BUILD UP OF ION ACOUSTIC VIBRATIONS IN AN ANISOTROPIC PLASMA

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The vibrations of an anisotropic plasma located in a magnetic field are investigated for small values of $\beta = 8\pi n T_\perp/H^2$. It is found that the ion acoustic waves may be unstable in this case.

USUALLY in the investigation of the stability of plasma, it is assumed that the mean Larmor radius of the ions r is small in comparison with the characteristic dimensions of the problem, for example the wavelength λ, inasmuch as this materially simplifies the problem. The stability of an anisotropic plasma has been investigated by us for $\lambda \sim r$ and $\lambda \ll r$. It is assumed that the ions have a Maxwellian distribution with different temperatures across and along the magnetic field. This is one of the simplest cases of a non-equilibrium distribution.

The effect of the temperature anisotropy on the build up of transverse vibrations in a homogeneous plasma under the assumption $r/\lambda \to 0$ was investigated by Vedenov and Sagdeev[1] and by Sagdeev and Shafranov.[2] They have shown that the plasma becomes more stable under a decrease in pressure. In fact, if the plasma pressure is much less than the pressure of the magnetic field, i.e., $\beta = 8\pi n T_\perp/H^2 \ll 1$, then the thermal velocity of the ions v_T is much smaller than the Alfven velocity v_A and consequently only a small fraction of the particles have velocities on the order of the phase velocity of the wave. But the vibrations can set into motion only particles which resonate with the wave, since only these particles effectively exchange their energy with it. Therefore, for $\beta = v_T^2/v_A^2 \ll 1$, the plasma will be practically stable.

The problem as to whether this is maintained for much smaller wavelengths, with $\lambda \ll r$, is of interest.

The dispersion equation for the frequency of the vibrations of a homogeneous plasma[5] contains components of the dielectric-constant tensor, obtained by Shafranov and Sagdeev[2] for an anisotropic plasma in the form of infinite sums. In the investigation of transverse vibrations it was assumed for simplicity that the electrons were cold, $T_e = 0$, the wavelength was much smaller than the mean Larmor radius of the ions, $kr \gg 1$ (k is the wave number), and the direction of propagation of the wave was perpendicular to the magnetic field, $kr \cos\theta \ll 1$. Under these assumptions, the expressions for the components of the tensor can be summed. Investigation shows that for small values of β the vibrations are stable. Thus, for $r/\lambda \gg 1$, just as for $r/\lambda \ll 1$, decrease of β has a stabilizing influence on the transverse vibrations.

Therefore, if $\beta \ll 1$, great interest attaches to the longitudinal vibrations. Harris[4] discovered instability of the plasma in relation to the Langmuir electron oscillations for certain simple types of non equilibrium distributions. We consider ion acoustic vibrations, which are also longitudinal. In a plasma with $T_e \gg T_i$, the phase velocity of these vibrations is determined by the temperature of the electrons; therefore, a change in β cannot have a substantial effect on their development. Consequently, it is natural to assume that just the oscillations may be unstable for small β.

The dispersion equation for the ion sound reduces to the equation

$$A = A_i + A_e = \varepsilon_{11}\sin^2\theta + 2\varepsilon_{13}\sin\theta\cos\theta + \varepsilon_{33}\cos^2\theta = 0,$$

$$\varepsilon_{ik} = \varepsilon_{ik}^{i} + \varepsilon_{ik}^{e}, \quad \theta = \widehat{\mathbf{kH}},$$

where

$$A_i = \frac{\varepsilon_0}{b}\sum_{n=-\infty}^{\infty}\left\{\frac{aX}{2\sqrt{d}}Y_n + \frac{n}{2\sqrt{d}}(1-X)Y_n - X\right\}e^{-b}I_n(b)$$

is the contribution of the ions to the dispersion equation. Here we have introduced the notation

$$a = \omega/\Omega, \quad b = T_\perp k_\perp^2/M\Omega^2, \quad d = T_\parallel k_\parallel^2/2M\Omega^2,$$

$$\varepsilon_0 = 4\pi e^2 n/M\Omega^2, \qquad \Omega = eH/Mc, \quad X = T_\perp/T_\parallel,$$

$$Z_n = (a-n)/2\sqrt{d},$$

$$Y_n = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty} e^{-t^2}\frac{dt}{Z_n - t} = \begin{cases} -i\sqrt{\pi} + Z_n + \ldots, & Z_n \ll 1 \\ -i\sqrt{\pi}\exp(-Z_n^2) + Z_n^{-1} + \frac{1}{2}Z_n^{-3}, & Z_n \gg 1 \end{cases}.$$

On the boundary of the region of instability,

$$\operatorname{Im} a = 0,$$

$$\operatorname{Im} A_i = \frac{\varepsilon_0}{b}\sqrt{\frac{\pi}{2d}}\sum_{n=-\infty}^{\infty}[(a-n)X + n]\,e^{-Z_n^2}I_n(b)\,e^{-b}.$$

For $T_\perp = T_\parallel$, when the vibrations are damped, $\operatorname{Im} A_i > 0$. Thus the instability is possible in those regions of the values of the parameters a, b, d, and X where Im $A_i < 0$.

It follows from the expression for the imaginary part that the zero term always gives a positive contribution (Landau damping). The build up of the vibrations is brought about by the remaining terms. It is not difficult to see that Im A_i can be negative only when $X > 1$ and $n + 1 > a > n + 1/2$ (it is taken into account here that the contributions of the n-th and (n + 1)-th exponents are the largest). The most dangerous in the sense of instability is the region $1 > a > 1/2$, which is therefore investigated in the present work.

The contribution of the electrons can be written down similarly. However, since $m \ll M$, it reduces to

$$A_e = \varepsilon_0 X Z_0 b^{-1} \{\mu + i\sqrt{\pi\mu^3 m/M}\}, \quad \mu = T_{\parallel}/T_{\parallel e}.$$

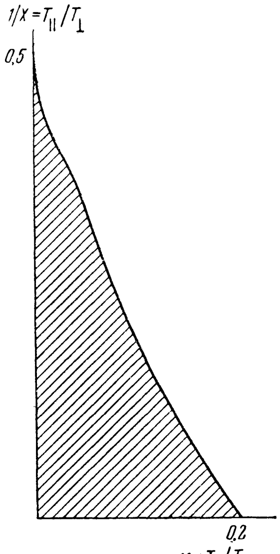
It is found that in order to find the characteristic part of the instability region (μ, X ~ 1) it is necessary to consider the values b, d ~ 1. The equation is simplified here because only the zeroth and first terms need be considered, since I_n and Y_n decrease with increasing n. Subsequent checking shows that such an approximation is correct. Thus, on the boundary of the instability, Im a = 0, we have

$$(Z_0 \operatorname{Re} Y_0 - 1) I_0 e^{-b} + (|Z_1| |\operatorname{Re} Y_1| - 1) I_1 e^{-b} - X^{-1}(Z_0 + |Z_1|) |\operatorname{Re} Y_1| I_1 e^{-b} = \mu$$

$$Z_0 e^{-Z_0^2} I_0 e^{-b} - |Z_1| e^{-Z_1^2} I_1 e^{-b} + X^{-1} (Z_0 + |Z_1|) e^{-Z_1^2} I_1 e^{-b} + Z_0 \sqrt{\mu^3 m/M} = 0.$$

In the present research we attempted to compute the maximum values of $\mu(X^{-1})$ (in Z_0, Z_1, b) satisfying the system. The region of instability is shown in the drawing (shaded). The maximum value of μ for which instability is possible is $\mu_{max} \approx 0.2$, while the minimum occurs at $X_{min} \approx 2$. Close to $\mu = 0$, the role of the electrons increases, since the contribution of the ions to the imaginary part is decreased.

It is not difficult to see that the build up of ion acoustic vibrations in an anisotropic nonisothermal plasma comes about as a result of cyclotron resonance. Such resonance is most effective when the wavelength λ is of the order r, and vanishes as $r/\lambda \to 0$. Here the vibration frequency is of the order of the cyclotron frequency of the ions.



We have thus shown that an anisotropic, nonisothermal plasma, μ, $X \neq 1$, is unstable under ion sound vibrations, while, in contrast to the instability for the Alfven branch, these vibrations are also sent into motion for.

$$\beta = P_{\text{plasma}}/P_{\text{mag. field}} \ll 1.$$

Consequently, the ion acoustic vibrations can be set in motion even in the case of relatively large magnetic fields. It should be noted that in the characteristic parts of the instability region the increment can be large, reaching values of the order of the cyclotron frequency; therefore, the ion acoustic vibrations develop first.

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[1] A. A. Vedenov and R. Z. Sagdeev, Физика плазмы и проблема управляемых термоядерных реакций Collection: Physics of Plasma and the Problem of Controlled Thermonuclear Reactions), vol. 3. Academy of Sciences Press, (1958), page 278.

[2] R. Z. Sagdeev and V. D. Shafranov, JETP **39**, 181 (1960), Soviet Phys. JETP **12**, 000 (1961).

[3] B. A. Trubnikov, loc. cit. ref. 1, page 104.

[4] E. G. Harris, Phys. Rev. Lett. **2**, 34 (1959).

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