PERIPHERAL INTERACTIONS BETWEEN π MESONS AND NUCLEONS AT HIGH ENERGIES

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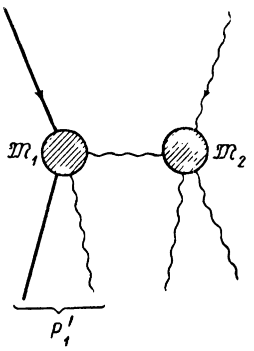
The inelastic interaction between a π meson and a nucleon is considered, taking into account the exchange of one virtual π meson. The cross section of a process in which two mesons are produced is computed. Relations between the cross sections for various channels of the process $\pi^- + p \rightarrow N + 3\pi$ are given.

In the present paper we consider the peripheral interaction between a π meson and a nucleon (πN) using the method proposed earlier in reference 1. In this method the cross section for the peripheral interaction is expressed approximately in terms of the cross sections of two real processes which are either known experimentally or about which reasonable assumptions can be made on the basis of known experiments. We compute the cross section for an inelastic πN interaction which leads to three π mesons in the final state. It is assumed in the calculation that the interaction is effected through the exchange of a single virtual π meson while the nucleon and the meson go into excited states with the masses $\mathfrak{M}_1 \geq m + \mu$ and $\mathfrak{M}_2 \geq 2\mu$, respectively, i.e., we consider the Feynman graph shown in the figure (m is the mass of the nucleon and μ the mass of the meson). This method is of interest in view of the fact that there is experimental information on the important role of such processes at high energies.[2,3]



In analogy with Eq. (3) of reference 1, we can write the cross section for the process under consideration in the form*

$$E_1E_2J\sigma_{\pi N} = \frac{1}{\pi^4}\int d^4P_1'(k^2+\mu^2)^{-2}\,\omega E_1J_1\sigma_{\pi\pi}\omega E_2J_2\sigma_2(\omega), \quad (1)$$

*Unless noted otherwise, we use the notation of reference 1.

where E_1 and E_2 are the energies of the colliding particles, P_1' is the momentum of the isobar, k is the four-momentum of the virtual meson, $k^2 = \mathbf{k}^2 - \omega^2$, J is the current density of the particles, and $\sigma_{\pi\pi}$ is the cross section for the interaction of a π meson with another π meson. Here (as in reference 1) we neglect the difference between the Lorentz-invariant expressions $\omega EJ\sigma$ and the corresponding expressions for the real π meson. To obtain from this information about the cross section for the πN interaction, we must make some definite assumptions concerning the cross section $\sigma_{\pi\pi}$.

We assume that the cross section for the $\pi\pi$ interaction is constant, i.e., independent of the energy of the π meson. We note, however, that if there really is a resonance in the $\pi\pi$ cross section for $p^2 \sim 3.5\mu^2$ (reference 4), where p^2 is the square of the momentum in the center-of-mass system of the two π mesons, this implies that the cross section for the πN interaction will have a maximum at $\epsilon \sim 5$ Bev (ϵ is the energy of the π meson in the laboratory system), since the function under the integral sign also has a maximum at $p^2 \sim 3.5\mu^2$ for such energies (see below).

With these assumptions we have, in the center-of-mass system of the π meson and the nucleon [writing $k^2 = \kappa_1^2 + 2p_0P_1'(1-\cos\vartheta)$],

$$\frac{\sigma_{\pi N}}{\sigma_{\pi\pi}} = \frac{2p_0}{\pi^3 m^2 \varepsilon^2}\int_{m\mu}^{z_{max}} dz \int_{\mu^2}^{y_{max}} dy \int_{\beta}^{1} d(\cos\vartheta)$$

$$\times \frac{P_1'\sqrt{z^2-m^2\mu^2}\sqrt{y^2-\mu^4}\,\sigma(z/m)}{[\mu^2+\varkappa_1^2+2p_0P_1'(1-\cos\vartheta)]^2},$$

$$z = (\mathfrak{M}_1^2 - m^2 - \mu^2)/2, \qquad y = (\mathfrak{M}_2^2 - 2\mu^2)/2.$$

Expanding the quantity κ_1^2 in terms of the ratios $z/E^2 \ll 1$, and $y/E^2 \ll 1$, we find

$$\varkappa_1^2 = 4zy/E^2 + 4y(z+y)(2z+m^2)/E^4 \equiv ay^2 + by,$$

where E is the total energy of the system in the center-of-mass system. The limits of integration z_{max}, y_{max}, and β are determined from the requirement that the degree of "virtualness" of the π meson be smaller than a certain given value: $k^2 \le \delta^2$. It is clear that this method is valid for $\delta \sim \mu$ and invalid for $\delta \gg \mu$. However, at the present moment it is impossible to give the exact theoretical value of δ corresponding to the upper limit up to which our method is applicable. A separate paper will be devoted to the discussion of this question. We propose here a method of obtaining δ from the experimental data on peripheral πN and NN collisions. After integrating, we rewrite (2) in the form

$$\frac{\sigma_{\pi N}}{\sigma_{\pi\pi}} \approx \frac{1}{2\pi^3 p_0^2 E^2} \int_{m\mu}^{z_{max}} dz \sqrt{z^2 - m^2\mu^2}$$

$$\times \left[\frac{1}{a} \ln \frac{\delta^2(\delta^2+\mu^2)}{by_{max}(by_{max}+\mu^2)} - \frac{y_{max}^2 - \mu^4}{\delta^2+\mu^2}\right] [2\sigma_{3/2}(z) + \sigma_{1/2}(z)],$$

$$ay_{max}^2 + by_{max} - \delta^2 = 0. \qquad (3)$$

We have made four different assumptions with respect to the interaction amplitudes in the different isotopic states of the two π mesons. According to the experimental data, probably the most correct assumption is that the amplitudes are equal in magnitude and mutually orthogonal (see the Appendix). We note that in this case the cross section for the inelastic peripheral interaction with nucleons is the same for π^- and π^+ mesons. Expression (3) was integrated numerically* for the π meson energy $\epsilon = 5$ Bev and the cut-off parameter $\delta^2 = 4\mu^2$. This yields $\sigma_{\pi N}/\sigma_{\pi\pi} \approx 0.13$. If we assume that the cross section for the $\pi\pi$ interaction is equal to the geometrical value, i.e., $\sigma_{\pi\pi} \approx 65$ mb, we obtain $\sigma_{\pi N} \approx 8.5$ mb. This value is in satisfactory agreement with the data of references 2 and 3.

It must be noted, however, that we can speak only of a good qualitative, but not of a strict quantitative, agreement, because there is some arbitrariness in the choice of the value of the $\pi\pi$ interaction cross section, and moreover, expression (3) depends strongly on δ (approximately like δ^3).

An analogous integration for the case of the NN interaction was performed in reference 1, but these authors did not take for the cut-off parameter the degree of "virtualness" of the π meson, but the average value of the perpendicular component of the momentum transfer, which was taken from experiment; this is, of course, less rigorous on account of the large experimental errors. If in this case the cut-off is also determined by the degree of "virtualness", one easily obtains the formula

$$\sigma_{NN} = \frac{1}{(2\pi)^3 p_0^2 E_0^2} \int_{m\mu}^{z_{max}} dz$$

$$\times \int_{m\mu}^{y_{max}} dy \sqrt{z^2 - m^2\mu^2} \sqrt{y^2 - m^2\mu^2} \left[\frac{1}{\varkappa^2+\mu^2} - \frac{1}{\delta^2+\mu^2}\right]$$

$$\times \left\{ \begin{matrix} {}^{10}/_9\sigma_{3/2}\sigma_{3/2} + {}^{16}/_9\sigma_{3/2}\sigma_{1/2} + {}^{1}/_9\sigma_{1/2}\sigma_{1/2} \\ {}^{14}/_9\sigma_{3/2}\sigma_{3/2} + {}^{8}/_9\sigma_{3/2}\sigma_{1/2} + {}^{5}/_9\sigma_{1/2}\sigma_{1/2} \end{matrix} \right\}. \qquad (4)$$

The top row in the curly brackets corresponds to processes of the type $p + p \rightarrow N + N + 2\pi$, the lower row corresponds to processes of the type $n + p \rightarrow N + N + 2\pi$, and $\sigma_i = \sigma_{i\ el}$. The values of z_{max} and y_{max} are determined from the requirement $\kappa^2 = \delta^2$.

We note the following relation

$$\sigma_{\pi N}/\sigma_{NN} = \sigma_{\pi\pi}/\sigma_{geom},$$

where α is a function only of the energy of the incident particles in the πN and NN interactions and is independent of δ^2, as was shown by the numerical computation. Indeed, $\sigma_{\pi N}$ is given by a formula analogous to formula (4), the only difference being that one of the σ_i in the curly brackets is replaced by $\sigma_{\pi\pi}$. The difference in the energy dependence of σ_i and $\sigma_{\pi\pi}$ has practically no effect on the value of this integral if $\delta \ge 2\mu$. It would therefore be desirable to determine the ratio of the cross section for the peripheral πN interaction with formation of two π mesons over the cross section for the peripheral NN interaction also with formation of two mesons by an evaluation of the corresponding experimental data. This will enable one to determine the $\pi\pi$ interaction uniquely. The value of $\sigma_{\pi\pi}$ obtained in this way will, of course, indeed agree with the cross section for the $\pi\pi$ interaction, if the latter does not depend on the energy of the colliding π mesons or does so only very weakly.

The constancy of the value of $\sigma_{\pi\pi}$, as obtained from the above-quoted ratio of the πN and NN interactions for various energies, will serve as a criterion of the correctness of our assumptions.

Furthermore, the parameter δ is directly determined by formula (4) if the exact value of σ_{NN} is known for some fixed energy of the colliding particles. The experimental data quoted in the re-

*In this case the integration goes over the region where $\sigma_{1/2} \ll \sigma_{3/2}$. The numerical values of $\sigma_{3/2}$ and $\sigma_{1/2}$ were taken from the report of Piccioni.[5] The integration over y is equivalent to the integration over the energies of the π meson in the $\pi\pi$ loop up to $\varepsilon' \approx 3.5$ Bev; the function in the integrand has a maximum at the kinetic energy of the π meson $\varepsilon'_{kin} \approx 0.8$ Bev in the rest system of the other π meson.

Ratio of cross sections*	Theoretical values				Experimental data	
	a	b	c	d	[2]	[3]
p^{-00}/p^{+--}	0.5	4	0.23	0.43	$\leqslant 2$	$\leqslant 1.2$
n^{+-0}/p^{+--}	2	4	0.12	0.36	$\leqslant 2.5$	$\leqslant 1.1$
n^{000}/p^{+--}	1	0	0.02	0.05	0	$\leqslant 0.4$

*p^{-00} denotes the cross section for the process $\pi^- + p \to p + \pi^- + \pi^0 + \pi^0$, etc.

port of Veksler[6] indicate that $\delta \sim 3\mu$. For a unique choice of δ it is desirable to find its value from formula (3) with $\sigma_{\pi\pi}$ obtained by the above-mentioned method. The presently available experimental data are not sufficiently accurate to use this procedure.

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APPENDIX

The isotopic spin relations for the process under consideration were determined in a way similar to that proposed in the report of Tamm.[7] The calculations were carried out for four different assumptions with respect to the matrix elements of the $\pi\pi$ interaction. If we denote the matrix element of the interaction of two π mesons in a state with the total isotopic spin I by (I), these assumptions can be written in the following fashion:

a) $(0) \neq 0, \quad (1) = (2) = 0$[8];

b) $(1) \neq 0, \quad (0) = (2) = 0$[9];

c) $(2) \neq 0, \quad (0) = (1) = 0$;

d) states of the $\pi\pi$ system with different total isotopic spin are mutually orthogonal and $|(0)| = |(1)| = |(2)|$.

We note that the contribution of the excited state of the nucleon with isospin $\frac{1}{2}$ to processes of the type considered will be negligibly small for π meson energies of $\epsilon \sim 5$ Bev. With the above-mentioned assumptions we can therefore obtain exact relations between the cross sections for various processes. These relations and their comparison with experiment are given in the table. The data in this table definitely point to the important role played by the interaction of the two mesons in the state with total isotopic spin 2. However, it has not yet been possible to discriminate in a unique way between, for example, the assumptions c) and d).

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