EFFECT OF SUPERFLUIDITY OF HEAVY NUCLEI ON SINGLE-PARTICLE ELECTROMAGNETIC TRANSITIONS AND β DECAY

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Formulas are deduced for the probabilities of single-particle electromagnetic transitions and of β decay, account being made for pair correlation of the nucleons in heavy nuclei.

It is well known that single-particle electromagnetic transitions in nuclei are determined at low excitations by the transition of the nucleon with the weakest bond. We shall show how realignment of the Fermi surface due to formation of correlated nucleon pairs,[1,4] influences these transitions.

To describe the superfluid state of heavy nuclei, we can use the canonical transformation of Bogolyubov,[3] which takes into account the correlation of nucleons whose states differ only in the sign of the projection of the total momentum on the symmetry axis of the nucleus:[1,4]

$$a_{\nu+} = u_\nu \alpha_\nu + v_\nu \beta_\nu^+, \qquad a_{\nu-} = u_\nu \beta_\nu - v_\nu \alpha_\nu^+, \qquad u_\nu^2 + v_\nu^2 = 1,$$

$\alpha_{\nu\sigma}$ is the operator of nucleon annihilation in the state characterized by the totality of quantum numbers ν,σ (σ denotes the sign of the projection of the momentum with modulus equal to Ω). The wave functions $\psi_{\nu\sigma}$ are chosen such that the transition from $\psi_{\nu+}$ to $\psi_{\nu-}$ corresponds to time reflection.

The ground state of an even nucleus is a vacuum Φ_0 for quasi particles: $\alpha_\nu\Phi_0 = \beta_\nu\Phi_0 = 0$. The ground and excited (with energy less than 2Δ) states of an odd nucleus can be considered as the ground state of the corresponding even nucleus plus one quasi particle:

$$\alpha_\nu^+\Phi_0; \; \beta_\nu^+\Phi_0. \tag{1}$$

The excited states of the even nucleus are characterized by the presence of at least two (in general, an even number) quasi particles

$$\alpha_{\nu_1}^+\beta_{\nu_2}^+\Phi_0; \; \alpha_{\nu_1}^+\alpha_{\nu_2}^+\Phi_0; \; \beta_{\nu_1}^+\beta_{\nu_2}^+\Phi_0. \tag{2}$$

The probability of single-particle electromagnetic transitions is determined by the matrix elements of the operators of multipole moments of the system

$$\mathfrak{M}_{\lambda\mu\alpha} = \sum_{\nu\sigma,\,\nu'\sigma'} \langle \nu, \sigma \,|\, \mathfrak{M}_{\lambda\mu\alpha}^{(1)} \,|\, \nu', \sigma' \rangle\, a_{\nu\sigma}^+ a_{\nu'\sigma'},$$

where $\mathfrak{M}_{\lambda\mu\alpha}^{(1)}$ is the single-particle operator of the electric ($\alpha = 0$) or magnetic ($\alpha = 1$) multipole moment of order λ.

For a deformed odd nucleus we readily obtain the probability of multipole transition, $w_{12}(\lambda)$:

$$w_{12}(\lambda) = |\langle \nu_2 + | \mathfrak{M}_{\lambda\mu\alpha}^{(1)} | \nu_1 + \rangle|^2 (u_{\nu_1}u_{\nu_2} + (-)^{\alpha+1} v_{\nu_1}v_{\nu_2})^2 + |\langle \nu_2 + | \mathfrak{M}_{\lambda\mu\alpha}^{(1)} | \nu_1 - \rangle|^2 (u_{\nu_1}u_{\nu_2} + (-)^{\alpha} v_{\nu_1}v_{\nu_2})^2. \tag{3}$$

[The probability of transition is defined as $(1/2)\Sigma\,|(\mathfrak{M}_{\lambda\mu\alpha})_{12}|^2$, where the averaging and summation are over the states of form (1).] The factor $(-)^\alpha$ is due to the relations

$$\langle \nu + | \mathfrak{M}_{\lambda\mu\alpha}^{(1)} | \nu' + \rangle = (-)^\alpha \langle \nu' - | \mathfrak{M}_{\lambda\mu\alpha}^{(1)} | \nu - \rangle,$$
$$\langle \nu + | \mathfrak{M}_{\lambda\mu\alpha}^{(1)} | \nu' - \rangle = (-)^\alpha \langle \nu' + | \mathfrak{M}_{\lambda\mu\alpha}^{(1)} | \nu - \rangle,$$

in accordance with the behavior of the operator $\mathfrak{M}_{\lambda\mu\alpha}^{(1)}$ under time reversal.

For a deformed even nucleus, the probability of transition from a state such as (2) to the ground state is given by the expression

$$w_{12}(\lambda) = |\langle \nu_2 + | \mathfrak{M}_{\lambda\mu\alpha}^{(1)} | \nu_1 + \rangle|^2 (u_{\nu_1}v_{\nu_2} + (-)^{\alpha} v_{\nu_1}u_{\nu_2})^2 + |\langle \nu_2 + | \mathfrak{M}_{\lambda\mu\alpha}^{(1)} | \nu_1 - \rangle|^2 (u_{\nu_1}v_{\nu_2} + (-)^{\alpha+1} v_{\nu_1}u_{\nu_2})^2. \tag{4}$$

The probability of transition between the excited states such as (2) is given by a formula analogous to (3).

The selection rules yield $\lambda \geq |\Omega_1 - \Omega_2|$ for the first term in (3) and (4) and $\lambda \geq \Omega_1 + \Omega_2$ for the second. In heavy nuclei, transitions with $\lambda < \Omega_1 + \Omega_2$ are realized as a rule. In this case the influence of superfluidity of the nuclei on the probability of single-particle electromagnetic transitions is expressed by the factors $(u_{\nu_1}u_{\nu_2} + (-)^{\alpha+1}v_{\nu_1}v_{\nu_2})^2$ and $(u_{\nu_1}v_{\nu_2} + (-)^{\alpha}v_{\nu_1}u_{\nu_2})^2$, which can reduce the transition probability by one order of magnitude. (Near the Fermi surface, the factors $(u_{\nu_1}u_{\nu_2} - v_{\nu_1}v_{\nu_2})^2$ and $(u_{\nu_1}v_{\nu_2} - v_{\nu_1}u_{\nu_2})^2$

are of the order $(\rho_0\Delta)^{-2}$, where ρ_0 is the energy density of the levels at the Fermi surface).

A realignment of the Fermi surface changes also the probability of β decay. For β^- decay of an odd deformed nucleus without excitation of the collective-motion levels, we readily obtain, in analogy with (3),

$$W_{12} = w_{12}(u^n_{\nu_1}u^p_{\nu_2})^2. \qquad (5)$$

Here w_{12} is the probability of decay without allowance for "pairing"; n and p denote respectively the neutron and proton fluids; ν_1 and ν_2 characterize initial and final states of form (1).

The probability of β^- decay of an even-even (odd-odd) deformed nucleus from the ground states (to the ground states) is given by the expressions

$$W_{12} = w^{ee}_{12}(v^n_{\nu_1}u^p_{\nu_2})^2, \quad W_{12} = w^{oo}(u^n_{\nu_1}v^p_{\nu_2})^2. \qquad (6)$$

The transition to β^+ decay is made by the substitution n $\rightleftarrows$ p. For a decay from a ground state to a ground state, the factors occuring when superfluidity is taken into account are approximately $^1/_4$.

Formulas (3) — (6) can also be obtained in a different manner, by expressing W_{12} in terms of a single-particle Green's function and $w_{12}(\lambda)$ in terms of a two-particle Green's function. The latter is expressed, in turn, in terms of the single-particle Green's function and the correlation functions. The equations for these functions have been solved for the case of superfluidity in references 5 and 2.

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