KINETIC EQUATION FOR RAPIDLY VARYING PROCESSES

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A kinetic equation is considered for processes characterized by times which are small or comparable with collision times. In the most interesting case of a gas of charged particles in the absence of a magnetic field, and for frequencies which greatly exceed the Langmuir frequency ω_0, it is found that not the Debye screening range but the distance traversed by the particle during the period of the field is the maximal impact parameter in the Coulomb logarithm. The effective frequency of collisions between electrons and ions, which determines the dissipative part of the complex dielectric permittivity tensor for a plasma located in a strong, constant magnetic field is computed for the same case of high frequencies. If, moreover, the electron Larmor frequency is much greater than the Langmuir frequency, then there is a resonance character to the dependence of the maximal impact parameter in the Coulomb logarithm on the frequency of the variable field.

1. The Boltzmann kinetic equation is unsuitable under conditions in which the characteristic dimensions of the inhomogeneities are comparable with the characteristic impact parameters of the particles participating in the collisions, or when an initiated collision is not completed within the characteristic time of change in the distribution of the particles. A generalization of the Boltzmann equation to the case of extremely inhomogeneous problems was given by Chapman and Cowling[1] for the case of a gas of rigid spheres, and by Bogolyubov[2] in obtaining generalized Boltzmann equations for an arbitrary law of interaction of the gas particles.

The account below is devoted to obtaining the kinetic equation for gas particles with weak interaction, which is suitable for describing rapidly varying processes. An example of such a gas is a plasma in which the particles interact according to Coulomb's law, while an appreciable contribution to the scattering of the particles is made by long-range collisions for which the interaction can be regarded as weak. Because of the slowness in fall off of the Coulomb forces, the time during which long-range collisions take place lies in a wide range from $\tau_{\min} \sim e^2 m^{1/2} (\kappa T)^{-3/2} \sim 10^{-8}\, T^{-3/2}$ to $\tau_{\max} \sim \sqrt{m/4\pi e^2 n_0} = \omega_0^{-1} \sim 10^{-5} n_0^{-1/2}$. Here, T is the temperature, and n_0 is the density of electrons per unit volume; ω_0 is the Langmuir frequency. Under conditions in which the characteristic time of the process τ_{pr} satisfies the inequality

$$\tau_{min} \ll \tau_{\mathrm{pr}} \ll \tau_{max},$$

there is a wide region ($\tau_{\min} \ll \tau_{\max}$) in which not only can the interaction be regarded as weak, but we can also consider the collisions without regarding the time variation of the distribution. Also, there is a wide region ($\tau_{\mathrm{pr}} \ll \tau_{\max}$) in which the collisions are changed appreciably. Actually, in this latter region, the collisions are suppressed because of the rapid change in the particle distribution, and τ_{pr} plays the role of a maximum collision time.

The situation is somewhat more complicated in the case of a high-frequency process taking place in a plasma placed in a strong magnetic field. Here, under the condition that the variable frequency ω is close to the Larmor frequency of the electrons Ω and is much greater than the Langmuir frequency, the effective maximal collision time is equal to $(\omega\omega_0)^{-1/2}$ in order of magnitude, as shown below. If $|\omega \pm \Omega| \gg \omega_0$, then the effective maximal collision time is $\sim |\Omega^2 - \omega^2|^{-1/2}$.

2. We consider the interaction between gas particles to be weak. The kinetic equation for such a gas in the case of a slowly changing process was found by Landau.[3] To obtain the corresponding kinetic equation that describes rapidly changing processes, we make use of perturbation theory. It was shown by Bogolyubov[2] that, in the construction of such an equation for the distribution function $f(\mathbf{p}, \mathbf{r})$, it is necessary to determine the correlation function $g(\mathbf{p}, \mathbf{r}, \mathbf{p}', \mathbf{r}')$. For a known correlation function, the equation for the distribution function can be written in the form

$$\frac{\partial f}{\partial t}+\mathbf{v}\frac{\partial f}{\partial \mathbf{r}}-n\frac{\partial f}{\partial \mathbf{p}}\frac{\partial}{\partial \mathbf{r}}\int d\mathbf{r}'\,d\mathbf{p}' f(\mathbf{p}',\mathbf{r}')\,U(|\mathbf{r}-\mathbf{r}'|)$$
$$-n\frac{\partial}{\partial \mathbf{p}}\int d\mathbf{r}'\,d\mathbf{p}' g(\mathbf{p},\mathbf{r},\mathbf{p}',\mathbf{r}')\frac{\partial U(|\mathbf{r}-\mathbf{r}'|)}{\partial \mathbf{r}}=0. \quad (1)$$

Here $f(\mathbf{p}, \mathbf{r}, t)$ is the particle distribution function normalized to unity, n is the number of particles per unit volume, U is the potential of pair forces, which we shall assume to be central, $\mathbf{v}$ is the velocity of the particles.

Because of the small interaction of the particles, the correlation function can be described by the following approximate equation:

$$\frac{\partial g}{\partial t}+\mathbf{v}\frac{\partial g}{\partial \mathbf{r}}+\mathbf{v}'\frac{\partial g}{\partial \mathbf{r}'}$$
$$=\left[f(\mathbf{p}',\mathbf{r}')\frac{\partial f(\mathbf{p},\mathbf{r})}{\partial \mathbf{p}}-f(\mathbf{p},\mathbf{r})\frac{\partial f(\mathbf{p}',\mathbf{r}')}{\partial \mathbf{p}'}\right]\frac{\partial U(|\mathbf{r}-\mathbf{r}'|)}{\partial \mathbf{r}}. \quad (2)$$

Under the assumption of an adiabatic turning on of the interaction in the infinitely distant past, Eq. (2) can easily be written in the form

$$g(\mathbf{p},\mathbf{r},\mathbf{p}',\mathbf{r}',t)=\int_{-\infty}^{0}d\tau\left\{\frac{\partial}{\partial r_i}U(|\mathbf{r}-\mathbf{r}'-(\mathbf{v}-\mathbf{v}')\tau|)\right\}$$
$$\times\left\{\frac{\partial}{\partial p_i}+\frac{\partial v_j}{\partial p_i}\tau\frac{\partial}{\partial r_j}-\frac{\partial}{\partial p_i'}-\frac{\partial v_j'}{\partial p_j'}\tau\frac{\partial}{\partial r_j'}\right\}f(\mathbf{p},\mathbf{r}-\mathbf{v}\tau,t+\tau)$$
$$\times f(\mathbf{p}',\mathbf{r}'-\mathbf{v}'\tau,t+\tau). \quad (3)$$

Equation (3), along with Eq. (1), gives the generalized kinetic equation which is suitable for describing rapidly changing processes.

3. We consider in detail the case of a spatially homogeneous distribution. In this case the kinetic equation takes on the following form:

$$\frac{\partial f(\mathbf{p},t)}{\partial t}=n\frac{\partial}{\partial p_i}\int d\mathbf{p}'\int_{-\infty}^{0}d\tau\mathscr{E}_{ij}((\mathbf{v}-\mathbf{v}')\tau)$$
$$\times\left\{f(\mathbf{p}',t+\tau)\frac{\partial f(\mathbf{p},t+\tau)}{\partial p_j}-f(\mathbf{p},t+\tau)\frac{\partial f(\mathbf{p}',t+\tau)}{\partial p_j'}\right\}. \quad (4)$$

Here

$$\mathscr{E}_{ij}(\mathbf{v}\tau)=\int d\mathbf{r}\frac{\partial U(r)}{\partial r_i}\frac{\partial U(|\mathbf{r}-\mathbf{v}\tau|)}{\partial r_j}=\frac{1}{(2\pi)^3}\int d\mathbf{k}\nu^2(k)\,k_i k_j e^{i\mathbf{k}\mathbf{v}\tau},$$
$$\nu(k)=\int U(r)\,e^{i\mathbf{k}\mathbf{r}}d\mathbf{r}.$$

If the time dependence is weak, then one can neglect the difference between $t+\tau$ and t in the arguments of the distribution function on the right side of Eq. (4). Then Eq. (4) transforms into the Landau equation:

$$\frac{\partial f}{\partial t}=\frac{\partial}{\partial p_i}\int d\mathbf{p}'\,I_{ij}(\mathbf{v}-\mathbf{v}',0)\left\{f(\mathbf{p}')\frac{\partial f(\mathbf{p})}{\partial p_j}-f(\mathbf{p})\frac{\partial f(\mathbf{p}')}{\partial p_j'}\right\},$$
$$I_{ij}(\mathbf{v},0)=n\int_{-\infty}^{0}d\tau\,\mathscr{E}_{ij}(\mathbf{v}\tau)=\frac{v^2\delta_{ij}-v_iv_j}{v^3}A(0),$$
$$A(0)=\frac{n}{4\pi}\int_0^\infty q^3\nu^2(q)\,dq. \quad (5)$$

In order to make clear the difference between Eq. (4) and the usual equation of Landau (5), we consider a linearized Eq. (4), assuming that the nonequilibrium increment δf is small in comparison with the equilibrium distribution function f_0. Moreover, owing to the linearization, we can put the time dependence in the form $\exp(-i\omega t)$. Then the right-hand side of Eq. (4) takes the form

$$\frac{\partial}{\partial p_i}\int d\mathbf{p}'\,I_{ij}(\mathbf{v}-\mathbf{v}',\omega)\left\{f_0(\mathbf{p}')\frac{\partial}{\partial p_j}\delta f(\mathbf{p})+\delta f(\mathbf{p}')\frac{\partial}{\partial p_j}f_0(\mathbf{p})\right.$$
$$\left.-f_0(\mathbf{p})\frac{\partial}{\partial p_j'}\delta f(\mathbf{p}')-\delta f(\mathbf{p})\frac{\partial}{\partial p_j'}f_0(\mathbf{p}')\right\}.$$

The resultant equation differs from the corresponding right-hand side obtained in the linearization of the Landau equation in the frequency dependence of the kernel $I_{if}(\mathbf{v},\omega)$, which is determined by the following formula:

$$I_{ij}(\mathbf{v},\omega)=\frac{n}{(2\pi)^3}\int dk\nu^2(k)\,k_ik_j\left\{i\mathrm{P}\frac{1}{\mathbf{kv}-\omega}+\pi\delta(\mathbf{kv}-\omega)\right\}. \quad (6)$$

Here P denotes the principal value.

On the basis of Eq. (6), it is easy to conclude that the frequency dependence can become important only under conditions in which v/ω — the distance traversed by the particle during the period of oscillation — is small in comparison with the characteristic impact parameter $\rho \sim 1/k$.

The kernel of the collision integral (6) can be written in the form

$$I_{ij}(\mathbf{v},\omega)=v^{-3}(v^2\delta_{ij}-v_iv_j)[A'(\omega,v)+iA''(\omega,v)]$$
$$+v^{-3}v_iv_j[B'(\omega,v)+iB''(\omega,v)]. \quad (7)$$

It is easy to see that, by (6), the quantities A' and A'' (and, correspondingly, B' and B'') are connected by a dispersion relation of the usual type:

$$A''(\omega)=\frac{1}{\pi}\int d\omega\mathrm{P}\frac{A'(\omega')}{\omega'-\omega}. \quad (8)$$

4. To determine the dissipation it is necessary first to know A' and B'. We find these quantities for the case of Coulomb interaction, when $\nu(k) = 4\pi e^2/k^2$. In the consideration of the Coulomb case, it is necessary to cut off the integration over the impact parameter both above (ρ_{max}) and below (ρ_{min}). We have

$$A'(\omega,v)=\pi e^4 n\left[\ln\frac{\rho_{min}^{-2}+\omega^2/v^2}{\rho_{max}^{-2}+\omega^2/v^2}-\frac{1}{v^2/\omega^2\rho_{min}^2+1}\right.$$
$$\left.+\frac{1}{v^2/\omega^2\rho_{max}^2+1}\right],$$
$$B'(\omega,v)=2\pi e^4 n\,[(v^2/\omega^2\rho_{min}^2+1)^{-1}-(v^2/\omega^2\rho_{max}^2+1)^{-1}]. \quad (9)$$

In the low frequency region, where $\omega \ll v/\rho_{max}$, the quantity A' goes over into $A(0) \equiv A(0, v)$.

while B′ falls off as ω^2. By virtue of the fact that ρ_{max} is equal to the Debye radius in order of magnitude, the low-frequency region is obtained for $\omega \ll \omega_0 = \sqrt{4\pi e^2 n/m}$. In the opposite case, in which $\omega \gg \omega_0$, (but, at the same time, $\omega \ll v/\rho_{min}$, where $\rho_{min} \sim e^2/\kappa T$), we have

$$A'(\omega, v) \approx \pi e^4 n\,[\ln(\omega^2/v^2\rho_{min}^2) + 1],$$
$$B'(\omega, v) \approx -2\pi e^4 n. \tag{10}$$

In this expression for A′, it is v/ω, rather than ρ_{max}, that appears as the argument of the logarithm. This physically clear result was essentially obtained earlier, for example, in the calculation of the high frequency absorption in an interstellar electron gas (with the aid of the Einstein relations).[4]

It must be remarked that the approximate equation (2) which we have used is not suitable for the description of long-range correlation of particles interacting according to Coulomb's law for the case of frequencies which are small in comparison with the Langmuir frequency ω_0. Therefore, in consideration of the Coulomb case, one is obliged to cut off the integration at the Debye radius of screening. As was shown by Bogolyubov,[2] one can improve the accuracy of Eq. (2) so that it correctly describes the effects of screening. However, in this case, the kinetic equation is actually not obtained, inasmuch as it is not possible to solve the resulting equation for the correlation function. Such a situation occurs both in the case of consideration of slow processes and also in our case of rapidly changing processes.

5. We shall apply the above to the case of a gas in which there are charged particles located in a constant and homogeneous magnetic field, and also in a homogeneous but time dependent electric field. The distribution function of an alpha type of particles of such a gas then obeys the kinetic equation

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v}\frac{\partial f_\alpha}{\partial \mathbf{r}} + e_\alpha\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}\times\mathbf{H}]\right)\frac{\partial f_\alpha}{\partial \mathbf{p}}$$
$$-\frac{\partial f_\alpha}{\partial \mathbf{p}}\frac{\partial}{\partial \mathbf{r}}\int \cancel{d\mathbf{p}'}\, d\mathbf{r}' \sum_{\alpha'} n_{\alpha'} U_{\alpha\alpha'}(|\mathbf{r}-\mathbf{r}'|)\, f_{\alpha'}(\mathbf{p}', \mathbf{r}')$$
$$= J_\alpha \equiv \int d\mathbf{r}'\, d\mathbf{p}' \sum_{\alpha'} n_{\alpha'} \frac{\partial U_{\alpha\alpha'}(|\mathbf{r}-\mathbf{r}'|)}{\partial \mathbf{r}} \frac{\partial}{\partial \mathbf{p}} g_{\alpha\alpha'}(\mathbf{p}, \mathbf{r}, \mathbf{p}', \mathbf{r}', t), \tag{11}$$

while the correlation function g has the form

$$g_{\alpha\alpha'}(\mathbf{p}, \mathbf{r}, \mathbf{p}', \mathbf{r}', t) = \int_{-\infty}^{0} d\tau \left\{\frac{\partial}{\partial \mathbf{r}} U_{\alpha\alpha'}(|\mathbf{r}-\mathbf{r}' + \mathbf{a}(\tau, \mathbf{v}, \mathbf{v}', \alpha, \alpha', t)|)\right\} \times \left\{\frac{\mathbf{H}}{H^2}\left[\mathbf{H}.\left(\frac{\partial}{\partial \mathbf{p}} - \frac{\partial}{\partial \mathbf{p}'}\right)\right] + \frac{1}{H^2}\mathbf{H}\times\left[\left(\cos\Omega_\alpha\tau\frac{\partial}{\partial \mathbf{p}}\right.\right.\right.$$
$$\left.\left. - \cos\Omega_{\alpha'}\tau\frac{\partial}{\partial \mathbf{p}'}\right)\times\mathbf{H}\right] - \frac{1}{H}\mathbf{H}\times\left[\sin\Omega_\alpha\tau\frac{\partial}{\partial \mathbf{p}} - \sin\Omega_{\alpha'}\tau\frac{\partial}{\partial \mathbf{p}'}\right] + \frac{1}{H}\mathbf{H}\times\left[\frac{1-\cos\Omega_\alpha\tau}{m_\alpha\Omega_\alpha}\frac{\partial}{\partial \mathbf{r}} - \frac{1-\cos\Omega_{\alpha'}\tau}{m_{\alpha'}\Omega_{\alpha'}}\frac{\partial}{\partial \mathbf{r}'}\right]$$
$$\left. - \frac{1}{H^2}\mathbf{H}\times\left[\frac{\sin\Omega_\alpha\tau}{m_\alpha\Omega_\alpha}\frac{\partial}{\partial \mathbf{r}} - \frac{\sin\Omega_{\alpha'}\tau}{m_{\alpha'}\Omega_{\alpha'}}\frac{\partial}{\partial \mathbf{r}'}\right]\times\mathbf{H} - \tau\frac{\mathbf{H}}{H}\mathbf{H}\times\left(\frac{1}{m_\alpha}\frac{\partial}{\partial r} - \frac{1}{m_{\alpha'}}\frac{\partial}{\partial r'}\right)\right\} f_{\alpha'}(\mathbf{P}', \mathbf{R}', t+\tau)\, f_\alpha(\mathbf{P}, \mathbf{R}, t+\tau), \tag{12}$$

where ($\Omega_\alpha = e_\alpha H/m_\alpha c$)

$$\mathbf{P} = \frac{\mathbf{H}}{H}(\mathbf{pH}) + \frac{\mathbf{H}\times[\mathbf{p}\times\mathbf{H}]}{H^2}\cos\Omega_\alpha\tau$$
$$+ \frac{[\mathbf{p}\times\mathbf{H}]}{H}\sin\Omega_\alpha\tau + e_\alpha\int_t^{t+\tau} dt'\left\{\frac{\mathbf{H}}{H}(\mathbf{HE}(t'))\right.$$
$$+ \frac{\mathbf{H}\times[\mathbf{E}(t')\times\mathbf{H}]}{H^2}\cos\Omega_\alpha(\tau + t - t')$$
$$\left. + \frac{[\mathbf{E}(t')\times\mathbf{H}]}{H}\sin\Omega_\alpha(\tau + t - t')\right\},$$
$$\mathbf{R} = \mathbf{r} + \frac{1}{m_\alpha}\int_t^{t+\tau} dt'\,\mathbf{P}(t'), \qquad \mathbf{a} = \int_t^{t+\tau} dt'\left[\frac{\mathbf{P}(t')}{m_\alpha} - \frac{\mathbf{P}'(t')}{m_{\alpha'}}\right].$$

In the case of a spatially homogeneous distribution, the right side of the kinetic equation (11) takes the form

$$J_\alpha(p, t) = \sum_{\alpha'} n_{\alpha'} \frac{\partial}{\partial p_i}\int d\mathbf{p}' \int_{-\infty}^{0} d\tau \left\{\mathscr{E}_{ij}^{\alpha\alpha'}(\mathbf{v}, \mathbf{v}', \tau, t)\frac{\partial}{\partial p_j}\right.$$
$$\left. - \mathscr{E}_{ij}^{\alpha'\alpha}(\mathbf{v}', \mathbf{v}, \tau, t)\frac{\partial}{\partial p'_j}\right\} f_{\alpha'}(\mathbf{P}', t+\tau)\, f_\alpha(\mathbf{P}, t+\tau), \tag{13}$$

$$\mathscr{E}_{ij}^{\alpha\alpha'}(\mathbf{v}, \mathbf{v}', \tau, t) = \int d\mathbf{r}\, \frac{\partial U_{\alpha\alpha'}(r)}{\partial r_i} \frac{\partial U_{\alpha\alpha'}(|\mathbf{r} + \mathbf{a}|)}{\partial r_l}$$
$$\times\left\{\frac{H_l H_j}{H^2} + \left(\delta_{lj} - \frac{H_l H_j}{H^2}\right)\cos\Omega_\alpha\tau - e_{lmj}\frac{H_m}{H}\sin\Omega_\alpha\tau\right\}$$
$$= \frac{1}{(2\pi)^3}\int d\mathbf{k}\nu_{\alpha\alpha'}^2(k)\, e^{i\mathbf{ka}} \times \left\{\frac{H_l H_j}{H^2} + \left(\delta_{lj} - \frac{H_l H_j}{H^2}\right)\cos\Omega_\alpha\tau\right.$$
$$\left. - e_{lmj}\frac{H_m}{H}\sin\Omega_\alpha\tau\right\}. \tag{14}$$

For the case of Coulomb interaction, $\nu_{\alpha\alpha'}(\mathbf{k}) = 4\pi e_\alpha e_{\alpha'} k^{-2}$, and therefore

$$\mathscr{E}_{ij}^{\alpha\alpha'}(\mathbf{v}, \mathbf{v}', \tau, t) = \frac{2\pi e_\alpha^2 e_{\alpha'}^2}{a}\left(\delta_{il} - \frac{a_i a_l}{a^2}\right)$$
$$\times\left\{\frac{H_l H_j}{H^2} + \left(\delta_{lj} - \frac{H_l H_j}{H^2}\right)\cos\Omega_\alpha\tau - e_{lmj}\frac{H_m}{H}\sin\Omega_\alpha\tau\right\}, \tag{15}$$

$$\mathbf{a}(\tau, \mathbf{v}, \mathbf{v}', \alpha, \alpha', t) = \frac{\mathbf{H}}{H}(\mathbf{v} - \mathbf{v}', \mathbf{H})\,\tau + \mathbf{H}\times\left[\left(\frac{\mathbf{v}\sin\Omega_\alpha\tau}{H^2\Omega_\alpha}\right.\right.$$
$$\left.\left. - \frac{\mathbf{v}'\sin\Omega_{\alpha'}\tau}{H^2\Omega_{\alpha'}}\right)\times\mathbf{H}\right] + \left[\mathbf{v}\,\frac{1-\cos\Omega_\alpha\tau}{H\Omega_\alpha} - \mathbf{v}'\,\frac{1-\cos\Omega_{\alpha'}\tau}{H\Omega_{\alpha'}}\right]\times\mathbf{H}$$
$$+ \int_t^{t+\tau} dt' \int_{t'}^{t'+\tau} dt''\left\{\frac{\mathbf{H}}{H}(\mathbf{HE}(t''))\left(\frac{e_\alpha}{m_\alpha} - \frac{e_{\alpha'}}{m'_\alpha}\right)\right.$$
$$+ \left(\frac{e_\alpha\cos\Omega_\alpha(\tau + t' - t'')}{m_\alpha H^2} - \frac{e_{\alpha'}\cos\Omega_{\alpha'}(\tau + t' - t)}{m_{\alpha'} H^2}\right)$$
$$\times[\mathbf{H}\times[\mathbf{E}(t'')\times\mathbf{H}]] + [\mathbf{E}(t'')\times\mathbf{H}]\left(\frac{e_\alpha}{m_\alpha H}\sin\Omega_\alpha(\tau + t' - t'')\right.$$
$$\left.\left. - \frac{e_{\alpha'}}{m_{\alpha'} H}\sin\Omega_{\alpha'}(\tau + t' - t'')\right)\right\}. \tag{16}$$

It should be noted that the integration over τ in the case of Coulomb interaction must be carried out from τ_{max} to τ_{min}, which corresponds to the cutting off of the impact parameter integral at both high and low values.

6. We use the kinetic equation obtained in Sec. 5 for determining the dissipation of electromagnetic waves brought about by collisions of electrons with ions. In this case it is expedient to carry out the calculation with logarithmic accuracy, keeping in mind the occurrence of a large Coulomb logarithm. In such an approximation,

$$\mathscr{E}_{ij}^{\alpha\alpha'}(\mathbf{v}, \mathbf{v}', \tau) = 2\pi e_\alpha^2 e_{\alpha'}^2 [\delta_{ij}(\mathbf{v}-\mathbf{v}')^2 - (v_i - v_i')(v_j - v_j')] |\tau|^{-1} |\mathbf{v}-\mathbf{v}'|^{-3} \cos\Omega_\alpha \tau. \quad (17)$$

Comparatively simple calculations, carried out in detail in the determination of the dielectric permittivity tensor for frequencies below the Langmuir frequency ω_0, show that we can use in our case for the complex dielectric permittivity tensor the usual expression,[5] in which the dissipation processes are characterized by an effective collision frequency ($\nu_{\text{eff}\,i}$). The only difference is that in the expression for the effective collision frequency between electrons and ions

$$\nu_{\text{eff}\,i} = \frac{4}{3}\sqrt{2\pi}\, e^2 e_i^2 n_i m^{-1/2} (\varkappa T)^{-3/2} L' \quad (18)$$

we have, in place of the usual Coulomb logarithm $L = \ln(\kappa T e^{-2} \rho_{max})$ (here, $\rho_{max} = \sqrt{\kappa T/m}\, \tau_{max}$ is the maximal impact parameter, equal in order of magnitude, to the Debye radius) the following expression:

$$L' = \int_{\tau_{min}}^{\tau_{max}} \frac{d\tau}{\tau} \cos\Omega\tau \cos\omega\tau \qquad \left(\Omega = \frac{eH}{mc}\right). \quad (19)$$

In particular, for $|\omega \pm \Omega| \gg \omega_0$,

$$L' \approx \ln\left(\frac{\varkappa T}{e^2}\sqrt{\frac{\varkappa T}{m|\Omega^2 - \omega^2|}}\right). \quad (20)$$

If the frequency of the electromagnetic field is close to the gyroscopic frequency $|\Omega - \omega| \ll \omega_0$, but is at the same time larger than the Langmuir frequency, the Coulomb logarithm is shown to be equal to

$$L' \approx \ln\left(\frac{\varkappa T}{e^2}\sqrt{\frac{\varkappa T}{m 2\omega\omega_0}}\right). \quad (21)$$

Under the conditions in which $\omega \ll \omega_0$ and $\Omega \ll \omega_0$, Eq. (19) evidently yields the usual Coulomb logarithm.

I take this opportunity to express my thanks to V. L. Ginzburg for his interest in the present research and for useful discussions.

APPENDIX

THE COLLISION INTEGRAL IN A STRONG ELECTRIC FIELD

In a constant electric field, the collision integral (13) takes the form

$$J_\alpha(\mathbf{p}, t) = \sum_{\alpha'} n_{\alpha'} \frac{\partial}{\partial p_i} \int d\mathbf{p}' \int_{-\infty}^{0} d\tau \mathscr{E}_{ij}^{\alpha\alpha'}(\mathbf{v}-\mathbf{v}', \tau) \times \left\{\frac{\partial}{\partial p_j} - \frac{\partial}{\partial p_j'}\right\} f_{\alpha'}(\mathbf{p}' + e_{\alpha'}\mathbf{E}\tau, t+\tau) f_\alpha(\mathbf{p} + e_\alpha \mathbf{E}\tau, t+\tau), \quad (A.1)$$

where, for the case of Coulomb interaction,

$$\mathscr{E}_{ij}^{\alpha\alpha'}(\mathbf{v}, \tau) = 2\pi e_\alpha^2 e_{\alpha'}^2 |\tau|^{-1} (\delta_{ij} V^2 - V_i V_j) V^{-3},$$
$$\mathbf{V} = \mathbf{v} - (e_\alpha / m_\alpha - e_{\alpha'}/m_{\alpha'})\mathbf{E}\tau. \quad (A.2)$$

For slowly changing processes, taking the weakness of the interaction into account, we can transform in (A.1) in the distribution functions f from $t + \tau$ to t (which is completely analogous to what was done in reference 2). Then the collision integral takes the form

$$J_\alpha(p, t) = \sum_{\alpha'} \frac{\partial}{\partial p_i} \int d\mathbf{p}' \left\{ f_{\alpha'}(\mathbf{p}', t) \frac{\partial f_\alpha(\mathbf{p}, t)}{\partial p_j} - f_\alpha(\mathbf{p}, t) \frac{\partial f_{\alpha'}(\mathbf{p}', t)}{\partial p_j'} \right\} \times I_{ij}^{\alpha\alpha'}(\mathbf{v}-\mathbf{v}'), \quad (A.3)$$

$$I_{ij}^{\alpha\alpha'}(\mathbf{v}) = n_{\alpha'} \int_{-\tau_{max}}^{-\tau_{min}} d\tau \mathscr{E}_{ij}^{\alpha\alpha'}(\mathbf{v}, \tau). \quad (A.4)$$

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