CERENKOV RADIATION OF DIPOLES IN A MEDIUM WITH SPATIAL DISPERSION

G. A. BEGIASHVILI and É. V. GEDALIN

Tbilisi State University, Institute of Physics, Academy of Sciences, Georgian S.S.R.

Submitted to JETP editor July 13, 1959; resubmitted February 17, 1960

J. Exptl. Theoret. Phys. (U.S.S.R.) 38, 1738-1739 (June, 1960)

An analysis is made of the Cerenkov radiation of electric dipoles and magnetic dipoles in an isotropic non-gyrotropic medium with spatial dispersion. The Cerenkov radiation of a current loop in such a medium is also considered.

THE Cerenkov radiation of a charged particle in a medium with spatial dispersion has been considered by a number of authors.[1,2] In the present paper we consider the Cerenkov energy losses of particle bunches for particles with dipole moments (electric and magnetic) in an isotropic non-gyrotropic medium with spatial dispersion. It is assumed that the bunch radiates as a point particle with dipole moment corresponding to the entire bunch. We also consider the Cerenkov radiation of a closed current ring in a medium of this kind. These analyses are desirable for investigating the possibility of excitation of new kinds of waves (which arise when spatial dispersion is considered) by means of the Cerenkov effect.[1]

1. In frequency regions which lie close to one of the natural resonant frequencies of the medium, we can use the well-known reciprocal dispersion expansion*

$$\mathbf{E} = (1/\varepsilon_0 + \beta n^2)\,\mathbf{D}. \tag{1}$$

Then, the Cerenkov radiation loss for an electric dipole with arbitrary orientation is given by

$$dF = \frac{\omega^3\,d\omega}{c^2 v}\sum_i \left\{p_z^2 + \frac{p_r^2}{2}\left(\frac{v^2}{c^2}n_i^2 - 1\right)\right\}\left(1 - \frac{c^2}{v^2 n_i^2}\right)|1 + \beta n_i^4|^{-1}, \tag{2}$$

$$n_{1,2}^2 = -1/\varepsilon_0\beta \pm \sqrt{(1/\varepsilon_0\beta)^2 + 1/\beta}. \tag{3}$$

Cerenkov radiation is excited at a frequency ω only when

$$v > c/n_i(\omega) \tag{4}$$

and is distributed over two cones which are defined by the condition

$$\cos\vartheta_i = c/vn_i(\omega) \tag{5}$$

(ϑ_i is the angle between the direction of motion of the dipole† and the radiation direction).

Ginzburg has shown[1] that when $\beta > 0$ one of the roots of Eq. (3) is always smaller than unity and the radiation condition (4) is not satisfied for it. In this case the Cerenkov radiation is distributed over the surface of one (ordinary) cone. When $\beta < 0$ the radiation condition (4) is satisfied for both roots and the radiation is distributed over the surfaces of two cones.

It is of interest to consider the intensity distribution of the Cerenkov radiation over these cones. Let I_1 be the intensity of the radiation for the "ordinary" cone and I_2 the intensity for the "new" cone; then we have

$$\frac{I_2}{I_1} = \frac{(1 - c^2/v^2n_2^2)\{p_z^2 + {}^1/_2 p_r^2 (v^2 n_2^2/c^2 - 1)\}\,|1 + \beta n_1^4|}{(1 - c^2/v^2n_1^2)\{p_z^2 + {}^1/_2 p_r^2 (v^2 n_1^2/c^2 - 1)\}\,|1 + \beta n_2^4|}. \tag{6}$$

When $\epsilon_0^2|\beta| \ll 1$ and $n_2^2 \gg n_1^2$, for a dipole which is oriented along the direction of motion we find $I_2/I_1 \ll 1$, that is, the Cerenkov radiation is almost completely concentrated in the "ordinary" cone. For a dipole which is perpendicular to the direction of motion we have $I_2/I_1 \sim 1$.

2. In considering the Cerenkov radiation of magnetic dipoles we will distinguish between current dipoles and "true" dipoles;[4] "true" magnetic dipoles are dipoles formed from magnetic poles.

For the frequency region in which the expansion in (1) applies, we have for a current magnetic dipoles

$$dF = \frac{\omega^3 d\omega}{c^2 v}\sum_i n_i^2\left\{\mu_r^2\left[\left(1 - \frac{1}{n_i^2}\right)^2 - \frac{1}{2}\left(1 - \frac{c^2}{v^2 n_i^2}\right)\right.\right.$$
$$\left.\left.\times\left(1 - \frac{v^2}{c^2 n_i^2}\right)\right] + \mu_z^2\left(1 - \frac{c^2}{v^2 n_i^2}\right)\right\}|1 + n_i^4\beta|^{-1}, \tag{7}$$

while for a true magnetic dipole

$$dF = \frac{\omega^3 d\omega}{c^2 v}\sum_i \left(1 - \frac{c^2}{v^2 n_i^2}\right)$$
$$\times\, n_i^2\left[\mu_z^2 + \frac{\mu_r^2}{2}\left(n_i^2\frac{v^2}{c^2} - 1\right)\right]|1 + n_i^4\beta^{-1}|. \tag{8}$$

*In the present paper we use the notation of references 1 and 3.

†The direction of motion is taken along the z axis.

The intensity distribution of the Cerenkov radiation over the cones is different for the current and true dipoles. If $\epsilon_0^2 |\beta| \ll 1$ we have $I_2/I_1 \sim 1$ for any orientation of a current dipole. For true magnetic dipoles $I_2/I_1 \sim 1$ for a moment oriented along the direction of motion and $I_2/I_1 \gg 1$ for a moment oriented perpendicularly to the direction of motion. This difference in the radiation distribution over the cones for current and true magnetic dipoles is due to the fact that moving true dipoles are not equivalent to moving current magnetic moments if $\epsilon \neq 1$.[4]

We now give the expression for the Cerenkov radiation loss of a current loop which moves through a medium of this kind where it is assumed that the plane of the current loop is perpendicular to the direction of motion. If (1) applies we have

$$dF = \frac{8\pi^2 a^2 I_0^2 \omega d\omega}{c^2 v} \sum_i I_1^2 \left(a \frac{n_i \omega}{c} \sqrt{1 - \frac{c^2}{v^2 n_i^2}} \right) |1 + n_i^4 \beta|^{-1}, \quad (9)$$

where a is the radius of the loop, I_0 is the current strength in the loop, and I_1 is a Bessel function. It is apparent that for a point current loop ($a \rightarrow 0$) Eq. (9) becomes the formula for the Cerenkov radiation of a magnetic dipole.

In frequency regions of the Cerenkov radiation which are remote from resonance frequencies of the medium it is not necessary to take spatial dispersion into account.

The authors are indebted to N. M. Polievktova-Nikoladze for her interest in this work.

[1] V. L. Ginzburg, JETP **34**, 1593 (1958), Soviet Phys. JETP **7**, 1096 (1958).

[2] Agranovich, Pafomov, and Rukhadze, JETP **36**, 238 (1959), Soviet Phys. JETP **9**, 160 (1959).

[3] B. M. Bolotovskiĭ, Usp. Fiz. Nauk **62**, 201 (1957).

[4] V. L. Ginzburg and V. Ya. Éĭdman, JETP **35**, 1508 (1958), Soviet Phys. JETP **8**, 1055 (1959).

Translated by H. Lashinsky
337