EFFECT OF PAIR CORRELATION OF NUCLEONS ON THE PROBABILITIES OF ELECTROMAGNETIC TRANSITIONS IN THE NUCLEUS

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The effect is considered of the pair correlation of nucleons on the probabilities of electromagnetic transitions in deformed nuclei. It is shown that the single quasiparticle electric multipole transitions near the Fermi surface will be forbidden by a factor $\sim A^{-2/3}$ compared to single particle transitions.

It is well known that pair correlation strongly affects the distribution of nucleons near the Fermi surface and the energy spectrum of particles in the nucleus.[1,2] Therefore, the problem arises of investigating the effect of pair interaction on the electromagnetic transitions in the nucleus.

We consider an axially deformed nucleus. Then the internal state of the nucleons is characterized by the component of the total angular momentum Ω of the nucleon along the nuclear symmetry axis. We denote by $\pm\lambda$ the set of $\pm\Omega$ and the other characteristics of the state. The states $+\lambda$ and $-\lambda$ in a deformed nucleus are degenerate, and differ by the sign of the component of the total angular momentum of the nucleus along the axis along which the nucleus is elongated. As has been shown by Belyaev,[1] the wave function for a system with an odd particle in the state $+\lambda_0$ has the form

$$\Phi(+\lambda_0) = \prod_{\lambda\neq\lambda_0} (u_\lambda + v_\lambda b^+_{+\lambda} b^+_{-\lambda})\, b^+_{+\lambda_0} | \Phi_0^0 \rangle,$$

$$u_\lambda = \sqrt{{}^1/_2 (1 + \varepsilon_\lambda / E_\lambda)}, \quad v_\lambda = \sqrt{{}^1/_2 (1 - \varepsilon_\lambda / E_\lambda)}, \qquad (1)$$

where b_λ and b_λ^+ are the operators for the annihilation and the creation of a particle in the state λ, ϵ_λ is the energy of the nuclear level referred to the Fermi surface ϵ_0, and $E\lambda = \sqrt{\epsilon_\lambda^2 + \Delta^2}$. As has been shown earlier[2] the quantity Δ is determined from the difference of the mass defects by means of the formula

$$\Delta = {}^1/_2 | 2E_0(N+1) - E_0(N+2) - E_0(N) |,$$

where $E_0(N)$ is the energy of the ground state of a system of N particles, while $| \Phi_0^0 \rangle$ is the wave function of the ground state for a system of noninteracting particles.

Let the system make a transition from a state with the odd particle $+\lambda_2$ to the state with an odd particle $+\lambda_1$. These states have different values of Δ, and we shall denote this by priming u_λ and v_λ. We evaluate the matrix element for the transition between these states:

$$M_{+\lambda_2, +\lambda_1} = \langle \Phi^*(+\lambda_2), \hat{A}\Phi(+\lambda_1) \rangle, \qquad (2)$$

where $\hat{A}$ is the operator for the electromagnetic transition in the second-quantization formalism. Then we have

$$M_{+\lambda_2, +\lambda_1} = [u_{\lambda_1} u'_{\lambda_2} A_{+\lambda_2, +\lambda_1} - v_{\lambda_1} v'_{\lambda_2} A_{-\lambda_1, -\lambda_2}] \times [(u_{\lambda_1} u'_{\lambda_1} + v_{\lambda_1} v'_{\lambda_1})(u_{\lambda_2} u'_{\lambda_2} + v_{\lambda_2} v'_{\lambda_2})]^{-1} \prod_\lambda (u_\lambda u'_\lambda + v_\lambda v'_\lambda), \qquad (3)$$

where $A_{\lambda,\lambda'}$ is the single particle matrix element for the transition between the states λ and λ'. In the present case $\Delta' - \Delta = \delta \ll \Delta$ since the maximum value of δ is given by[2] $\delta = \Delta_e - \Delta_o \sim 1/\rho_0 \sim \epsilon_0/A$, where ρ_0 is the level density at the Fermi surface, while the quantities Δ_e and Δ_o denote the values of Δ respectively for the neighboring even and odd nuclei. Therefore we have the expression $u_\lambda u'_\lambda + v_\lambda v'_\lambda = 1 - \epsilon_\lambda^2 \delta^2 / 8E_\lambda^4$. On assuming that the interval Δ contains many levels so that we can go over from summing over λ to integration we obtain

$$N = \prod_\lambda (u_\lambda u'_\lambda + v_\lambda v'_\lambda) = \exp\{-\tfrac{1}{16}\pi\rho_0\Delta(\delta/\Delta)^2\}.$$

Since $\delta_{max}/\Delta \sim A^{-1/3}$ and $\rho_0\Delta \sim A^{1/3}$ for deformed nuclei, N differs from unity by a quantity of the order of magnitude of $A^{-1/3}$. We shall therefore, neglect the variation in the quantity Δ. Then formula (3) assumes the form

$$M_{+\lambda_2, +\lambda_1} = (u_{\lambda_2} u_{\lambda_1} \mp v_{\lambda_1} v_{\lambda_2}) A_{+\lambda_2, +\lambda_1}. \qquad (4)$$

The matrix element is $A_{-\lambda_1, -\lambda_2} = \pm A_{+\lambda_2, +\lambda_1}$ depending on the behavior of the single particle operator $\hat{A}$ under time inversion. We therefore choose in formula (4) the minus or plus sign for electric or magnetic transitions respectively.

Formula (4) means that the so called "single particle" transitions in an odd nucleus are in fact

Nucleus	Transition energy in kev	w_N/w_{exp}	w'_N/w_{exp}
Dy^{161}	74.5	47	~2.3
Np^{237}	26.4	29	~1.8
	59.6	50	~3

single quasiparticle transitions. The matrix element for such a transition will be a superposition of matrix elements for the transitions of a particle from the state $+\lambda_2$ to the state $+\lambda_1$ and of a hole from the state $-\lambda_1$ to the state $-\lambda_2$ taken with appropriate weighting factors. Thus, the pair correlation essentially alters the form of the matrix element for the transition, and consequently also the transition probability.

We consider an electric multipole transition between states close to the Fermi surface when $\epsilon_\lambda \ll \Delta$. Then (4) assumes the form

$$M_{+\lambda_2, +\lambda_1} = A_{+\lambda_2, +\lambda_1}(\varepsilon_{\lambda_1} + \varepsilon_{\lambda_2})/2\Delta. \qquad (5)$$

Since $\epsilon_\lambda \sim \epsilon_0/A$, and $\Delta \sim \epsilon_0/A^{2/3}$, the probability of this transition will be decreased by a factor of $\sim A^{-2/3}$ compared to the single particle transition.

Recently Voĭkhanskiĭ and Gnedin[3] calculated the single particle transition probabilities, using Nilsson's model. It was found that the calculated probabilities differed from the experimental ones on the average by one order of magnitude. Account of nucleon pair correlation by means of formula (4) improves the agreement between theory and experiment, at least in the case of purely electric transitions which depend on the nuclear model to a smaller extent than the magnetic ones. The results of calculations for the E1 transitions in Dy^{161} and Np^{237} are listed in the table, where w_N, w'_N and w_{exp} are respectively the transition probabilities calculated according to Nilsson's model, those corrected for the pair correlation and the experimental ones.

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