonging to each of the protons, and M is the proton mass. It is easily seen that w(E) has a maximum when x = 0.5, i.e., when the energies of the two protons are equal. In this case the quantity in the exponent is the same as for the subbarrier emission of the diproton of energy E_{pp} as a whole.

Thus, the break-up of the diproton under the barrier does not change the penetrability of the barrier if we disregard any supplementary interactions (for example, Coulomb repulsion) between two protons. The probability $w_x(E)$ of such a two-proton decay, in which the one proton has an energy $(0.5+\kappa)E_{pp}$ the other $(0.5-\kappa)E_{pp}$, with $\kappa \ll 0.5$, is connected with $w_{max}(E) = w_0(E)$, by the relation

$$\frac{w_\kappa(E)}{w_0(E)} \approx \exp\left\{-\frac{6\pi(Z-2)e^2\sqrt{M}}{\hbar\sqrt{E_{pp}}}\kappa^2\right\}.$$

It is obvious that the energy correlation between the two protons, in the case of two-proton decay that leads to nearly equal proton energies, is very strong. This correlation (the closeness of the energies of the subbarrier protons) can be noticed even in the almost instantaneous 2p decay, so that the probability of observing a two-proton radioactivity, say in emulsions, is greatly increased.

The interesting question of two-proton radioactivity of neutron-deficient isotopes of even elements is undoubtedly worthy of a more detailed special examination.

The most realistic way of obtaining neutron-deficient isotopes of light nuclei is to bombard the lightest stable isotopes of the neighboring nuclei with protons or He^3 nuclei of near-threshold energy and, in particular, to use reactions induced by heavy ions. It must be borne in mind, however, that owing to the smallness of the energy of the Coulomb barrier, the 'boiling off' of the protons will not be suppressed, and consequently the cross sections for the production of neutron-deficient isotopes should prove to be small.

In conclusion we note that in the case of a neutron excess we can observe a phenomenon analogous to two-proton radioactivity. The excitation energy of nuclei with an even number of protons and a large excess of neutrons, at which the emission of two neutrons is possible, may prove to be lower than the threshold for the emission of a single neutron, owing to pairing effects. Therefore there may exist for such nuclei an interval of excitation energies corresponding to the emission of pairs of neutrons correlated in angle and in energy, without emission of single neutrons. Cases which are even more frequently realizable are those in which the strongly excited states of neutron-rich nuclei with an even number of neutrons can disintegrate, with emission of both a single neutron and a correlated neutron pair. If similar excited states occur after the preceding β decay, they can be detected by the coincidences of the delayed neutron pairs.

I take this opportunity to thank Ya. B. Zel'dovich for useful discussions.

[1] A. I. Baz', Атомная энергия (Atomic Energy) **6**, 571 (1959).

[2] Ya. B. Zel'dovich, JETP **38**, 1123 (1960), Soviet Phys. JETP **11**, 812 (1960).

[3] V. I. Gol'danskiĭ, Nucl. Phys., in press.

[4] N. Swamy and A. Green, Phys. Rev. **112**, 1719 (1958).

[5] Bogdanov, Vlasov, Kalinin, Rybakov, and Sidorov, Атомная энергия (Atomic Energy) **3**, 204 (1957).

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