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SOME EXPERIMENTAL POSSIBILTIES FOR VERIFICATION OF THE MODEL OF NONAXIAL NUCLEI ROTATIONAL SPECTRUM

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It is shown that detection of E0 transitions of the $I^+ \to I^+$ type (I is the nuclear spin) in even-even nuclei is a crucial test of the model of nonaxial nuclei with a rotational spectrum. The isotopic shift predicted by the model is considered.

THE energy spectrum of individual even-even nuclei in the ranges $60 \lesssim A \lesssim 160$ and $180 \lesssim A \lesssim 240$ is interpreted by Davydov and Filippov[1] as the rotational spectrum of a nonaxial deformed nucleus whose surface is determined by two deformation parameters β and γ. In what follows we shall call this model the DF model.

The comparison with experimental data of the results of calculations in the DF model of the level spectrum and of the probabilities of the M1 and E2 transitions, carried out by Davydov and his coworkers,[1-4] shows that some even-even nuclei are accurately described by the DF model.

The DF model embraces the regions in the neighborhood of the magic nuclei and also the regions in which the nuclei have an equidistant level spectrum which was interpreted earlier as the spectrum of vibrational excitations of the nucleus.[5,6] The fundamental laws of M1 and E2 transitions are also sufficiently well described (qualitatively) in this region by the vibration model of the nucleus. In near magic regions of atomic weights, it appears that good agreement with experiment can be obtained in the shell model with residual interaction between the nucleons. We note that in our view there is at the present time no completely convincing theoretical foundation for the DF model of a nonaxial nucleus with a rotation interaction spectrum. Therefore, the problem of a model which most adequately describes the nuclear spectrum remains open, and more experimental data are necessary for its solution by means of any sort of model.

In particular, it will be shown below that the detection of E0 transitions of the type $I^+ \to I^+$ ($I \neq 0$; I is the nuclear spin) is a crucial test for the DF model in which these transitions are strictly forbidden.*

The additional data make possible a study of the isotopic shifts of the levels of the atomic electrons.

1. E0 Transitions in the DF Model

In the DF model, the wave function of the n-th rotational state with spin I and projection M is the product of the functions of the rotational motion of the nucleus $\Psi_{nIM}(\theta_i)$, where θ_i are the Euler angles, by the function of the state of the core, which depends on the deformation parameters, $\varphi_{\beta\gamma}(r'_k)$. It is assumed that in the different rotational states of the nucleus the internal state of the core remains unchanged:[1]

$$\Psi_{nIM}(\beta, \gamma, \theta_i) = \Psi_{nIM}(\theta_i)\,\varphi_{\beta\gamma}(r'_k). \quad (1)$$

Inasmuch as the operator of the E0 transition

$$\widehat{E0} = \sum_{i=1}^{Z} r_i^2 \quad (1a)$$

is a scalar, on going to a system of coordinates rotating with the nucleus, we obtain for the matrix element of the E0 transition ($n_1IM \to n_2IM$)

$$\langle n_2IM \,|\, \widehat{E0} \,|\, n_1IM\rangle = \left\langle \varphi^*_{\beta\gamma} \,\middle|\, \sum_{i=1}^{Z} (r'_i)^2 \,\middle|\, \varphi_{\beta\gamma} \right\rangle \int \Psi^*_{n_2IM}(\theta_i)\,\Psi_{n_1IM}(\theta_i)\,d\tau. \quad (2)$$

Since the functions of the rotational motion of the nucleus for different states are orthogonal

$$\int \Psi^*_{n_2IM}(\theta_i)\,\Psi_{n_1IM}(\theta_i)\,d\tau = \delta_{n_2n_1}, \quad (2a)$$

the matrix element of the E0 transition in the DF model is exactly equal to zero.

We note that in a model of the nucleus with a vibrational spectrum of excitation E0 transitions of the $I^+ \to I^+$ type ($I \neq 0$) are not forbidden. In particular, for E0 transitions ($2^+ \to 2^+$) between

*The necessity for investigating E0 transitions in even-even nuclei with equally spaced levels was noted earlier by the author.[7] We again turn to this problem in connection with the appearance of new experimental data on E0 transitions.[8] The author is grateful to I. S. Shapiro who directed his attention to this research of Gerholm and Petterson.

the second and third excited levels of the nucleus, the following has been obtained:[7]

$$\langle \chi_{2\mu}^{c+} | \widehat{E0} | \chi_{2\mu}^{1} \rangle = \frac{3}{4\pi} Z R_0^2 \cdot 5 \sqrt{\frac{10}{4\pi}} C_{2020}^{20} \left(\frac{\hbar\omega}{2C_2}\right)^{3/2}, \quad (3)$$

where $\hbar\omega$ is the energy of the photon (the energy of the first excited level of the nucleus); C_2 is the stiffness of the nucleus relative to quadrupole deformations of the surface. For example, it is assumed in reference 4 that the nucleus Pt^{196} is nonaxial ($\gamma \approx 30°$) and has a rotational spectrum of levels. But is was established by Gerholm,[8] by observation of the angular correlation of the conversion electron with the γ quantum, that an E0 transition takes place between the levels $2^+ \to 2^+$, where the matrix element of the E0 transition

$$\rho = \left\langle 2.2 \left| \sum_{i=1}^{Z} (r_i / R_0)^2 \right| 1.2 \right\rangle \quad (3a)$$

has the order of magnitude $0.017 \le |\rho| \le 0.05$. This value of $|\rho|$ is comparable with $|\rho| = 0.09$ for Ge^{70} and $|\rho| = 0.06$ for Zr^{90}.[9] Consequently, there is no basis for assuming the nucleus Pt^{196} to fit the DF model.

2. The Isotopic Shift of Electron Levels According to the DF Model

The isotopic displacement of the levels of atomic electrons in heavy elements is determined by the change in the mean square radius of the proton distribution in the ground state of the nucleus

$$\langle r^2 \rangle = \left\langle 0 \left| \sum_{i=1}^{Z} r_i^2 \right| 0 \right\rangle. \quad (3b)$$

In the DF model[1] it is assumed that the protons are uniformly distributed over the volume of the deformed nucleus. Taking this into account, we obtain for $\langle r^2 \rangle_{DF}$:

$$\langle r^2 \rangle_{DF} = \frac{3}{5} Z R_0^2 \left\{ 1 + \frac{5}{4\pi} \beta^2 + \frac{15}{8\pi} \beta^3 \cos\gamma \, [1 - 4\sin^2\gamma] \right\}. \quad (4)$$

For comparison we write down $\langle r^2 \rangle_{vibr}$, obtained in the drop model with vibration spectrum:

$$\langle r^2 \rangle_{vibr} = \frac{3}{5} Z R_0^2 \left\{ 1 + \frac{5}{4\pi} \beta_{vibr}^2 \right\}, \quad (5)$$

where

$$\beta_{vibr} = 5\hbar\omega / 2C_2. \quad (6)$$

The quantities β^2 and β_{vibr}^2 are determined from the given probability of E2 transitions from the first excited level of the nucleus to the ground state:

$$B(E2; 21 \to 0) \approx 9 Z^2 e^2 R_0^4 \beta^2 / 80\pi^2 \quad (7)$$

and similarly for β_{vibr}^2.

As is seen from (4), we have for nuclei with equally spaced spectrum ($\gamma = 30°$)

$$\langle r^2 \rangle_{DF} = \frac{3}{5} Z R_0^2 \{1 + 5\beta^2 / 4\pi\}, \quad (7a)$$

which coincides with $\langle r^2 \rangle_{vibr}$.

The change in $\langle r^2 \rangle_{DF}$ on going from one isotope to another is determined by the change in the parameters β and γ. As is seen from (4), the change of γ no longer contributes to the shift for nuclei with an equally spaced spectrum, but in the regions of transitions to magic nuclei and to axially symmetric nuclei with a rotational level spectrum the parameter γ changes abruptly. For example, if we consider the isotopes Gd^{152}, Gd^{154} and Gd^{156} within the framework of a DF model, then we have here for γ the values 30, 13.7 and 0°, respectively. A similar situation holds for the isotopes Sm^{150}, Sm^{152}, Sm^{154} and, apparently, for Dy^{160}, Dy^{162} and Dy^{164}. The study of the isotopic mixture for these nuclei is important for verification of the DF model.

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