DYNAMIC STABILIZATION OF A PLASMA RING

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A method is proposed for dynamic stabilization of long-wave instabilities of a plasma filament. The stability conditions are presented and the frequency range of the rapidly varying stabilizing magnetic fields is determined.

IT has been shown earlier,[1] that a current-carrying plasma ring can have stable equilibrium in a magnetic field of definite configuration. The equilibrium condition given in reference 1, reduces to a relation between the intensity of the magnetic field on the equilibrium orbit and the average value of intensity of the field linked with the ring. The stability conditions impose limitations on the rate of decrease of the magnetic field along the radius in the region of the equilibrium orbit. These conditions are valid when the ring moves as a unit, and when its radius changes but its form remains circular. It is known, however, that a plasma current ring is unstable when its shape is distorted. In the present paper we consider the possibility of obtaining a ring which is stable under disturbances that distort the form of the ring, when the wavelength of the perturbations is sufficiently large compared with the radius of the ring cross section.

A ring with current is acted upon by an electrodynamic force made up of the forces due to the interaction between the current and the external magnetic field and the forces due to the interaction between the current and its own field. The first of these forces is of the form

$$F_1 = 2\pi R\,[\mathbf{J}\times\mathbf{H}]/c. \tag{1}$$

The second force can be represented as the derivative of the potential function of the current $U = LJ^2/2c^2$ with respect to the corresponding coordinate. Then the component of this force, acting in the plane of the ring, will be

$$F_R = -\frac{\partial U}{\partial R} = \frac{2\pi J^2}{c^2}\left(\ln\frac{8R}{r_0} - 1\right). \tag{2}$$

Here r_0 is the radius of the ring section, R is the radius of the ring, and H is the intensity of the field on the orbit.

If the ring is in equilibrium, there are no forces acting in the vertical plane along the z axis. Therefore the equilibrium condition reduces to the relation

$$F_1 + F_R = \frac{2\pi RJ}{c}\left[H + \frac{J}{cR}\left(\ln\frac{8R}{r_0} - 1\right)\right] = 0. \tag{3}$$

Expressing J in terms of the flux linked with the ring, we arrive at Eq. (9) of reference 1

$$H = \frac{\overline{H}}{4}\left(1 + \frac{2}{l}\right), \qquad l = 2\left(\ln\frac{8R}{r_0} - 2\right). \tag{4}$$

Here l is the inductance of one centimeter of length of the ring, and $\overline{H}$ is the average value of the field inside the ring.

If the ring deviates little from the equilibrium position, the electrodynamic force acting in the plane of the ring can be represented in the form

$$(\partial F_1/\partial R - \partial^2 U/\partial R^2)\,\Delta R,$$

while the force acting in a plane perpendicular to the ring is given by

$$(\partial F_1/\partial z - \partial^2 U/\partial z^2)\,\Delta z,$$

where the values of F_1 and of $\partial U/\partial R$ correspond to the equilibrium position.

In order to introduce the force that arises when the ring is distorted, we use for simplicity the model of a straight current-carrying conductor. If the wavelength of the distortion is considerably less than the radius of the ring, such a model does not introduce any significant error. The expression for the force acting when a straight current-carrying conductor is bent, has the following form (see, for example, reference 2).

$$\frac{J^2}{c^2}\left(\frac{2\pi}{\lambda}\right)^2 \zeta \ln\frac{\pi\lambda}{r_0}, \tag{5}$$

where ζ is the magnitude of the deviation, which is small compared with the wavelength of the distortion λ.

In addition to this force, the following forces will act in the plane of the ring

$$\frac{\partial F_R}{\partial R}\Delta R = \frac{2\pi J^2}{c^2 R}\Delta R, \qquad \frac{\partial F_1}{\partial R}\Delta R = \frac{2\pi J}{c}\left(H + R\frac{\partial H}{\partial R}\right)\Delta R, \tag{6}$$

while the force

$$\frac{\partial F_1}{\partial z}\Delta z = -\frac{2\pi RJ}{c}\frac{\partial H}{\partial R}. \qquad (7)$$

will act in the plane perpendicular to the ring. We use here the condition curl $\mathbf{H} = 0$, and consequently $\partial H_R/\partial z = \partial H_z/\partial R$. Since all the expressions contain only the z component of the intensity of the magnetic field, we drop the subscript. It is clear that the fields and the derivatives of the fields with respect to coordinates are taken at the point corresponding to the equilibrium position.

We can now write the complete equations of motion of a ring that deviates from the equilibrium position. For the radial deviations we have

$$MN\frac{d^2\Delta R}{dt^2} = \left[\frac{J(H + R\partial H/\partial R)}{cR} + \frac{J^2}{c^2R^2} + \frac{J^2}{c^2}\left(\frac{2\pi}{\lambda}\right)^2 \ln\frac{\lambda}{\pi r_0}\right]\Delta R \qquad (8)$$

and for vertical deviations we have

$$MN\frac{d^2\Delta z}{dt^2} = -\left[\frac{J}{c}\frac{\partial H}{\partial R} - \frac{J^2}{c^2}\left(\frac{2\pi}{\lambda}\right)^2 \ln\frac{\lambda}{\pi r_0}\right]\Delta z. \qquad (9)$$

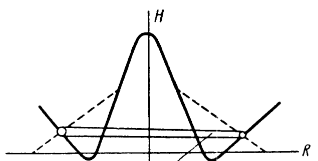
Here M is the mass of the ion and N is the number of the ions in the transverse section of the ring per unit length.

The value of the current J flowing through the ring in equilibrium is connected with the magnitude of the field by the relation

$$J = -2cRH/(l + 2) \qquad (10)$$

[See Eqs. (4) and (9) of reference 1].

If we substitute this expression into (8) and (9), it follows from the latter that when $2\pi R/\lambda > 1$, it becomes impossible to satisfy simultaneously the stability conditions in the horizontal and vertical planes. In order for the ring to be stable under deviations in the plane of the ring, the form of the field must be such as shown by the solid curve in the figure. The field near the equilibrium orbit should increase with increasing radius and its rate of increase should be the greater, the shorter the wavelength of the distortion which we wish to stabilize. On the other hand, to stabilize the ring



in the plane perpendicular to the ring, the field must diminish with the radius, i.e., its shape should be approximately as shown dotted in the figure.

This contradiction can be resolved by using a method, which we shall call "dynamic stabilization." The shape of the field near the equilibrium state of the plasma ring is varied periodically in such a way, that at some definite instant of time it has the form shown in the figure by the solid line, and after half a cycle it changes to the form represented by the dotted line. In this case the expression for the field in the region of existence of the ring can be approximately written as

$$H = H_0 + [H_\sim + (R - R_0)(\partial H/\partial R)_\sim]\sin\omega t.$$

Here H_0 is the field component which is constant in space and in time, $H_\sim$ is the amplitude of the field which is constant in space but is alternating in time, $(\partial H/\partial R)_\sim$ is the amplitude of the rate of spatial change of the ac component of the field, R is the running coordinate, and R_0 is the radius of the equilibrium orbit.

It can be assumed that the magnitude of the current is determined essentially by the quasi-stationary, slowly varying component of the flux linked with the ring, while the rapidly varying component determines only the instantaneous value of the density of the field near the equilibrium orbit and affects little the value of the current. Then the equations of motion, for example Eq. (9), can be written in the form

$$\frac{d^2\Delta z}{dt^2} = \left[\frac{J^2}{MNc^2}\left(\frac{2\pi}{\lambda}\right)^2 \ln\frac{\lambda}{\pi r_0} - \frac{J}{MNc}\left(\frac{\partial H}{\partial R}\right)_\sim \sin\omega t\right]\Delta z. \qquad (11)$$

This is none other than the equation of the small oscillations of an inverted pendulum on a vibrating suspension. The equation of motion of such a pendulum is

$$d^2\theta/dt^2 = (gL^{-1} - aL^{-1}\omega^2\sin\omega t)\,\theta, \qquad (12)$$

where g is the acceleration due to gravity, L the length of the pendulum, a the amplitude of the oscillations of the point of suspension, and ω the angular frequency of the oscillations of the suspension.

The stability condition for systems described by such an equation is given by Jeffreys[3] and Kapitza:[4] Jeffreys solves the problem by the method of infinite determinants, while Kapitza uses the method of averaging. The results of their calculations are identical, and the stability condition reduces to the inequality

$$a^2\omega^2 \geqslant 2gL. \qquad (13)$$

In order for the methods used in the investigation of (12) to be applicable, it is necessary to satisfy also the condition

$$a/L \ll 1, \qquad (14)$$

which ensures the convergence of the solution.

On going over to the system described by

Eq. (11), conditions (13) and (14) reduce to

$$\frac{\omega^2 MN}{J} \gg \left(\frac{\partial H}{\partial R}\right)_\sim \gg \omega \frac{2\pi}{\lambda}\left(2MN \ln \frac{\lambda}{\pi r_0}\right)^{1/2}. \quad (15)$$

From this we obtain the condition for the frequency of the stabilizing field

$$\omega \gg \frac{4\pi v_T}{\lambda} \sqrt{\ln \frac{\lambda}{\pi r_0}}, \quad (16)$$

where the well-known relation

$$J^2 = 4c^2 N\varkappa T \quad (17)$$

is used, $v_T = \sqrt{2\kappa T/M}$ denotes the thermal velocity of the ions, and κ is Boltzmann's constant.

The condition for the rate of change of the magnetic field along the radius is written in the form

$$\left(\frac{\partial H}{\partial R}\right)_\sim \gg \frac{2J}{c}\left(\frac{2\pi}{\lambda}\right)^2 \ln \frac{\lambda}{\pi r_0}. \quad (18)$$

It follows from this expression that the dynamic stabilization is more effective for distortions with relatively long wavelengths, whereas the stabilizing action of a longitudinal stationary magnetic field is more effective for short-wave distortion (see, for example, reference 2). It seems natural therefore to use a relatively weak longitudinal stationary field to stabilize the short-wave distortions in conjunction with dynamic stabilization of the long-wave instabilities.

An analysis of the stability of the plasma ring under deformations in the plane of the ring [Eq. (8)] leads to relations similar to (16) and (17), except that in this case the only components of the magnetic field that can be effective are the one constant in space and the one constant in time. These components must be chosen in such a manner, that at the initial instant of time, upon formation of the discharge, the plasma ring is on an equilibrium orbit. Later on, in view of the finite active resistance of the plasma (see reference 5), the plasma ring will contract towards the center, but if the shape of the field is chosen, for example, as shown by the solid line in the figure, the plasma ring cannot go outside the region of positive values of H, and will be located, in the limit, near the zero value of H. Consequently the system described here permits, in principle, realization of an equilibrium loop over a time much longer than the time determined by the active resistance of the plasma.

Inequality (18) shows that in the case of dynamic stabilization the required intensity of the stabilizing field depends little on the degree of compression of the plasma, i.e., on the radius of the cross section of the ring, r_0, which enters only under the logarithm sign. It appears therefore that dynamic stabilization will make it possible to carry out investigations with a relatively larger particle density; consequently, relatively short lifetimes of the particle in the plasma are permissible here. This last circumstance may prove to be very important, since instabilities of other types, which appear in the quasi-stationary mode with low particle density (with the exception of the macroscopic instability considered here), may result in too large a disturbance to the magnetic thermal insulation of the plasma.

Instabilities of this kind are essentially connected with the deviation of the particle distribution function from equilibrium (Maxwellian) distribution and to the appearance of groups of particles with considerable ordered velocities. The action of rapidly varying fields on the plasma should prevent the development of effects which are due to the presence of ordered motions in the plasma.

The use of high-frequency fields in research on hot thermally insulated plasma entails considerable difficulties. These difficulties are due, on the one hand, to the need of constructing apparatus with power ratings much higher than those presently available. On the other hand, compared with quasi-stationary fields, the use of rapidly varying fields leads to additional energy losses in the external circuit. These energy losses are connected with the presence of skin effect, which makes the active resistance of the high frequency circuit, and consequently the losses in the circuit, assume an unusually large part in the general energy balance. Consequently the Q of the circuit in existing equipment, in the range of frequency of interest to us, cannot exceed several hundred. The generator power P necessary to supply a system with a given high-frequency circuit voltage, is given by the relation

$$PQ = \int_V \frac{H^2}{8\pi} \omega dV, \quad (19)$$

where the integration is over the entire volume. If we neglect the dissipation and represent the working volume in the form of a torus with a large radius R and a small radius a, and use conditions (16) and (18), taking $(\partial H/\partial R)_\sim$ constant from zero to a, we arrive at the condition

$$P \gg \frac{\pi J^2}{c^2 Q} v_T \frac{R}{a}\left(\frac{2\pi}{\lambda}\right)^5 \left(\ln \frac{\lambda}{\pi r_0}\right)^{5/} .$$

This inequality, using (17), can be expressed in terms of the total number of particles in the working volume N and their temperature. Then

$$P \geqslant \frac{6NT^{3/2}}{aQM^{1/2}}\left(\frac{2\pi a}{\lambda}\right)^5\left(\ln\frac{\lambda}{\pi r_0}\right)^{5/2}\cdot 10^{-31}, \qquad (20)$$

where P is in megawatts and T in electron volts.

The following example illustrates the order of magnitude of the required power. Let us take gas — deuterium — with $M \approx 3 \times 10^{-24}$g, and let us stabilize the wavelengths greater than those for which $2\pi a/\lambda \approx 1$. A typical value is $\ln(\lambda/\pi r_0) \approx 2$; let there be $N = 10^{18}$ particles in the entire working volume and let a = 5 cm. Here

$$P \geqslant 0.5T^{3/2}/Q.$$

For an installation of this kind, which is feasible in the present technology, we assume as the characteristic values P = 50 megawatts and Q = 200. Then the temperature to which the plasma can be raised is 500 ev, which is fully adequate for experiments aimed at explaining the principal physical problems connected with this trend of research.

[1] S. M. Osovets, in Физика плазмы и проблема управляемых термоядерных реакций (Plasma Physics and the Problem of Controllable Thermonuclear Reactions), vol. 2, M., Atomizdat, 1958.

[2] M. A. Leontovich, ibid., vol. 2, M., Atomizdat, 1958.

[3] H. and B. S. Jeffreys, Methods of Mathematical Physics, Cambridge, 1950.

[4] P. L. Kapitza, JETP **21**, 588 (1951).

[5] S. M. Osovets and N. I. Shchedrin, op. cit. ref. 1, vol. 3, M., Atomizdat, 1958.

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