MAGNETIC BREMSSTRAHLUNG OF A CONFINED PLASMA

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Magnetic bremsstrahlung of a high-temperature plasma located in a strong magnetic field is considered.

AN investigation of the magnetic bremsstrahlung of electrons of a high-temperature plasma is of interest in connection with the problem of controllable thermonuclear reactions (energy balance in a thermonuclear reaction, microwave diagnostics of plasma).

In the present paper we consider magnetic bremsstrahlung from a confined plasma under conditions where the magnetic pressure p_H is considerably greater than the pressure of the electron gas p_e. The condition $p_H \gg p_e + p_i$ (p_i is the pressure of the ion gas) is necessary to create equilibrium plasma configurations and to ensure their stability (plasma ring stabilized by a strong magnetic field,[1] stellarator,[2] etc.).

A plasma electron in a magnetic field **H**, moving in a helix along a magnetic force line, radiates electromagnetic waves of frequency

$$\omega = s\omega_H/(1 - v_\parallel n_j \cos\theta/c), \quad s = 1, 2, \ldots, \tag{1}$$

where $\omega_H = eH/mc$ is the gyrofrequency for the electron; e, m and $v_\parallel$ are the charge, mass, and projection of the electron velocity **v** on the direction of the magnetic field; n_j are the refractive indices of the two normal waves (ordinary and extraordinary) that can propagate in the plasma; and θ is the angle between the direction of propagation of the wave and the magnetic field. The n_j are defined by the well-known expressions

$$n_{1,2}^2 = (-B \pm \sqrt{B^2 - 4AC})/2A, \quad A = 1 - u - v + uv\cos^2\theta,$$
$$B = (2 - v)u - 2(1 - v)^2 - uv\cos^2\theta,$$
$$C = (1 - v)[(1 - v)^2 - u], \tag{2}$$

where $v = \Omega^2/\omega^2$, $u = \omega_H^2/\omega^2$, $\Omega = (4\pi e^2 n_0/m)^{1/2}$ is the Langmuir frequency for the electrons, and n_0 is the number of electrons per unit volume.

We assume that $\beta = v_t/c = (T_e/mc^2)^{1/2} \ll 1$, where T_e is the temperature of the electron gas. In addition, we assume that the electromagnetic waves corresponding to the first harmonics of (1), for which $n_j^2 > 0$, can propagate in the plasma. This takes place when $\Omega/\omega \lesssim 1$. In this case $n_j \sim 1$, so that $\omega \approx s\omega_H$ in (1). Thus, the total radiation per unit volume of plasma is found to consist of individual lines near the frequencies ω_H, $2\omega_H$, ..., which are Doppler-broadened with a half-width $\Delta\omega \sim s\omega_H\beta\cos\theta$. For large values of s ($s \gtrsim 1/\beta$) the neighboring harmonics coalesce, and when $\omega \gg \omega_H/\beta$ the radiation spectrum becomes continuous. Our analysis pertains only to the case of non-overlapping harmonics. The case when the major fraction of the radiation intensity of the confined plasma occurs at the frequencies ω corresponding to overlapping harmonics was considered by Trubnikov[3,4] (the case of large plasma dimensions and high temperatures). He considered radiation in the direction $\theta = \pi/2$, when there is no ordinary (non-relativistic) Doppler effect and relativistic effects must be taken into account.

We note that inhomogeneities in the magnetic field lead to additional line broadening. We shall assume that the inhomogeneities in the magnetic field are sufficiently small, $\Delta H/H \lesssim \beta$, so that the magnetic field can be considered homogeneous.

Electromagnetic waves with frequencies close to $s\omega_H$ are strongly absorbed in a high-temperature plasma[5] (cyclotron absorption, due to the presence of thermal motion of the plasma electrons). The distance in which the intensity of the electric field of a plane electromagnetic wave of frequency $\omega \approx s\omega_H$ is reduced by a factor e = 2.7 ... is

$$l_s = \lambdabar/\varkappa_s, \quad \lambdabar = c/s\omega_H, \tag{3}$$

where κ_s is the imaginary part of the complex index of refraction (damping factor).

When s = 2, 3, ... and $s^2\beta^2 n_j^2 \sin^2\theta \ll 1$, the propagation of an electromagnetic wave in a plasma is characterized by the dielectric-permittivity tensor

$$(\varepsilon_{ij}) = \begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} & 0 \\ -\varepsilon_{12} & \varepsilon_{11} & 0 \\ 0 & 0 & \varepsilon_{33} \end{pmatrix}, \qquad \varepsilon_{ij} = \varepsilon^{(0)} + \varepsilon_{ij}^{(1)}, \tag{4}$$

where $\epsilon_{ij}^{(0)}$ are certain konwn expressions for ϵ_{ij}, obtained without allowance for the thermal motion of the electrons. The small additions $\epsilon_{ij}^{(1)}$, which cause the cyclotron absorption of the waves, are

$$\varepsilon_{11}^{(1)} = i\varepsilon_{12}^{(1)} = 2i\sigma_s, \qquad \varepsilon_{33}^{(1)} \approx 0;$$

$$\sigma_s = \frac{\sqrt{\pi}\, s^{2s-2} \sin^{2s-2}\theta\, \Omega^2}{2^{s+1/2} s! \cos\theta\, \omega_H^2} (\beta n)^{2s-3} e^{-z_s^2}, \qquad z_s = \frac{1 - s\omega_H/\omega}{\sqrt{2}\beta n \cos\theta}.$$

If the frequency ω is determined by expression (1), then $z_s = v_{\parallel}/\sqrt{2}v_t$. Taking (4) into account, we readily find

$$\begin{aligned} \varkappa_s &= \sigma_s n_j (1-u)(2C + Bn_j^2)^{-1} \\ &\times \Big\{ n_j^4 \sin^2\theta - (1-v)(1+\cos^2\theta) n_j^2 \\ &+ 2\Big(1 - \frac{v}{1-u} + \frac{v\sqrt{u}}{1-u}\Big)(1 - v - n_j^2 \sin^2\theta)\Big\}. \end{aligned} \tag{5}$$

We must put $n = n_j$ in the right half of (5). In order of magnitude,

$$l_s = a_s \lambda\!\!\!^{-} (\omega/\Omega)^2 \beta^{-(2s-3)} e^{-z_s^2}, \tag{6}$$

where, for the first harmonics, $a_s \sim 1$.

When $s = 1$, the tensor ϵ_{ij} is

$$\begin{aligned} &\varepsilon_{11} = i\sigma + 1 - v/4, \quad \varepsilon_{12} = \sigma - iv/4 - \alpha, \\ &\varepsilon_{22} = i\sigma + 1 - v/4 - 2i\alpha, \\ &\varepsilon_{33} = 1 - v + \varepsilon_{33}', \quad \varepsilon_{33}' = \frac{v \sin^2\theta}{\sqrt{2}\cos\theta} \beta n z_1 (1 + i\sqrt{\pi} z_1 w(z_1)), \\ &\varepsilon_{13} = \varepsilon_{31} = -i\varepsilon_{23} = i\varepsilon_{32} = \frac{v \sin\theta}{2\cos\theta}(1 + i\sqrt{\pi} z_1 w(z_1)), \\ &\sigma = \sqrt{\frac{\pi}{8}} \frac{v w_1(z_1)}{\beta n \cos\theta}, \quad \alpha = \beta^2 \sigma n^2 \sin^2\theta, \\ &w(z) = e^{-z^2}\Big(1 + \frac{2i}{\sqrt{\pi}} \int_0^z e^{t^2} dt\Big). \end{aligned} \tag{7}$$

The damping factor κ_1 can be determined in two limiting cases. If $v|w(z_1)|/\beta n_j \cos\theta \gg 1$ and $\theta^3 \gg \beta$, then*

$$\begin{aligned} \varkappa_1 &= \mathrm{Re} \sqrt{\frac{2}{\pi}} \frac{\beta\cos\theta}{v w(z_1)} Q (2n^2 \sin^2\theta - 2 + 2v - \sin^2\theta)^{-1}, \\ Q &= n^4 \Big[1 - v\Big(\cos^2\theta + \frac{1}{4}\sin^2\theta\Big) + 2\varepsilon_{13}\cos\theta\sin\theta\Big] \\ &+ n^2 [\varepsilon_{13}(v-2)\cos\theta\sin\theta + (1+\cos^2\theta)(\varepsilon_{13}^2 - (1-v) \\ &\times (1 - v/4) - i\sigma\varepsilon_{33}') - \sin^2\theta (1 - v/2)] \\ &+ i\sigma\varepsilon_{33}'(2-v) + (1-v)(1-v/2) + \varepsilon_{13}^2 (v-2). \end{aligned} \tag{8}$$

In order of magnitude, $l_1 \sim l_1 \sim (\lambda\!\!\!^{-}/\beta) \exp(v_{\parallel}^2/2v_T^2)$. The refractive index n_j is determined in this case by Eq. (2), in which we must put $u = 1$. If $\theta^3 \lesssim \beta$, then the electromagnetic wave of frequency $\omega \approx \omega_H$ is strongly damped: $n \sim \kappa \sim \beta^{-1/2}$ when $|1 - \omega_H/\omega| \lesssim \beta^{2/3}$.

If $v \ll 1$ and $v|w(z_1)|/\beta \cos\theta \ll 1$ (low-density plasma), we have $n_j = 1$ and

$$\varkappa_1 = \sqrt{\frac{\pi}{2}} \frac{v \exp(-z_1^2)}{4\beta\cos\theta} (1 + \cos^2\theta). \tag{9}$$

With this

$$l_1 \sim \beta \lambda\!\!\!^{-} (\omega^2/\Omega^2) \exp(v_{\parallel}^2/2v_T^2).$$

Using Kirchhoff's law and expressions (5), (8), and (9) for the absorption coefficients, we can determine the radiating ability of the plasma, after which one can readily estimate the energy of the magnetic bremsstrahlung from a confined plasma. A clearer estimate is obtained, however, in the following manner. The radiation intensity of an individual plasma electron, for the harmonics $s = 2, 3, \ldots,$ is[6]

$$w_s = b_s(\theta) \frac{s^4 n_j^3}{(s!)^2 2^{2s}} w \Big(\frac{v_\perp}{c} s n_j \sin\theta\Big)^{2s-2}, \tag{10}$$

where $b_s(\theta) \sim 1$, $v_\perp$ is the electron-velocity component perpendicular to **H**, and $w = 2e^2\omega_H^2 v_\perp^2/3c^2$ is the intensity of radiation of the electron in vacuum.

Since the radiation of an electron of velocity v is absorbed at a distance $l_s = l_s(v_{\parallel})$, the contribution of the s-th harmonic to the radiation intensity will be produced by electrons in the layer $l_s < L$, where L is the plasma dimension. We shall assume that when $v_{\parallel} \gg 2v_t$ we have $l_s \gg L$. The number of such electrons is exponentially small, and their contribution to the overall intensity of the s-th harmonic can be disregarded. Therefore the overall intensity of the s-th harmonic from a unit plasma surface will be on the order of $W_s \sim \overline{w_s l_s} n_0$, where $\overline{w_s l_s}$ is the value of $w_s l_s$ averaged over the Maxwellian distribution. The integration over $v_{\parallel}$ is carried out here up to the velocity v_{lim}, for which $l_s(v_{\mathrm{lim}}) \sim L$, i.e., $v_{\mathrm{lim}}^2 \sim 2v_t^2 \ln (L\Omega^2\beta^{2s-3}/\lambda\!\!\!^{-}\omega^2 a_s)$; v_{lim} is of order of several times v_t. It is precisely the value of v_{lim} which determines the line width of the radiation emitted from the plasma: $\Delta\omega \sim s\omega_H v_{\mathrm{lim}} c^{-1} \cos\theta$. When $z_s = 0$ ($\omega = s\omega_H$) the line intensity reaches a maximum corresponding to the radiation from the surface of an absolutely black body. Thus

$$W_s = q_s \frac{m\beta^3 s^3 \omega_H^3}{4\pi^3} \frac{v_{\mathrm{lim}}}{v_T} \sim \frac{\omega^2 T_e}{8\pi^3 c^2} \Delta\omega, \tag{11}$$

where $q_s \sim 1$.

*The expression for κ_1, derived in reference 5 for $|z_1| \ll 1$, does not include terms proportional to ε_{13}.

The radiation intensity of the electron at the first harmonic is determined by the following expressions

$$w_1 = \tfrac{2}{3} w (1 + \cos^2\theta) \text{ for } v\,|w(z_1)|/\beta\cos\theta \leqslant 1, \qquad (12)$$

$$w_1 \sim w\beta^2 \text{ for } v\,|w(z_1)|/\beta n_j \cos\theta \gg 1. \qquad (13)$$

The result (13) was obtained by Ginzburg and Zheleznyakov[7] (see also reference 8). However, since the expression used in reference 7 for the tensor ϵ_{ij} does not take the thermal motion of the electrons into account, the results of that paper pertain only to electrons with velocities $v_{\|} \gg \sqrt{2}v_t$. Taking (12), (13) and (8), (9) into account, we obtain also expression (11), from which it follows that the intensity of radiation from a unit plasma surface depends very little (logarithmically) on the density, and is proportional to $T^{3/2}$ and H^3.

Let, for example, $n_0 \sim 10^{13}\text{cm}^{-3}$, $H \sim 10^4$ Gauss, $T \sim 5$ kev, and $L \sim 10^2$ cm. In this case $\Omega/\omega_H \sim 1$, $\beta \sim 0.1$, $\lambda\!\!\!^{-} \sim 0.1$ cm, $l_1 \sim l_2 \sim 1$ cm, $l_3 \sim 10^2$ cm, and $W_{1-3} \sim 10^2 - 10^4$ erg-sec^{-1} cm^{-2}. The energy released in a nuclear reaction in a DT mixture is in this case $W_{\text{nuc}} \sim 10^3$ erg-sec^{-1} cm^{-3}. Thus, with the reactor size L ranging from 10 to 100 cm, the energy released in the nuclear reaction is greater than the energy carried away by the magnetic bremsstrahlung.

The radiation intensities corresponding to different harmonics depend on n_0 to an equal degree (if the logarithmic dependence is disregarded). This is due to the fact that the drop in the intensity of radiation from a single electron due to increasing s is offset by the increase in l_s due to the increase in the number of electrons radiated to the outside. Expressions (11) are valid up to $s = s^*$, where $l_{s^*} \sim L$. For $s \geq s^* + 1$ we have $l_s \gg L$, and instead of (11) we obtain

$$W_s \sim \overline{w}_s L n_0. \qquad (14)$$

It is obvious that $W_s \sim W_{s^*}\beta^{2(s-s^*)} \ll W_{s^*}$ for $s > s^*$.

The presence of density inhomogeneities, which are particularly large on the boundary between the plasma and the vacuum, will cause the rays to be reflected and curved. In addition, for s such that $l_s \sim L$, the value of W_s will depend on the plasma configuration. An accounting of these circumstances is essential for an accurate determination of the coefficients q_s.

Expression (11) and the estimate given show that even in strong magnetic fields the energy lost to magnetic bremsstrahlung is not very large in thermonuclear reactors using the DT reaction. This is explained primarily by the fact that when $v|w(z_1)|/\beta n \cos\theta \gg 1$ the intensity of radiation of the first harmonic in the plasma decreases by a factor $1/\beta^2 \gg 1$ compared with the intensity of radiation in vacuum. Secondly, the radiation of the first harmonics is strongly absorbed in the plasma.

In conclusion, we consider the question of the intensity of thermal radiation near resonant frequencies* $\omega \approx \omega_{1,2}$, where

$$\omega_{1,2}^2 = \tfrac{1}{2}(\Omega^2 + \omega_H^2) \pm \tfrac{1}{2}[(\Omega^2 + \omega_H^2) - 4\Omega^2\omega_H^2\cos^2\theta]^{1/2}.$$

When $\omega \to \omega_{1,2}$ the coefficient $A = (1 - \omega_1^2/\omega^2) \times (1 - \omega_2^2/\omega^2)$ tends to zero, and one of the refractive indices (2) tends to infinity:

$$n^2 = -B/A \gg 1. \qquad (15)$$

However, (15) can be used only when $1 \gg |A|^2 \gg \beta^2$. In the general case $|A| \ll 1$ and the value of n^2 is (see reference 5 and also reference 9)

$$n^2 = [-A \pm \sqrt{A^2 - 4\beta^2 A_1 B}]/2\beta^2 A_1 B \gg 1,$$

$$A_1 = -3v\left[\cos^4\theta(1-u) + \frac{6-3u+u^2}{(1-u)^2}\cos^2\theta\sin^2\theta + \frac{\sin^4\theta}{1-4u}\right]. \qquad (16)$$

When $1 \gg |A|^2 \gg \beta^2$, one of the refractive indices (16) turns into (15). The second refractive index describes the plasma wave. The damping factor is

$$\varkappa = -n(P+R)(4\beta^2 A_1 n^2 + 2A)^{-1}(1-u), \qquad (17)$$

where

$$P = \sqrt{\pi} v z_0 \sum_{s=-\infty}^{\infty} \frac{\mu^{|s|}}{2^{|s|}|s|!} e^{-z_s^2}\left(\frac{s^2\sin^2\theta}{\mu} + 2z_s^2\cos^2\theta\right), \qquad (18)$$

$$R = \nu(1 + 2uv(1-u)^{-2}\sin^2\theta), \qquad (19)$$

where $\mu = \beta^2 n^2 u^{-1}\sin^2\theta \ll 1$, and ν is the electron-ion collision frequency. The individual terms of (18) are in general of different order of magnitude, and the largest of these should be taken in the sum (18). In the derivation of (16) and (17) it is assumed that $\kappa \ll n$ and $\beta^2 n^2 \ll 1$.

The direction of beam propagation, i.e., the direction of the energy flux transferred by the plane wave with wave vector **k**, is not the same as **k** in an anisotropic medium. If the angle between **H** and **k** is θ, then the angle ξ between the beam and **H** is determined from the following equation[10]

$$\frac{1}{n(\theta)}\frac{\partial n(\theta)}{\partial\theta} = \tan(\xi - \theta).$$

The absorbing ability of the medium (the damping

*This question was considered by Ginzburg and Zheleznyakov.[7]

coefficient of the energy flux along the beam) is

$$\alpha(\xi, \omega) = 2\omega c^{-1}\varkappa(\theta, \omega)\cos(\theta - \xi).$$

In the preceding estimate of the energy flux we disregarded the difference between α and $2\omega\kappa/c$, for this difference is small when ω is not close to $\omega_{1,2}$. However, when $\omega \approx \omega_{1,2}$, the quantity $n^{-1}dn/d\theta$ is very large, so that the difference $|\theta - \xi|$ is close to $\pi/2$

$$\frac{1}{n}\frac{\partial n}{\partial \theta} \approx \frac{uv\cos\theta\sin\theta}{A + 2\beta^2 A_1 n^2} = \tan(\theta - \xi) \approx \frac{1}{\cos(\theta - \xi)} \gg 1.$$

Therefore α is much less than $2\omega\kappa/c$:

$$\alpha(\xi, \omega) = 2\varkappa\frac{\omega}{c}\frac{A + 2\beta^2 A_1 n^2}{uv\cos\theta\sin\theta}. \qquad (20)$$

If the dimensions of the plasma are sufficiently large, then the plasma radiates like a "black" body when $\omega = \omega_{1,2}$. The width of the radiation spectrum can be estimated from the condition $\alpha(\xi, \omega_{1,2} + \Delta\omega) L = 1$ (it is assumed that the plasma is transparent when ω is not close to $\omega_{1,2}$ or to $s\omega_H$, i.e., $\alpha(\omega) L \ll 1$). The intensity of radiation will be $W(\xi) = I_0|\Delta\omega|$, where $I_0 = \omega^2 T_e / 8\pi^3 c^2$. The question of the emergence of the energy of the plasma wave from the plasma calls for a separate consideration.

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