INSTABILITY OF PLASMA WITH ANISOTROPIC DISTRIBUTION OF ION AND ELECTRON VELOCITIES

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A study is made of the propagation of magnetohydrodynamic waves in an infinite rarefied plasma with an anisotropic distribution of charged particle velocities. The conditions are derived under which instability appears as a result of the anisotropy of the distribution function. It is shown that the kinetic treatment leads to an enlargement of the instability region compared with that obtained in the quasihydrodynamic approximation.

1. It was shown by Rudakov and Sagdeev[1] that the anisotropy of the pressure in a rarefied plasma can lead to instability. Their results were generalized by Polovin and Tsintsadze[2] to the case in which the Alfven velocity is of the order of the speed of light. The considerations given in the works mentioned were based on the quasihydrodynamic approximation of Chew, Goldberger, and Low.[3] However, the quasihydrodynamical approximation correctly describes only such plasma motions in which there is no transfer of pressure along the force lines of the magnetic field. In the present research, the low frequency vibrations of an infinite plasma are considered for the case of an anisotropic distribution of ion and electron velocities; these considerations are made on the basis of the kinetic equation. Such an analysis for certain special cases was carried out previously by Vedenov and Sagdeev,[4] and also by Chandrasekhar, Kaufman, and Watson.[5]

2. The kinetic equation for small deviations of the distribution function $f_\alpha(\mathbf{r}, \mathbf{v}, t)$ of particles of type α from the equilibrium distribution has the form

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v}\frac{\partial f_\alpha}{\partial \mathbf{r}} + \frac{e_\alpha}{m_\alpha}\mathbf{E}\frac{\partial f_{0\alpha}}{\partial \mathbf{v}} + \frac{e_\alpha}{m_\alpha c}[\mathbf{v}\times\mathbf{H}]\frac{\partial f_{0\alpha}}{\partial \mathbf{v}} + \frac{e_\alpha}{m_\alpha c}[\mathbf{v}\times\mathbf{H}_0]\frac{\partial f_\alpha}{\partial \mathbf{v}} = 0, \quad (1)$$

where $\mathbf{E}$ and $\mathbf{H}$ are the intensities of the electric and magnetic fields which are produced as a result of the departure of the plasma from the state of equilibrium; $\mathbf{H}_0$ is the intensity of the external magnetic field; $f_{0\alpha}$ is the equilbrium distribution function, the index $\alpha = e$ or i for electrons and ions, respectively; $m_e = m$, and $m_i = M$.

The equilibrium distribution function evidently depends only on $\mathbf{v}_\perp$ and $\mathbf{v}_\parallel$ ($\mathbf{v}_\parallel$ and $\mathbf{v}_\perp$ are the components of the velocity $\mathbf{v}$ parallel and perpendicular to $\mathbf{H}_0$). We shall assume no fluxes of charged particles along $\mathbf{H}_0$ in the equilibrium state, so that $f_{0\alpha} = f_{0\alpha}(v_\perp^2, v_\parallel^2)$. We shall also assume that the functions $f_{0\alpha}$ fall off monotonically for increase in $\mathbf{v}_\parallel^2$. In references 4 and 5, the function $f_{0\alpha}$ was chosen in the form

$$f_{0\alpha} = \frac{n_{0\alpha} m_\alpha^{3/2}}{(2\pi)^{3/2} T_{\perp\alpha} T_{\parallel\alpha}^{1/2}} \exp\left(-\frac{m_\alpha v_\parallel^2}{2T_{\parallel\alpha}} - \frac{m_\alpha v_\perp^2}{2T_{\perp\alpha}}\right), \quad (2)$$

where $T_{\parallel\alpha}$ and $T_{\perp\alpha}$ are the "longitudinal" and "transverse" temperatures, and $n_{0\alpha}$ is the number of particles of type α per unit volume.

Solving Eq. (1) and the Maxwell equations by the Fourier-Laplace method, we can show that after a sufficient time the Fourier components of the quantities $\mathbf{E}$ and $\mathbf{H}$ will be proportional to $\exp(-i\omega' t)$, where the complex frequency $\omega' = \omega - i\gamma$ is determined from the dispersion equation. If the dispersion equation has the solution with $\gamma < 0$, then the equilibrium state is unstable. To obtain the dispersion equation, we substitute the values of f_α, $\mathbf{E}$ and $\mathbf{H}$ in the form of plane waves in Eq. (1) and in the Maxwell equations. We consider perturbations corresponding to magnetohydrodynamic waves for which the following relations hold:

$$|\omega'| \ll \omega_{Hi}, \qquad k^2\overline{v_i^2} \ll \omega_{Hi}^2, \quad (3)$$

where $\mathbf{k}$ is the wave vector, $\omega_{H\alpha} = e_\alpha H_0/m_\alpha c$, and the bar over a quantity indicates averaging over the equilibrium distribution. We shall also assume that the Alfven velocity $V_A = H_0(4\pi n_0 M)^{-1/2}$ is small in comparison with the speed of light c ($n_0 = n_{0e} = n_{0i}$). In the approximations of (3), the dispersion equation for electromagnetic waves in plasma divides into two equations:

$$n^2 \cos^2\theta - \varepsilon_{11} = 0, \tag{4}$$

$$n^2 - \varepsilon_{22} - \varepsilon_{23}^2/\varepsilon_{33} = 0, \tag{5}$$

where ϵ_{ij} is the dielectric permittivity tensor, $n = kc/\omega'$, and θ is the angle between $\mathbf{k}$ and $\mathbf{H}_0$. The quantities ϵ_{ij} have the form

$$\varepsilon_{11} = \sum_\alpha \frac{\Omega_\alpha^2}{\omega_{H\alpha}^2}[1 + n^2\cos^2\theta\,(\overline{v_{\parallel\alpha}^2}/c^2 - \overline{v_{\perp\alpha}^2}/2c^2)],$$

$$\varepsilon_{22} - \varepsilon_{11} = -\sum_\alpha \frac{\Omega_\alpha^2}{\omega_{H\alpha}^2} n^2 \sin^2\theta\,(\overline{v_{\perp\alpha}^2}/c^2) \times\left[1 - \int_C \frac{k_\parallel v_\parallel \Phi_\alpha(v_\parallel^2)}{k_\parallel v_\parallel - \omega'} dv_\parallel\right],$$

$$\varepsilon_{33} = \sum_\alpha \frac{\Omega_\alpha^2}{\omega'^2} \int_C \frac{v_\parallel^2 F_\alpha(v_\parallel^2)}{1 - k_\parallel v_\parallel/\omega'} dv_\parallel,$$

$$\varepsilon_{23} = i\sum \frac{\Omega_\alpha^2}{\omega'\omega_{H\alpha}} \frac{\sin\theta}{\cos\theta} \int_C \frac{k_\parallel v_\parallel \Psi_\alpha(v_\parallel^2)}{k_\parallel v_\parallel - \omega'} dv_\parallel, \tag{6}$$

where

$$\Omega_\alpha^2 = 4\pi e^2 n_0/m_\alpha, \qquad k_\parallel = k\cos\theta,$$

$$n_0\Phi_\alpha(v_\parallel^2) = -\pi\left(\overline{v_{\perp\alpha}^2}\right)^{-1} \frac{\partial}{\partial v_\parallel^2} \int_0^\infty v_\perp^5 f_{0\alpha}(v_\perp^2, v_\parallel^2)\, dv_\perp,$$

$$n_0\Psi_\alpha(v_\parallel^2) = -2\pi \frac{\partial}{\partial v_\parallel^2} \int_0^\infty v_\perp^3 f_{0\alpha}(v_\perp^2, v_\parallel^2)\, dv_\perp,$$

$$n_0 F_\alpha(v_\parallel^2) = 4\pi \frac{\partial}{\partial v_\parallel^2} \int_0^\infty v_\perp f_{0\alpha}(v_\perp^2, v_\parallel^2)\, dv_\perp.$$

The integration over $v_\parallel$ in (6) is carried out along the contour C, which runs along the real axis from $-\infty$ to $+\infty$ for $\gamma < 0$; if $\gamma > 0$, then the contour C bypasses the singularity $v_\parallel = \omega'/k_\parallel$ below for $k_\parallel > 0$ and above for $k_\parallel < 0$.

We write out the asymptotic forms of the integrals appearing in (6):

$$\int_C \frac{k_\parallel v_\parallel \Phi_\alpha(v_\parallel^2)}{k_\parallel v_\parallel - \omega'} dv_\parallel = \begin{cases} -k_\parallel^2 \overline{v_{\perp\alpha}^4}/4\omega'^2\overline{v_{\perp\alpha}^2}, & x_\alpha \ll 1, \\ q_\alpha + i\pi\omega'|k_\parallel|^{-1}\Phi_\alpha(0), & x_\alpha \gg 1; \end{cases}$$

$$\int_C \frac{v_\parallel^2 F_\alpha(v_\parallel^2)}{1 - k_\parallel v_\parallel/\omega'} dv_\parallel = \begin{cases} -1 - 3k_\parallel^2 \overline{v_{\parallel\alpha}^2}(\omega')^{-2}, & x_\alpha \ll 1, \\ \omega'^2 k_\parallel^{-2}(1/u_\alpha^2 - i\pi\omega'|k_\parallel|^{-1}F_\alpha(0)), & x_\alpha \gg 1; \end{cases}$$

$$\int_C \frac{k_\parallel v_\parallel \Psi_\alpha(v_\parallel^2)}{k_\parallel v_\parallel - \omega'} dv_\parallel = \begin{cases} -k_\parallel^2 \overline{v_{\perp\alpha}^2}/2\omega'^2, & x_\alpha \ll 1, \\ \eta_\alpha + i\pi\omega'|k_\parallel|^{-1}\Psi_\alpha(0), & x_\alpha \gg 1, \end{cases}$$

where

$$x_\alpha = k_\parallel^2 \overline{v_{\parallel\alpha}^2}/\omega'^2, \quad q_\alpha = \int_{-\infty}^{\infty} \Phi_\alpha(v_\parallel^2)\, dv_\parallel,$$

$$\eta_\alpha = \int_{-\infty}^{\infty} \Psi_\alpha(v_\parallel^2)\, dv_\parallel, \quad \frac{1}{u_\alpha^2} = -\int_{-\infty}^{\infty} F_\alpha(v_\parallel^2)\, dv_\parallel.$$

In order of magnitude, $\eta_\alpha \sim q_\alpha \sim \overline{v_{\perp\alpha}^2}/\overline{v_{\parallel\alpha}^2}$, $u_\alpha^2 \sim \overline{v_{\parallel\alpha}^2}$.

3. Equation (4) determines the frequency of the ordinary magnetohydrodynamic wave, which is the analogue of the Alfven wave in magnetohydrodynamics. From (4) we find, with the aid of (6),

$$\omega'^2 = k^2 V_A^2 \cos^2\theta\,(1 + (\overline{v_{\perp i}^2} + \mu\overline{v_{\perp e}^2})/2V_A^2 - (\overline{v_{\parallel i}^2} + \mu\overline{v_{\parallel e}^2})/V_A^2), \tag{7}$$

where $\mu = m/M$. This result can be obtained in the quasihydrodynamic approximation also. Equation (7) was obtained for $\theta = 0$ previously.[1,4] It follows from (7) that the plasma is unstable if

$$\overline{v_{\parallel i}^2} + \mu\overline{v_{\parallel e}^2} > V_A^2 + \tfrac{1}{2}(\overline{v_{\perp i}^2} + \mu\overline{v_{\perp e}^2}). \tag{8}$$

Equation (5) determines the frequencies of the extraordinary magnetohydrodynamic and "sound" waves, which are the analogues of the fast and slow magnetoacoustic waves in magnetohydrodynamics. We note that the component $\epsilon_{23}^2/\epsilon_{33}$ in (5) appears as the result of consideration of the component of the electric field $\mathbf{E}$ parallel to $\mathbf{H}_0$. This component is essential in the consideration of waves with small phase velocities when $|\omega'/k_\parallel| \lesssim |\overline{v_{\parallel i}}|$.

The study of the dispersion equation (5) is materially simplified in a number of limiting cases.

a) In the case of a strong magnetic field, when $V_A^2 \gg \overline{v_i^2}$ and $V_A^2 \gg \mu\overline{v_e^2}$, we get from (5) the following expressions for the frequency and damping coefficient of the extraordinary magnetohydrodynamic wave:

$$\omega = kV_A, \tag{9}$$

$$\gamma/\omega = \frac{1}{2}V_A^{-2}\mu\sin^2\theta\left\{\pi\omega\overline{v_{\perp e}^2}|k_\parallel|^{-1}\Phi_e(V_A^2/\cos^2\theta) - u_e^2 \times \operatorname{Im}\frac{\left[\eta_e + \mathrm{P}\int_{-\infty}^{\infty}\omega\Psi_e(v_\parallel^2)(k_\parallel v_\parallel - \omega)^{-1}dv_\parallel + i\pi\omega|k_\parallel|^{-1}\Psi_e(V_A^2/\cos^2\theta)\right]^2}{1 - u_e^2\omega|k_\parallel|^{-1}\,\mathrm{P}\int_{-\infty}^{\infty}F_e(v_\parallel^2)(v_\parallel - \omega/k_\parallel)^{-1}dv_\parallel - i\pi\omega u_e^2|k_\parallel|^{-1}F_e(V_A^2/\cos^2\theta)}\right\}. \tag{9'}$$

For the anisotropic Maxwell distribution (2), Eq. (9') is simplified:

$$\frac{\gamma}{\omega} = \sqrt{\frac{\pi m}{8M}}\frac{\sin^2\theta\sqrt{T_{\parallel e}/M}}{|\cos\theta|V_A}\left(\frac{T_{\perp e}}{T_{\parallel e}}\right)^2 \exp\left\{-\frac{mV_A^2}{2T_{\parallel e}\cos^2\theta}\right\}.$$

The frequency of the "sound" wave in the case of a strong magnetic field ($V_A^2 \gg \overline{v_i^2}, \mu\overline{v_e^2}$) is determined from the equation

$$\varepsilon_{33}(\omega') = 0. \tag{10}$$

If $\overline{v_{\parallel i}^2} \gtrsim \mu\overline{v_{\parallel e}^2}$, then the "sound" wave is strongly damped as the result of Cerenkov absorption in the ionized gas: $\gamma \sim \omega \sim |k_\parallel\overline{v_{\parallel i}}|$. If $\overline{v_{\parallel i}^2} \ll \mu\overline{v_{\parallel e}^2}$, then

$$\omega = k_{\parallel} \sqrt{\mu u_e^2}. \tag{11}$$

The damping coefficient for θ not too close to $\pi/2$ is

$$\gamma = -(\pi u_e^2/2\,|k_{\parallel}|)\,F_e(0)\,\omega^2. \tag{11'}$$

In order of magnitude, $\gamma/\omega \sim \sqrt{\mu} \ll 1$. The damping in this case is brought about by Cerenkov absorption in the electron gas. If the equilibrium distribution has the form (2), then we get for the frequency and the damping coefficient of the "sound" wave

$$\omega = k_{\parallel} \sqrt{T_{\parallel e}/M}, \quad \gamma = |\omega| \sqrt{\pi m/8M}.$$

The analysis just given shows that a plasma with an anisotropic distribution of electron and ion velocities in a strong magnetic field is stable.

b) For a strongly non-isothermal plasma, in which the electrons are heated more strongly than the ions ($\mu\overline{v_{\parallel e}^2} \gg \overline{v_{\parallel i}^2}, \overline{v_{\perp i}^2}$ or $\mu\overline{v_{\perp e}^2} \gg \overline{v_{\parallel i}^2}, \overline{v_{\perp i}^2}$), we get a solution of the dispersion equation (5) in the form

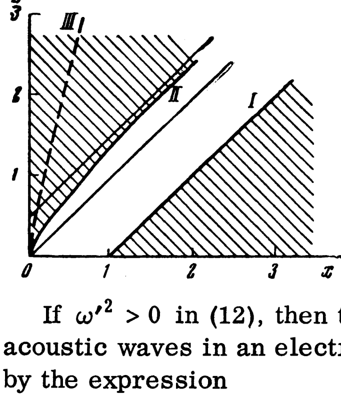
$$\omega'^2 = \tfrac{1}{2} k^2 (a \pm \sqrt{a^2 - 4b}),$$

$$a = V_A^2 + \mu \cos^2\theta\,(\overline{v_{\perp e}^2}/2 - \overline{v_{\parallel e}^2}) + \mu \sin^2\theta\,(1 - q_e)\,\overline{v_{\perp e}^2} + \mu u_e^2 \cos^2\theta + \mu u_e^2 \eta_e^2 \sin^2\theta,$$

$$b = \mu u_e^2 \cos^2\theta\,[V_A^2 + \mu\cos^2\theta\,(\overline{v_{\perp e}^2}/2 - \overline{v_{\parallel e}^2}) + \mu \sin^2\theta\,(1 - q_e)\,\overline{v_{\perp e}^2}]. \tag{12}$$

In the derivation of Eq. (12), it was assumed that $|\omega'|^2 \gg k_{\parallel}^2\overline{v_{\parallel i}^2}$ and $|\omega'|^2 \ll k^2\overline{v_{\parallel e}^2}$.

The condition for instability $b < 0$ becomes especially simply for an anisotropic Maxwellian distribution (2). The regions of instability in this case are shown as shaded areas in the drawing. The dashed line in the drawing corresponds to the boundary of the region of instability, according to Rudakov and Sagdeev.[1] As is seen from the drawing, kinetic consideration leads to an increase in the region of instability in comparison with the region of instability obtained in the quasihydrodynamic approximation.



Region of instability is shaded. The curves correspond to the equations I: $y = x - 1$, II: $y = \frac{1}{2}(x + \sqrt{x^2 + 2x})$, III: $y = 3x + \sqrt{9x^2 + 3x}$, where $x = T_{\parallel e}/MV_A^2$, $y = T_{\perp e}/MV_A^2$.

If $\omega'^2 > 0$ in (12), then the damping of magnetoacoustic waves in an electron gas is determined by the expression

$$\gamma = \tfrac{1}{2}\pi k \sin^2\theta\,\omega^2\,|\cos\theta|^{-1}\,\{\Phi_e(0)\,\mu\overline{v_{\perp e}^2}\,(1 - k_{\parallel}^2\mu u_e^2/\omega^2) - 2\Psi_e(0)\,\mu u_e^2\eta_e - \eta_e^2 F_e(0)\,\mu u_e^4\,(1 - k_{\parallel}^2\mu u_e^2/\omega^2)^{-1}\} \times \Big\{2\omega^2 - k^2\Big[V_A^2 + \mu\cos^2\theta\,\big(\tfrac{1}{2}\overline{v_{\perp e}^2} - \overline{v_{\parallel e}^2}\big) + \mu\sin^2\theta\,(1 - q_e)\,\overline{v_{\perp e}^2} + \mu u_e^2\,(\cos^2\theta + \eta_e^2\sin^2\theta)\Big]\Big\}^{-1}. \tag{13}$$

In the case of an anisotropic Maxwellian distribution (2), Eq. (13) takes the form

$$\gamma = \sqrt{\pi m/8M}\,\sin^2\theta\,\sqrt{T_{\parallel e}/M}\,|\cos\theta|^{-1}\,(T_{\perp e}/T_{\parallel e})^2 \times [\omega^2(1 - k_{\parallel}^2 T_{\parallel e}/M\omega^2)^{-1} - 2k_{\parallel}^2 T_{\parallel e}/M]\,\{2\omega^2 - k^2[V_A^2 + T_{\perp e}/M + \sin^2\theta\, T_{\perp e}(1 - T_{\perp e}/T_{\parallel e})/M]\}^{-1}. \tag{13'}$$

For $T_{\perp e} = T_{\parallel e} = T_e$, Eqs. (12) and (13) for the frequency and the damping of the magnetoacoustic waves transform to the equations obtained in reference 6.

c) We now consider the case in which the energy of thermal motion of the particles in a direction parallel to $\mathbf{H}_0$ significantly exceeds the energy of thermal motion in the direction perpendicular to $\mathbf{H}_0$, i.e., $\overline{v_{\parallel\alpha}^2} \gg \overline{v_{\perp\alpha}^2}$. In this case, one can neglect the quantity $\epsilon_{23}^2/\epsilon_{33}$ in (5) in comparison with $\epsilon_{22} \approx \epsilon_{11}$. As a result, we get the following expression for the frequency of the extraordinary magnetohydrodynamic wave

$$\omega'^2 = k^2\,[V_A^2 - \cos^2\theta\,(\overline{v_{\parallel i}^2} + \mu\overline{v_{\parallel e}^2})]. \tag{14}$$

This formula also follows from the expressions obtained in references 1 and 2.

If $\omega'^2 > 0$ in (13), then we have for the damping coefficient

$$\gamma = -\omega V_A^2\,\mathrm{Im}\,(\varepsilon_{22} - \varepsilon_{11} + \varepsilon_{23}^2/\varepsilon_{33})/2c^2. \tag{14'}$$

Here $\epsilon_{22} - \epsilon_{11}$, ϵ_{23} and ϵ_{33} are determined by Eqs. (6) for $\omega' = \omega$. If $\omega^2 \sim k_{\parallel}^2\overline{v_{\parallel i}^2}$ then, in order of magnitude, $\gamma/\omega \sim (\overline{v_{\perp i}^2}/\overline{v_{\parallel i}^2})^2 \ll 1$.

d) Let the energy of the thermal motion of the particles in the direction perpendicular to $\mathbf{H}_0$ significantly exceed the thermal energy of motion of the particles in the direction parallel to $\mathbf{H}_0$, i.e., $\overline{v_{\perp i}^2} \gg \overline{v_{\parallel i}^2}, \mu\overline{v_{\parallel e}^2}$ or $\mu\overline{v_{\perp e}^2}q_e \gg \mu\overline{v_{\parallel e}^2}, \overline{v_{\parallel i}^2}$. Assuming that $|\omega'|^2 \gg k_{\parallel}^2\overline{v_{\parallel i}^2}$ and $|\omega'|^2 \ll k_{\parallel}^2 v_{\parallel e}^2$, we obtain the expression for the frequency of the "magnetoacoustic" waves:

$$\omega'^2 = \tfrac{1}{2} k^2 (c \pm \sqrt{c^2 - 4d}),$$

$$c = V_A^2 + \big(1 - \tfrac{1}{2}\cos^2\theta\big)\,(\overline{v_{\perp i}^2} + \mu\overline{v_{\perp e}^2}) + \sin^2\theta\,(\mu u_e^2\eta_e^2 - \mu\overline{v_{\perp e}^2}q_e),$$

$$d = -\sin^2\theta\cos^2\theta\,\Big[\tfrac{1}{4}\overline{v_{\perp i}^4} + \mu u_e^2\eta_e\overline{v_{\perp i}^2} + \mu\overline{v_{\perp e}^2}\mu u_e^2 q_e\Big]. \tag{15}$$

It is evident that instability always takes place in the case under consideration.

If now $\overline{v_{\perp e}^2} \gg \overline{v_{\parallel e}^2}$ and $\mu\overline{v_{\perp e}^2}q_e \gg \overline{v_{\perp i}^2}, \overline{v_{\parallel i}^2}$, which corresponds to a sufficiently large value of the energy of thermal motion of the electrons in the direction perpendicular to $\mathbf{H}_0$, then

$$\omega'^2 = k^2 [V_A^2 + \mu \sin^2\theta (u_e^2\eta_e^2 - \overline{v_{\perp e}^2}q_e)]. \qquad (16)$$

In the quasihydrodynamic approximation for $\mu\overline{v_{\perp e}^2} \gg \mu\overline{v_{\parallel e}^2}, \overline{v_{\perp i}^2}, \overline{v_{\parallel i}^2}$, we have $\omega'^2 \sim k^2\mu\overline{v_{\perp e}^2}$, in contrast to (16), where $\omega'^2 \sim k^2\mu\overline{v_{\perp e}^2}q_e (q_e \gg 1)$. If $\omega'^2 > 0$ in (16), then we get for the coefficient of damping of this wave

$$\gamma = \frac{\pi m k \sin^2\theta}{2M|\cos\theta|} [\eta_e^2 u_e^4 F_e(0) + 2\eta_e^2 u_e^2 \Psi_e(0) - \overline{v_{\perp e}^2}\Phi_e(0)]. \qquad (17)$$

We write out the expressions (16) and (17) in the case in which the equilibrium distribution has the form (2):

$$\omega'^2 = k^2 (V_A^2 - \sin^2\theta T_{\perp e}^2 / T_{\parallel e} M), \qquad (16')$$

$$\gamma = \sqrt{\pi m / 8M}\, k \sin^2\theta |\cos\theta|^{-1} \sqrt{T_{\parallel e}/M}\, (T_{\perp e}/T_{\parallel e})^2. \qquad (17')$$

e) Above we considered the magnetohydrodynamic waves with high frequencies, in which $|\omega'|^2 \gg k_{\parallel}^2\overline{v_{\parallel i}^2}$. We now find the solution of Eq. (5) for frequencies satisfying the condition $|\omega'| \ll |k_{\parallel}\overline{v_{\parallel i}}|$. Making use of the expressions (6), we get for θ not close to zero and $\pi/2$.

$$\gamma = |k_{\parallel}| A/\pi B,$$

$$A = \{V_A^2 + \cos^2\theta \left[\tfrac{1}{2}(\overline{v_{\perp i}^2} + \mu\overline{v_{\perp e}^2}) - \overline{v_{\parallel i}^2} - \mu\overline{v_{\parallel e}^2}\right] + \sin^2\theta [\overline{v_{\perp i}^2}(1 - q_i) + \mu\overline{v_{\perp e}^2}(1 - q_e)]\} \mu u_e^2 u_i^2/(\mu u_e^2 + u_i^2) + (\eta_i - \eta_e)^2 \sin^2\theta,$$

$$B = \sin^2\theta [\Phi_i(0)\overline{v_{\perp i}^2}\mu u_e^2 u_i^2/(\mu u_e^2 + u_i^2) + 2(\eta_e - \eta_i)\Psi_i(0)] + F_i(0)\left\{V_A^2 + \cos^2\theta \left[\tfrac{1}{2}(\overline{v_{\perp i}^2} + \mu\overline{v_{\perp e}^2}) - \overline{v_{\parallel i}^2} - \mu\overline{v_{\parallel e}^2}\right] + \sin^2\theta [\overline{v_{\perp i}^2}(1 - q_i) + \mu\overline{v_{\perp e}^2}(1 - q_e)]\right\}. \qquad (18)$$

For $\theta = 0$, the frequency ω' is determined from the relation (7).

It follows from (18) that even for weak anisotropy, when $|1 - \eta_\alpha| \ll 1$, $|1 - q_\alpha| \ll 1$ and $|1 - 2\overline{v_{\parallel\alpha}^2}/\overline{v_{\perp\alpha}^2}| \ll 1$, it is possible to produce instabilities, if the intensity of the magnetic field is sufficiently small ($V_A^2 \ll \overline{v_{\perp i}^2}, \mu\overline{v_{\perp e}^2}$). However, the intensity of the magnetic field in this case cannot be so arbitrarily small because of conditions (3).

Expression (18) is valid not only in the case of weak anisotropy. It can also be used when the angle θ is close to θ_0 determined from the condition A = 0:

$$\sin^2\theta_0 = \frac{\overline{v_{\parallel i}^2} + \mu\overline{v_{\parallel e}^2} - \tfrac{1}{2}(\overline{v_{\perp i}^2} + \mu\overline{v_{\perp e}^2}) - V_A^2}{\overline{v_{\parallel i}^2} + \mu\overline{v_{\parallel e}^2} + \overline{v_{\perp i}^2}(\tfrac{1}{2} - q_i) + \mu\overline{v_{\perp e}^2}(\tfrac{1}{2} - q_e) + (\eta_i - \eta_e)^2\mu u^2 u_i^2/(\mu u_i^2 + u_i^2)} \qquad (19)$$

As θ varies, γ changes sign at the point $\theta = \theta_0$.

It was shown above that the plasma is unstable if the inequality (8) holds. Let the condition (8) not be observed. Then it follows from (18) and (19) that the equilibrium state is unstable if the inequality

$$V_A^2 + \overline{v_{\perp i}^2}(1 - q_i) + \mu\overline{v_{\perp e}^2}(1 - q_e) + \mu u_e^2 u_i^2 (\eta_i - \eta_e)^2/(\mu u_e^2 + u_i^2) < 0 \qquad (20)$$

is satisfied. In the case of an anisotropic Maxwellian distribution (2) for $T_{\perp e} = T_{\perp i} = T_{\perp}$ and $T_{\parallel e} = T_{\parallel i} = T_{\parallel}$, the inequality (20) takes the form

$$V_A^2 + 4T_{\perp}(1 - T_{\perp}/T_{\parallel})/M < 0 \qquad (20')$$

The regions of instability for $T_{\perp e} = T_{\perp i} = T_{\perp}$ and $T_{\parallel e} = T_{\parallel i} = T_{\parallel}$ are shown in the drawing (here the quantity $x = 2T_{\parallel}/MV_A^2$ is plotted along the abscissa and $y = 2T_{\perp}/MV_A^2$ along the ordinate).

We note here that the condition analogous to (20′) is written incorrectly in the work of Vedenov and Sagdeev[4] (the coefficient 2 appears instead of the coefficient 4.)

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