INVESTIGATION OF THE SPECTRA OF NEUTRONS EMITTED IN THE DISINTEGRATION OF DEUTERONS BY PROTONS

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We calculate the energy spectra of neutrons emitted at 0 and 180 deg in the c.m.s. from the reaction $p + d \rightarrow p + p' + n$, at an approximate total reaction energy of 4 Mev, with pair interaction of the nucleons in the final state taken into account. Applications of this method to the calculation of the energy distributions of reaction products when several particles are emitted are considered from the standpoint of clarifying the role of interactions of particles in final states.

TWO maxima, one at the upper boundary of the spectrum and one at a neutron energy ~ 0.7, have been observed[1,2] in the spectrum of neutrons from the reaction $p + d \rightarrow p + p' + n$, emitted at zero degrees to the direction of incident 8.9-Mev protons. These maxima can be attributed to the interaction between the nucleons produced in the final state.[3] This spectrum of neutrons from the reaction $p + d \rightarrow p + p' + n$ was investigated theoretically by Heckrotte and McGregor,[4] but only with allowance for the interaction between two protons in the final state, and in the approximation of a zero proton interaction radius, so that the form of the neutron spectrum could not be fully explained.

In the present paper we calculate the energy distributions from the reactions $(p + d)$ emitted at 0 and 180 deg to the direction of the incident protons in the c.m.s., for a total reaction energy of approximately 4 Mev. It is assumed in the calculation that two out of three nucleons produced in the reaction interact with each other. Thus, the three-body problem is reduced to a two-body problem, that of a virtual biproton and a deuteron, which symbolize pairs of nucleons interacting in the final state. (The use of the terms "virtual particle" does not imply their existence.) The interaction between nucleons was taken into account accurately, and its parameters were taken from the experimental data on nucleon-nucleon scattering. The Born approximation was used to account for the interaction between the third nucleon and the virtual particle.

It must be noted that, strictly speaking, the use of the Born approximation is not justified at such low energies. However, this approximation has been used successfully in that energy region for the investigation of the angle and energy distributions in stripping which is quite similar to the process investigated here.[5] One can therefore expect the use of the Born approximation for the interaction of a pair of particles with small relative momenta and a third particle to yield in this case angle and energy distributions that are close to the real ones.

The orbital momentum of the relative motion of the proton and deuteron in the investigated disintegration of a deuteron by protons was assumed to be zero, for a total reaction energy of 4 Mev. The orbital momentum of the relative motion of the nucleons produced by the reaction, joined in a virtual biproton and deuteron, is also assumed to be zero. Then, by virtue of the conservation of the total momentum of the system and of the exclusion principle, the total momentum of the system can be either $J = 1/2$ or $J = 3/2$, and the spin of the virtual particles can be either $S = 0$ or $S = 1$. Consequently, the differential cross section of the disintegration of the deuteron by protons should be determined by the expression

$$\frac{d\sigma}{dE_n d\Omega_n} = \frac{M_n M_0}{(2\pi\hbar^2)^2} \frac{\rho(E_n)}{k_0} \left\{\frac{2}{3}|H_1|^2 + \frac{1}{3}|H_2|^2 + \frac{1}{3}|H_3|^2\right\}, \quad (1)$$

where M_0 and M_n are the reduced masses of the incident proton and of the neutron produced by the reaction, $\rho(E_n)$ is the density of the neutron energy states, H_1, H_2, and H_3 are the matrix elements of the transition, corresponding to $J = 3/2$, $s = 1$; $j = 1/2$, $s = 1$; and $j = 1/2$, $s = 0$.

The general form of the matrix element of the transition is

$$H = \langle (1 - P_{13}) \psi_f \chi_f | V_{pp} + V_{pn} | \psi_i \chi_i \rangle, \quad (2)$$

where $(1 - P_{13})$ is the particle rearrangement operator. Expansion of the expression for H leads to six integrals, of which two correspond to the formation of a virtual biproton (I_{pp}) and four to the formation of a virtual deuteron (I_{pn}). Since the spin state of the two interacting protons can be only singlet in this case, while the spin states of the interacting proton and neutron can be singlet and triplet, the matrix elements H_1 and H_2 include integrals I^t_{pn} corresponding to a triplet interaction of a neutron and a proton in the final state, while H_3 can contain integrals I^s_{pp} and I^s_{pn}, corresponding to the formation of virtual particles in the singlet state.

The integral I_{pp} is written in the form

$$I_{pp} \sim \int \varphi_{2p}(\rho)\chi_{2p}\varphi(\mathbf{r}_n)\chi_n(V_{pp}+V_{pn})\psi_d\chi_d\psi(\mathbf{r}_p)\chi_p d\tau, \quad (3)$$

where ψ_d is the internal wave function of the deuteron, taken in the form of a Hulthen function; $\psi(\mathbf{r}_p)$ and $\varphi(\mathbf{r}_n)$ are the wave functions of the incoming proton and the emitted neutron, taken in the form of plane waves; χ_d and χ_p are the spin functions of the deuteron and the incident protons; χ_{2p} and χ_n are the spin functions of two protons in the singlet state and a neutron; $\varphi_{2p}(\rho)$ is the wave function of the relative motion of two protons, taken with allowance for their scattering by each other in the singlet states.

If the nuclear interaction of the two protons is chosen in the form of a potential well with $\rho_0 = 2.65 \times 10^{-13}$ cm, the radial part of the wave function $\varphi_{2p}(\rho)$ assumes the following form for $\rho < \rho_0$:

$$\varphi^{(1)}_{2p}(\rho) = A\rho^{-1}\sin k\rho,$$

where $\mathbf{k}$ is the wave vector of the relative motion of two protons within the potential well. For $\rho > \rho_0$, account was taken of the Coulomb interaction of the two protons, and

$$\varphi^{(2)}_{2p}(\rho) = G(f\rho) + \cot\delta\, F(f\rho),$$

where $G(f\rho)$ and $F(f\rho)$ are Coulomb functions, and cot δ is a factor that takes nuclear interaction into account. The coefficient A and the absolute value of the wave value $\mathbf{k}$ are determined by making the logarithmic derivatives of the functions $\varphi^{(1)}_{2p}(\rho)$ and $\varphi^{(2)}_{2p}(\rho)$ continuous at $\rho = \rho_0$. Variation of the parameter ρ_0 from 2.4×10^{-13} to 2.8×10^{-13}, values known from experiments on proton-proton scattering, does not change substantially the magnitude and form of the differential cross section of the reaction. In Eq. (3) V_{pp} and V_{pn} are the potentials of interaction between the incoming proton and the nucleons of the deuteron, V_{pn} was taken in the form of a δ function, while V_{pp} was chosen in the form of a potential well of radius $\rho_0 = 2.65 \times 10^{-13}$ cm. It was impossible to take V_{pp} in the form of a δ function in this case, since $\varphi_{2p}(\rho)$ depends on the same parameter ρ. Since we deal with S waves only in this analysis, the choice of the type of interaction, V_{pp} or V_{pn}, is immaterial, for it is determined only by the one parameter — the scattering length.

For the case of interaction of a neutron and proton in the singlet or triplet final state, i.e., in the case of production of a virtual deuteron, the integrals I_{pn} have the form

$$I_{pn} \sim \int \varphi_{pn}(\rho)\chi_{pn}\varphi(\mathbf{r}_{p'})\chi_{p'}(V_{pp}+V_{pn})\psi_d\chi_d\psi(\mathbf{r}_p)\chi_p d\tau, \quad (4)$$

where $\varphi_{pn}(\rho)$ is the wave function of the relative motion of a proton and neutron, taking into account scattering in the triplet or singlet states.

If the interaction between the proton and neutron is chosen in the form of a potential well with $\rho_0 = 2.65 \times 10^{-13}$ cm, the radial part of $\varphi_{pn}(\rho)$ has the form

$$\varphi^{(1)}_{pn}(\rho) = A'\rho^{-1}\sin k'\rho, \qquad \rho < \rho_0;$$

$$\varphi^{(2)}_{pn}(\rho) = 2\sqrt{\pi}(f\rho)^{-1}\sin f\rho + 4\pi f(\theta)\rho^{-1}e^{if\rho}, \qquad \rho > \rho_0,$$

where $f(\theta)$ is the amplitude of scattering of a proton by a neutron in the singlet or triplet states;[6] A and k' are determined by making the logarithmic derivatives of the functions $\varphi^{(1)}_{pn}(\rho)$ and $\varphi^{(2)}_{pn}(\rho)$ continuous at $\rho = \rho_0$. The potential V_{pp} in the integrals I_{pn} was chosen in the form of a δ function, while V_{pn} was chosen either in the form of a δ function for $\varphi_{pn}(\rho)$ independent of the coordinates of the incident proton, or in the form of a potential well with $\rho_0 = 2.65 \times 10^{-13}$ cm for $\varphi_{pn}(\rho)$ dependent on the coordinates of the incident proton.

Calculation has shown that the integrals I^t_{pn}, which take into account the triplet interaction of the neutron with the proton in the final state, are much smaller than the integrals I_{pn} corresponding to the interaction in the singlet state. Consequently, the differential cross section of the reaction should be determined by the square of the matrix element H_3 and can be written in the form $d\sigma = d\sigma_{pn} + d\sigma_{pp} + d\sigma_{interf}$.

Figure 1 shows the experimental c.m.s. spectrum of the neutrons from the reaction $p + d \rightarrow p + p' + n$, emitted at zero degrees to the direction of the incident 8.9-Mev protons.[1,2] Figure 2 shows the experimental c.m.s. spectrum of the neutrons from the reaction $d + p \rightarrow p + p' + n$, emitted at zero degrees to the direction of the incident 18.6-Mev deuterons.[2] This last spectrum can be considered as a spectrum of neutrons emitted at 180 deg to the direction of the incident protons from the reaction $p + d \rightarrow p + p' + n$, since the total energy of either reaction is of order 4 Mev. The figures show also

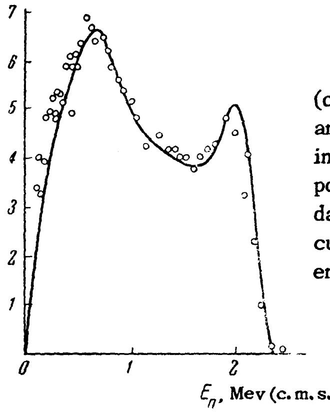

FIG. 1. Spectrum of neutrons (c.m.s.) produced at a zero angle in the (p + d) reaction by incoming 8.9-Mev protons. The points are the experimental data of reference 1, and the curve is calculated in the present work.

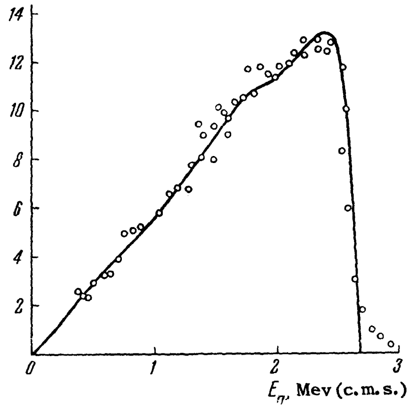

FIG. 2. Spectrum of neutrons (c.m.s.) produced at a zero angle in the (d + p) reaction by incoming 18.6-Mev deuterons. The points are the experimental data of reference 2, and the curve is calculated in the present work.

the neutron energy spectra, calculated in the present paper, for neutrons from the reaction p + d ⟶ p + p′ + n, emitted at 0 and 180 deg to the direction of the incoming protons, for the corresponding total reaction energies.

The calculated spectrum of the neutrons emitted at zero degrees has two maxima, like the experimental one. The position and shape of the maximum at 0.7 Mev neutron energy is determined essentially by the contribution of $d\sigma_{pn}$ to $d\sigma$, i.e., by the interaction between the neutron and the proton in the final singlet state. The maximum at the upper boundary of the neutron spectrum is determined by the contribution of $d\sigma_{pp}$ to $d\sigma$, i.e., by the interaction of two protons in the final singlet state. The maximum at $E_n \approx 0.7$ Mev is considerably broader and higher than the maximum at the upper boundary of the neutron spectrum. This may be due to the high probability of pickup of a neutron (deuteron) by the incoming proton and their emission forward.

The spectrum of the neutrons emitted at 180 deg has one maximum at the upper boundary. The form of this spectrum, as shown by calculation, is determined completely by $d\sigma_{pp}$, i.e., by the production of a virtual biproton in the singlet state, emitted forward.

The ratio of the areas under the experimental points is

$$\frac{d\sigma}{d\Omega}(180°)\Big/\frac{d\sigma}{d\Omega}(0°) = 1{,}8 \pm 0.3,$$

while the same ratio under the corresponding theoretical curves is 1.75. This is evidence that the angular distributions of the products of reactions accompanied by emission of several particles can be calculated by the foregoing method.

Because of isotopic invariance, one can expect the c.m.s. spectrum of protons from the reaction n + d ⟶ p + n + n′, emitted at zero degrees to the direction of the incident protons of energy ~10 Mev to have two maxima, like the corresponding neutron spectrum from the reaction p + d ⟶ p + p′ + n. The first maximum, at a proton energy of approximately 0.7 Mev, should correspond to the interaction between the neutron and proton in the final singlet state, while the second, that at the upper boundary of the spectrum, should correspond to the interaction between two neutrons in the final singlet state. However, great interest attaches to an experimental investigation of the spectrum of protons from the reaction n + d ⟶ n + n′ + p, emitted at zero degrees, since its theoretical analysis by the method given above yields the neutron-neutron scattering length.

The method developed in the present paper and in references 3 and 7 can be used to analyze the spectra of the products of the reactions (d + d), (d + T), (d + He^4), (T + T), and similar reactions, in order to explain the role of interaction of the products of these reactions in the final state and the possibility of the existence of excited states of He^4, He^5, and Li^5.

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