THE RELATIVISTICALLY COVARIANT SPIN STRUCTURE OF THE S MATRIX

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The relativistically covariant spin structure of the S matrix is examined for the reactions $a + b \rightarrow a' + b'$, $\gamma + b \rightarrow a' + b'$, $\gamma + b \rightarrow \gamma' + b'$, and $a + b \rightarrow 2\gamma$. The S matrix is represented in a form in which its individual terms correspond to transitions between states with definite initial and final spins of the system.

1. THE KINEMATICAL VARIABLES

Let us consider a reaction of the type $a + b \rightarrow a' + b'$. The state of a system of two particles a, b is completely determined by the three-dimensional momenta **p**, **q** and spin projections μ, ν of these particles. Instead of the momenta **p**, **q** (six independent variables) we can introduce the four-momenta* **p**, **q**, remembering the condition that their lengths are equal to the masses of the corresponding particles:

$$|p| = m_a, \qquad |q| = m_b. \tag{1}$$

Finally, instead of p, q we can use the four-momentum t of the system

$$t = p + q \tag{2}$$

and the four-momentum k

$$k = \tfrac{1}{2}(p - q) - \tfrac{1}{2}(p + q)(p^2 - q^2)/(p + q)^2, \tag{3}$$

which satisfy the invariant conditions

$$k^2 = [p^2q^2 - \tfrac{1}{4}(t^2 - p^2 - q^2)^2]t^{-2}, \qquad (tk) = 0 \tag{4}$$

and which, like the set of variables p and q, contain six independent variables. We note that the vector t is timelike and the vector k is spacelike; in the center-of-mass system (c.m.s.) t reduces to the total energy of the system and k to the relative momentum,

$$t_c = (0,\ i(p_{0c} + q_{0c})), \qquad k_c = (\mathbf{k}_c, 0). \tag{5}$$

In what follows we shall describe a system of particles a, b by the spin projections μ, ν and by the momenta p, q satisfying the conditions (1) or the momenta t, k satisfying the conditions (4). We shall describe the system a′, b′ by analogous quantities with a prime. If a particle is a photon, we shall describe its spin state not by the spin projection, but by the polarization four-vector e_μ. In virtue of gauge invariance we can require that the vector potential A_μ satisfy the Lorentz gauge condition $\partial A_\mu/\partial x_\mu = 0$, and choose its fourth component equal to zero in the c.m.s. Then the polarization vector e_μ will satisfy the invariant conditions

$$(ep) = (et) = 0, \tag{6}$$

where p is the four-momentum of the photon, and consequently also the condition

$$(ek) = 0. \tag{7}$$

In the c.m.s. the spacelike vector e_μ reduces to the three-dimensional vector **e** satisfying the condition $\mathbf{e}\cdot\mathbf{k} = 0$. It is also convenient to introduce a spacelike four-vector s_μ:

$$s_\mu = -i\varepsilon_{\mu\nu\lambda\sigma}k_\nu e_\lambda t^0_\sigma, \qquad t^0_\sigma = t_\sigma|t|^{-1}, \tag{8}$$

which satisfies the conditions

$$(se) = (sk) = (st) = (sp) = 0 \tag{9}$$

and reduces in the c.m.s. to the vector $\mathbf{s} = \mathbf{k} \times \mathbf{e}$.

Choosing the vectors t, k and t′, k′ to describe the reaction $a + b \rightarrow a' + b'$ and recalling the energy-momentum conservation law $t = t'$

$$t = p + q = p' + q', \tag{10}$$

we see without difficulty that we can describe the reaction $a + b \rightarrow a' + b'$ by only three independent four-vectors k, k′, t, which in virtue of the relations (4) contain only eight independent variables. Since the conditions (4) are of invariant character, we can construct from the vectors k, k′, t only two independent scalars*

*The following notation is used:
$a = (\mathbf{a}, a_4)$, $a_4 = ia_0$, $(ab) = a_\mu b_\mu = \mathbf{ab} + a_4b_4$,
$\hat{a} = \gamma_\mu a_\mu$, $\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\delta_{\mu\nu}$, $\gamma_\mu^+ = \gamma_\mu$, $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$.

*In the general case one can construct from n independent vectors $n(n + 1)/2$ independent scalars; but if m invariant conditions are imposed on the n vectors, then the number of independent scalars is $n(n + 1)/2 - m$.

$$t^2, \ (k'k). \tag{11}$$

These scalars have good physical meanings: $\epsilon = (-t^2)^{1/2}$ is the total energy of the system in the c.m.s. and $x = (kk')/|k||k'|$ is equal to the cosine of the scattering angle in the c.m.s.

From the vectors k, k′, t we can construct a spacelike pseudovector n_μ:

$$n_\mu = -i\varepsilon_{\mu\nu\lambda\sigma} k'_\nu k_\lambda t^0_\sigma, \tag{12}$$

which is orthogonal to the vectors k, k′, t:

$$(nk) = (nk') = (nt) = 0, \tag{13}$$

and consequently also to p, q, p′, q′, and reduces in the c.m.s. to the vector normal to the plane of the scattering, $\mathbf{n} = \mathbf{k} \times \mathbf{k}'$.

2. THE INVARIANT PROPERTIES OF THE S MATRIX AND OF THE OPERATOR $\mathfrak{S}(p'q';pq)$

The matrix element of the S matrix for the process $a + b \to a' + b'$ in which particles a, b with momenta p, q and spin projections μ, ν collide and particles a′, b′ with momenta p′, q′ and spin projections μ', ν' emerge in one of three forms

$$S(p'q'\mu'\nu';\ pq\mu\nu) = \bar{u}(p'\mu')\bar{u}(q'\nu')\,\mathfrak{S}(p'q';\ pq)\,u(p\mu)u(q\nu), \tag{14a}$$

$$S(p'q'\mu'\nu';\ pq\mu\nu) = \bar{v}(p\mu)\bar{v}(q\nu)\,\mathfrak{S}(p'q';\ pq)\,v(p'\mu')v(q'\nu'), \tag{14b}$$

$$S(p'q'\mu'\nu';\ pq\mu\nu) = \bar{u}(p'\mu')\bar{v}(q\nu)\,\mathfrak{S}(p'q';\ pq)\,u(p\mu)\,v(q'\nu'), \tag{14c}$$

where $u(p\mu)$, $\bar{u}(p\mu)$ and $v(p\mu)$, $\bar{v}(p\mu)$ are the spin functions of the particle (u) and antiparticle (v) with momentum **p** and spin projection μ in the initial $(u, \bar{v})$ and final $(\bar{u}, v)$ states, and $\mathfrak{S}(p'p';\ pq)$ is an operator in the spin space of the particles which depends on the momenta of the particles. For particles with spin 0 all the u, v are equal to unity, and for particles with spin ½

$$u\left(\mathbf{p}, \pm\tfrac{1}{2}\right) = N_p\begin{pmatrix}1\\ \sigma\mathbf{p}/(p_0+|p|)\end{pmatrix},$$

$$v\left(\mathbf{p}, \mp\tfrac{1}{2}\right) = N_p\begin{pmatrix}\sigma\mathbf{p}/(p_0+|p|)\\ 1\end{pmatrix},$$

$$\bar{u} = u^+\gamma_4, \qquad N_p = \sqrt{(p_0+|p|)/2|p|}.$$

Taken as a matrix in the indices μ', ν', μ, ν, the elements $S(\mathbf{p}'\mathbf{q}'\mu'\nu';\ \mathbf{pq}\mu\nu)$ form a matrix $S(\mathbf{p}'\mathbf{q}';\ \mathbf{pq})$, whose structure has been well studied in the c.m.s. The right members of the equations (14) also form matrices in the indices μ', ν', μ, ν, which we denote by $M(\mathbf{p}'\mathbf{q}';\ \mathbf{pq})$, $M(\mathbf{pq};\ \mathbf{p}'\mathbf{q}')$, and $M(\mathbf{p}'\mathbf{q};\ \mathbf{pq}')$, respectively. It is obvious that with the exception of $M(\mathbf{p}'\mathbf{q}';\ \mathbf{pq})$ these matrices are not in general the same as the matrices $S(\mathbf{p}'\mathbf{q}';\ \mathbf{pq})$. One can, however, obtain the matrices $M(\mathbf{pq};\ \mathbf{p}'\mathbf{q}')$ and $M(\mathbf{p}'\mathbf{q};\ \mathbf{pq}')$ in a simple way from the corresponding matrices $S(\mathbf{p}'\mathbf{q}';\ \mathbf{pq})$. Our problem is to find the structure of the operator $\mathfrak{S}(p'q';\ pq)$ from the structure of the matrix $S(\mathbf{p}'\mathbf{q}';\ \mathbf{pq})$ in the c.m.s. or from the structure of the corresponding matrix M. The analogous problem for a narrow class of reactions has been treated by Stapp[1] and also by Kawaguchi and Mugibayashi;[2] we note that the results of the latter authors are incorrect (see further discussion here, and reference 3).

We shall assume that the operator $\mathfrak{S}$ is invariant under Lorentz transformations, spatial rotations, reflections, and time reversal. The invariance of $\mathfrak{S}$ under the first three transformations means that the operator $\mathfrak{S}(p'q';\ pq)$ of a physical system K is connected with the analogous operator $\mathfrak{S}(p'_c q'_c;\ p_c q_c)$ of the transformed system K_c by relation

$$\mathfrak{S}(p'q';\ pq) = F_{a'}^{-1}F_{b'}^{-1}\mathfrak{S}(p'_c q'_c;\ p_c q_c)\,F_a F_b, \tag{15a}$$

$$\mathfrak{S}(p'q';\ pq) = F_{a}^{-1}F_{b}^{-1}\mathfrak{S}(p'_c q'_c;\ p_c q_c)\,F_{a'} F_{b'} \tag{15b}$$

$$\mathfrak{S}(p'q';\ pq) = F_{a'}^{-1}F_{b}^{-1}\mathfrak{S}(p'_c p'_c;\ p_c q_c)\,F_a F_{b'}, \tag{15c}$$

where F_a is the operator in the spin space of particle a that transforms the system K into the system K_c.

In what follows we shall present only relations of type a), since the corresponding relations of types b) and c) can be obtained from those of type a) by obvious interchanges [cf. Eqs. (14) and (15)].

The function $Fu(p\mu)$ obtained by the action of the operator F on the function $u(p\mu)$ of the system K can be expressed in terms of the function $u(p_c\mu_c)$ of the system K_c:

$$Fu\,(p\mu) = \sum_{\mu_c} u\,(p_c\mu_c)\,f\,(p_c\mu_c;\ p\mu),$$

$$\bar{u}(p\mu)\,F^{-1} = \sum_{\mu_c} f^{-1}(p\mu;\ p_c\mu_c)\,\bar{u}\,(p_c\mu_c). \tag{16}$$

In the indices μ_c, μ the coefficients $f(\mathbf{p}_c\mu_c;\ \mathbf{p}\mu)$ form a certain matrix $f(\mathbf{p}_c;\ \mathbf{p})$.

From the invariance of $\mathfrak{S}$ under the transformations F it follows that

$$M(\mathbf{p}'\mathbf{q}';\ \mathbf{pq}) = f^{-1}(\mathbf{p}', \mathbf{p}'_c)\,f^{-1}(\mathbf{q}';\ \mathbf{q}'_c)\,M(\mathbf{p}'_c\mathbf{q}'_c;\ \mathbf{p}_c\mathbf{q}_c)\,f(\mathbf{p}_c;\ \mathbf{p})\,f(\mathbf{q}_c;\ \mathbf{q}). \tag{17a}$$

For spinless particles $F = f = 1$, and for particles of spin ½ the Lorentz operator is

$$L(\mathbf{v}_c) = \exp[\tfrac{1}{2}i\chi\gamma_4\gamma\mathbf{n}]$$

(where $\mathbf{v}_c$ is the velocity of system K_c relative to K, $\tanh\chi = v_c$, $\mathbf{n} = \mathbf{v}_c/v_c$); the operator for rotation of the physical system around the axis **n** by the angle φ is

$$R(\mathbf{n}, \varphi) = \exp[{}^1/_2 \varphi \gamma_4 \gamma_5 \gamma \mathbf{n}];$$

the reflection (inversion) operator is $P = \gamma_4$; and the corresponding matrixes l, r, p are

$$l(\mathbf{p}_c; \mathbf{p}) = \frac{(p_{0c} + |p|)(p_0 + |p|) + (\sigma \mathbf{p}_c)(\sigma \mathbf{p})}{[2(p_{0c} + |p|)(p_0 + |p|)(p_{0c} p_0 + |p|^2 + \mathbf{p}_c \mathbf{p})]^{1/2}}, \quad (18)$$

$$r(\mathbf{p}_c; \mathbf{p}) = \exp[-\tfrac{1}{2} i\varphi(\sigma \mathbf{n})], \quad p(\mathbf{p}_c; \mathbf{p}) \equiv p(-\mathbf{p}; \mathbf{p}) = 1. \quad (19)$$

We note that the matrix $l(\mathbf{p}_C; \mathbf{p})$ has the following properties:

1) it is Hermitian, $l^+(\mathbf{p}_C; \mathbf{p}) = l(\mathbf{p}; \mathbf{p}_C)$;

2) $l(\mathbf{p}; \mathbf{q}) l^+(\mathbf{r}; \mathbf{q}) = l(\mathbf{p}; \mathbf{q}) l(\mathbf{q}; \mathbf{r}) = l(\mathbf{p}; \mathbf{r})$;

3) $l(\mathbf{p}; \mathbf{q}) = 1$ if $\mathbf{q}$ is parallel or antiparallel to $\mathbf{p}$;

4) $l^{-1}(\mathbf{p}; \mathbf{q}) = l(\mathbf{p}; \mathbf{q})$, and by definition $l^{-1}(\mathbf{p}; \mathbf{q}) = [l(\mathbf{q}; \mathbf{p})]^{-1}$.

The invariance of $\mathfrak{S}$ with respect to time reversal, under which the momentum $p = (\mathbf{p}, ip_0)$ goes over into $p_C = (-\mathbf{p}, ip_0)$, and so on, means that

$$\mathfrak{S}^*(p'q'; pq) = T_{a'}^{-1} T_{b'}^{-1} \mathfrak{S}^+(p_c q_c; p'_c q'_c)\, T_a T_b, \quad (20a)$$

where T is the time-reversal operator, which transforms the function $u(\mathbf{p}\mu)$ into the function $u(-\mathbf{p}\, -\mu)$:

$$T u^*(\mathrm{p}\mu) = \sum_{\mu_c} \bar{u}^+(\mathrm{p}_c \mu_c)\, t(\mu_c; \mu),$$

$$\bar{u}^*(\mathrm{p}\mu)\, T^{-1} = \sum_{\mu_c} t^{-1}(\mu; \mu_c)\, u^+(\mathrm{p}_c \mu_c). \quad (21)$$

From the invariance of $\mathfrak{S}$ with respect to time reversal it follows that

$$M^*(\mathrm{p}'\mathrm{q}'; \mathrm{pq}) = t_{a'}^{-1} t_{b'}^{-1} M^+(\mathrm{p}_c \mathrm{q}_c; \mathrm{p}'_c \mathrm{q}'_c)\, t_a t_b. \quad (22a)$$

For particles with spin zero $T = t = 1$, and for particles with spin $1/2$ we have $T = i\gamma_4\gamma_5 C$, $t = \pm i\sigma_2$ (the signs $\pm$ correspond to the functions u, v).

Let us now go on to the construction of the operator $\mathfrak{S}$ which satisfies the invariance conditions (15), (20) in terms of the S matrix in the c.m.s. (or of the M matrix in the c.m.s.), which satisfies the invariance conditions (17) and (22). For this purpose we write the right member of Eq. (14a) in the form

$$\bar{u}(0\mu')\, \bar{u}(0\nu') L^{-1}(\mathrm{p}'/p'_0) L^{-1}(\mathrm{q}'/q'_0)$$
$$\times \mathfrak{S}(p'q'; pq) L(\mathrm{p}/p_0) L(\mathrm{q}/q_0)\, u(0\mu)\, u(0\nu),$$

where $L(\mathbf{p}/p_0)$ is the Lorentz operator which transforms the spin function of a particle at rest into that of a particle with the momentum $\mathbf{p}$:

$$L(\mathrm{p}/p_0)\, u(0\mu) = u(\mathrm{p}\mu). \quad (23)$$

It is obvious that the operator $L^{-1}(\mathbf{p}'/p'_0) \times L^{-1}(\mathbf{q}'/q'_0)\, \mathfrak{S}(p'q'; pq)\, L(\mathbf{p}/p_0)\, L(\mathbf{q}/q_0)$ transforms the spin functions of the particles a, b at rest into those of the particles a′, b′ at rest. Since for a particle at rest $u_\alpha(0\mu) = \delta_{\alpha\mu}$, the matrix $M(\mathbf{p}'\mathbf{q}'; \mathbf{pq})$ with the matrix elements

$$\bar{u}(\mathrm{p}'\mu')\, \bar{u}(\mathrm{q}'\nu')\, \mathfrak{S}(p'q'; pq)\, u(\mathrm{p}\mu)\, u(\mathrm{q}\nu)$$

is equal to the matrix of the operator

$$L^{-1}(\mathrm{p}'/p'_0) L^{-1}(\mathrm{q}'/q'_0)\, \mathfrak{S}(p'q'; pq)\, L(\mathrm{p}/p_0) L(\mathrm{q}/q_0),$$

namely

$$M(\mathrm{p}'\mathrm{q}'; \mathrm{pq}) = L^{-1}(\mathrm{p}'/p'_0) L^{-1}(\mathrm{q}'/q'_0)\, \mathfrak{S}(p'q'; pq)\, L(\mathrm{p}/p_0) L(\mathrm{q}/q_0) \quad (24a)$$

Using the explicit expression for the matrix $M(\mathbf{p}_C\mathbf{q}_C; \mathbf{p}_C\mathbf{q}_C)$ in the c.m.s., we can find the operator $\mathfrak{S}(p'_C q'_C; p_C q_C)$:

$$\mathfrak{S}(p'_c q'_c; p_c q_c) = L(\mathrm{p}'_c/p'_{0c}) L(\mathrm{q}'_c/q'_{0c})$$
$$\times M(\mathrm{p}'_c \mathrm{q}'_c; \mathrm{p}_c \mathrm{q}_c) L^{-1}(\mathrm{p}_c/p_{0c}) L^{-1}(\mathrm{q}_c/q_{0c}), \quad (25a)$$

which has as its only elements (14) different from zero the elements (14a).

We now note that the matrix M and the operators L are not relativistically invariant in structure, whereas the expression (25a) as a whole must be of invariant structure. Therefore, if by means of the explicitly noninvariant expressions for the matrix $M(\mathbf{p}'_C\mathbf{q}'_C; \mathbf{p}_C\mathbf{q}_C)$ and the operators $L(\mathbf{p}_C/p_{0C})$, etc., we construct corresponding auxiliary relativistically-invariant operators $\mathfrak{M}(p'q'; pq)$ and $L(p, t)$, etc., which coincide with $M(\mathbf{p}'_C\mathbf{q}'_C; \mathbf{p}_C\mathbf{q}_C)$ and $L(\mathbf{p}_C/p_{0C})$, etc., then by substituting these relativistically-invariant operators into Eq. (25a) we shall get a relativistically-invariant expression for the operator $\mathfrak{S}(p'q'; pq)$. Thus in the construction of the relativistically-invariant operator $\mathfrak{S}(p'q'; pq)$ we can work from the formula

$$\mathfrak{S}(p'q'; pq) = L(p', t)\, L(q', t)\, \mathfrak{M}(p'q'; pq) L^{-1}(p, t) L^{-1}(q, t), \quad (26a)$$

where $\mathfrak{M}(p'q'; pq)$, $L(p, t)$, etc., are auxiliary relativistically-invariant operators that coincide in the c.m.s. with the matrix $M(\mathbf{p}'_C\mathbf{q}'_C; \mathbf{p}_C\mathbf{q}_C)$ and the Lorentz operators $L(\mathbf{p}_C/p_{0C})$, etc.

3. THE STRUCTURES OF THE MATRIX S(p′q′; pq) AND THE OPERATOR $\mathfrak{S}$(p′q′; pq)

For particles with spins 0 and $1/2$ the Lorentz operators $L(\mathbf{v})$ are respectively unity and $\exp[i\tfrac{1}{2}\chi\gamma_4\gamma\mathbf{n}]$. If $\mathbf{v} = \mathbf{p}/p_0$, where $p_0 = (\mathbf{p}^2 + |p|^2)^{1/2}$, then we can write the operator $L(\mathbf{p}/p_0)$ for a particle with spin $\tfrac{1}{2}$ in the form

$$L(\mathrm{p}/p_0) = (-i\hat{p}\gamma_4 + |p|)/\sqrt{2|p|(p_0 + |p|)}. \quad (27)$$

It is obvious that the corresponding relativistically-invariant operator $L(\mathbf{p}, \mathbf{t})$ that coincides with $L(\mathbf{p}/p_0)$ in the c.m.s. is the operator

$$L(p, t) = (-\hat{p}\hat{t} + |t||p|)/\sqrt{2|t||p|(|p||t| - (pt))}. \quad (28)$$

In the spin space of the system the matrix $S(\mathbf{p}'\mathbf{q}'; \mathbf{pq})$ has the structure

$$S(\mathbf{p}'\mathbf{q}'; \mathbf{pq}) = A + \sigma\mathbf{A}, \quad (29)$$

if the spin of the system is half-integral (i.e., is equal to $\frac{1}{2}$), and

$$S(\mathbf{p}'\mathbf{q}'; \mathbf{pq}) = (1 - \tfrac{1}{2}\mathbf{S}^2)A + \mathbf{T}_{01}\mathbf{B} + \mathbf{T}_{10}\mathbf{C} + \tfrac{1}{2}\mathbf{S}^2 D + \mathbf{S}\mathbf{D} + (S_iS_j + S_jS_i - \tfrac{2}{3}\mathbf{S}^2\delta_{ij})D_{ij}, \quad (30)$$

if the spin of the system is integral (i.e., is equal to 0 or 1). In the latter case the term in A corresponds to singlet-singlet transitions, the terms in **B** and **C** to triplet-singlet and singlet-triplet transitions, and the terms in D, **D**, and D_{ij} to triplet-triplet transitions. The functions A, **A**, **B**, **C**, D, **D**, D_{ij} depend on the vectors **k**, **k′**, **e** and the total energy of the system and are so constructed that the entire matrix $S(\mathbf{p}'\mathbf{q}'; \mathbf{pq})$ is a scalar or pseudoscalar, depending on whether the intrinsic parity of the system is conserved or changes in the reaction. Namely, if the intrinsic parity of the system is conserved, then

$$A = A_1, \quad \mathbf{A} = i\mathbf{n}A_2 \quad (\text{spin}\,{}^1/_2); \quad A = A_1, \quad \mathbf{B} = i\mathbf{n}B_1, \quad \mathbf{C} = i\mathbf{n}C_1, \quad D = D_1, \quad \mathbf{D} = i\mathbf{n}D_2,$$

$$D_{ij} = (k_i'k_j' - \tfrac{1}{3}\mathbf{k}'^2\delta_{ij})D_3 + (k_ik_j - \tfrac{1}{3}\mathbf{k}^2\delta_{ij})D_4 + (k_i'k_j + k_j'k_i - \tfrac{2}{3}(\mathbf{k}'\mathbf{k})\delta_{ij})D_5 \quad (\text{spin}\,0,1).$$

If, on the other hand, the parity changes, then

$$A = 0, \quad \mathbf{A} = \mathbf{k}'A_1 + \mathbf{k}A_2 \quad (\text{spin}\,{}^1/_2);$$

$$A = 0, \quad \mathbf{B} = \mathbf{k}'B_1 + \mathbf{k}B_2, \quad \mathbf{C} = \mathbf{k}'C_1 + \mathbf{k}C_2,$$

$$D = 0, \quad \mathbf{D} = \mathbf{k}'D_1 + \mathbf{k}D_2,$$

$$D_{ij} = (k_i'n_j + k_j'n_i)D_3 + (k_in_j + k_jn_i)D_4 \quad (\text{spin}\,0,1).$$

For details of the structure of the matrix $S(\mathbf{p}'\mathbf{q}'; \mathbf{pq})$ see reference 4.

An investigation shows that if the matrix $S(\mathbf{p}'\mathbf{q}'; \mathbf{pq})$ is put in the form (29) or (30), then the matrix M associated with it has the forms shown below for the three cases of Eq. (14). In these formulas the signs + or − placed on the two-rowed matrices I and σ indicate that the corresponding indices on the elements of these matrices refer to the particle (+) or the antiparticle (−); for example, σ_{-+} denotes a Pauli matrix for whose elements $(\sigma_{-+})_{\mu\nu}$ the index μ refers to the antiparticle and ν to the particle; σ^1 and σ^2 connect the particles a, a′ and b, b′ [see Eqs. (32, a, b, c) below] or the particles a, b and a′, b′ [see Eq. (32c′)].

Case a

$$M(\mathbf{p}'\mathbf{q}'; \mathbf{pq}) = A + \sigma_{++}\mathbf{A}; \quad (31a)$$

$$M(\mathbf{p}'\mathbf{q}'; \mathbf{pq}) = \tfrac{1}{4}(1 - \sigma^1_{++}\sigma^2_{++})A + \tfrac{1}{4}(\sigma^1_{++} - \sigma^2_{++}, \mathbf{B} + \mathbf{C}) - \tfrac{1}{4}i([\sigma^1_{++}, \sigma^2_{++}], \mathbf{B} - \mathbf{C}) + \tfrac{1}{4}(3 + \sigma^1_{++}\sigma^2_{++})D + \tfrac{1}{2}(\sigma^1_{++} + \sigma^2_{++}, \mathbf{D}) + \tfrac{1}{2}[\sigma^1_{++i}\sigma^2_{++j} + \sigma^1_{++j}\sigma^2_{++i} - \tfrac{2}{3}(\sigma^1_{++}\sigma^2_{++})\delta_{ij}]D_{ij}. \quad (32a)$$

Case b

$$M(\mathbf{pq}; \mathbf{p}'\mathbf{q}') = A - \sigma_{--}\mathbf{A}; \quad (31b)$$

$$M(\mathbf{pq}; \mathbf{p}'\mathbf{q}') = \tfrac{1}{4}(1 - \sigma^1_{--}\sigma^2_{--})A + \tfrac{1}{4}(-\sigma^1_{--} + \sigma^2_{--}, \mathbf{B} + \mathbf{C}) - \tfrac{1}{4}i([\sigma^1_{--}, \sigma^2_{--}], \mathbf{B} - \mathbf{C}) + \tfrac{1}{4}(3 + \sigma^1_{--}\sigma^2_{--})D - \tfrac{1}{2}(\sigma^1_{--} + \sigma^2_{--}, \mathbf{D}) + \tfrac{1}{2}[\sigma^1_{--i}\sigma^2_{--j} + \sigma^1_{--j}\sigma^2_{--i} - \tfrac{2}{3}(\sigma^1_{--}\sigma^2_{--})\delta_{ij}]D_{ij}. \quad (32b)$$

Case c

$$M(\mathbf{p}'\mathbf{q}; \mathbf{pq}') = A + \sigma_{++}\mathbf{A} \text{ or } A - (\sigma_{--}\mathbf{A}); \quad (31c)$$

$$M(\mathbf{p}'\mathbf{q}; \mathbf{pq}') = \tfrac{1}{4}(1 + \sigma^1_{++}\sigma^2_{--})A + \tfrac{1}{4}(\sigma^1_{++} + \sigma^2_{--}, \mathbf{B} + \mathbf{C}) + \tfrac{1}{4}i([\sigma^1_{++}, \sigma^2_{--}], \mathbf{B} - \mathbf{C}) + \tfrac{1}{4}(3 - \sigma^1_{++}\sigma^2_{--})D + \tfrac{1}{2}(\sigma^1_{++} - \sigma^2_{--}, \mathbf{D}) - \tfrac{1}{2}[\sigma^1_{++i}\sigma^2_{--j} + \sigma^1_{++j}\sigma^2_{--i} - \tfrac{2}{3}(\sigma^1_{++}\sigma^2_{--})\delta_{ij}]D_{ij} \quad (32c)$$

or

$$M(\mathbf{p}'\mathbf{q}; \mathbf{pq}') = \frac{1}{\sqrt{2}}I_{-+}A + \frac{1}{\sqrt{2}}\sigma_{-+}\mathbf{B} \text{ or } \frac{1}{\sqrt{2}}I_{+-}A + \frac{1}{\sqrt{2}}\sigma_{+-}\mathbf{C}; \quad (31c')$$

$$M(\mathbf{p}'\mathbf{q}; \mathbf{pq}') = \tfrac{1}{2}I^1_{-+}I^2_{+-}A + \tfrac{1}{2}\sigma^1_{-+}\mathbf{B} + \tfrac{1}{2}\sigma^2_{+-}\mathbf{C} + \tfrac{1}{2}\sigma^1_{-+}\sigma^2_{+-}D + \tfrac{1}{2}i([\sigma^1_{-+}, \sigma^2_{+-}]\mathbf{D}) - \tfrac{1}{2}[\sigma_{-+i}\sigma^2_{+-j} + \sigma^1_{-+j}\sigma^2_{+-i} - \tfrac{2}{3}(\sigma^1_{-+}\sigma^2_{+-})\delta_{ij}]D_{ij}. \quad (32c')$$

The matrices (31, a, b, c) refer to half-integral spin of the system, and the matrices (32, a, b, c, c′) and (31c′) to integral spin.

Each matrix M is two-rowed if only two of the particles in the reaction $a + b \rightarrow a' + b'$ have spin $\frac{1}{2}$ [see Eqs. (31, a, b, c, c′)], and is the direct product of two two-rowed matrices if all of the particles have spin $\frac{1}{2}$ [see Eqs. (31, a, b, c, c′)]. The corresponding relativistically invariant matrix $\mathfrak{M}$ will obviously be a four-rowed matrix or the direct product of two four-rowed matrices, and in the c.m.s. these four-rowed matrices reduce to the left upper (++) two-rowed matrices for processes of type a, to the right lower (− −) matrices for processes of type b, to the left upper or right lower for processes of type c, and to the left lower (− +) or right upper (+ −) for processes of type c′. In accordance with this, to construct the relativistically invariant operators $\mathfrak{M}(p'q'; pq)$, etc., that coincide in the c.m.s. with the matrices $M(\mathbf{p}'\mathbf{q}'; \mathbf{pq})$, etc., we have only to replace in the matrix $M(\mathbf{p}'\mathbf{q}'; \mathbf{pq})$ the three-dimensional vectors $\mathbf{k}$, $\mathbf{k}'$, $\mathbf{n}$, $\mathbf{e}$, $\mathbf{s}$ by the four-dimensional vectors k, k′, n, e, s and replace the two-rowed matrices $I_{\pm\pm}$, $I_{\pm\mp}$, $\sigma_{\pm\pm}$, $\sigma_{\pm\mp}$ by the respective four-rowed matrices

$$-\Lambda_\pm(t), \qquad -\Lambda_\pm(t)\gamma_5\Lambda_\mp(t) = -\gamma_5\Lambda_\mp(t),$$
$$\pm\Lambda_\pm(t)\, i\gamma_5\gamma_\mu\Lambda_\pm(t) = \pm i\gamma_5(\gamma_\mu \mp it_\mu^0)\Lambda_\pm(t),$$
$$\pm\Lambda_\pm(t)\, i\gamma_\mu\Lambda_\mp(t) = \pm i(\gamma_\mu \mp it_\mu^0)\Lambda_\mp(t).$$

The operators $\Lambda_\pm(t)$ are defined in the following way

$$\Lambda_\pm(t) = (|t| \mp i\hat{t})/2|t|, \qquad \Lambda_\pm^2(t) = \Lambda_\pm(t),$$
$$\Lambda_\pm(t)\Lambda_\mp(t) = 0, \quad \Lambda_\pm(t) + \Lambda_\mp(t) = 1 \tag{33}$$

and in the c.m.s. are equal to $\frac{1}{2}(1 \pm \beta)$. The signs ± for the operators $\Lambda_\pm(t)$ must be chosen in accordance with the form of the matrix M [cf. Eqs. (31) and (32)] and the statements made above.

Using now the formula (26) and the relation

$$\Lambda_\pm(t)L^{-1}(p,t) = K_p\Lambda_\pm(t)\Lambda_\pm(p),$$
$$K_p = [2|t|\,|p|/(|t|\,|p| - (tp))]^{1/2}, \tag{34}$$

we get the final formulas for the matrix $\mathfrak{S}(p'q'; pq)$ (we have further $K_{p'p} = K_{p'}K_p$, etc.):

$$\mathfrak{S}(p'q';\; pq) = K_{p'p}\Lambda_+(p')\Lambda_+(t)\mathfrak{R}(p'q';\; pq)\Lambda_+(t)\Lambda_+(p), \tag{35a, c}$$

$$\mathfrak{S}(p'q';\; pq) = K_{p'p}\Lambda_-(p)\Lambda_-(t)\mathfrak{R}(pq;\; p'q')\Lambda_-(t)\Lambda_-(p), \tag{35b, c}$$

$$\mathfrak{S}(p'q';\; pq) = K_{pq}\Lambda_-(q)\Lambda_-(t)\mathfrak{R}(q'p;\; pq')\Lambda_+(t)\Lambda_+(p)$$

or

$$K_{p'q'}\Lambda_+(p')\Lambda_+(t)\mathfrak{R}(qp';\; pq')\Lambda_-(t)\Lambda_-(q'), \tag{35c'}$$

if only two particles have spin $\frac{1}{2}$, and

$$\mathfrak{S}(p'q';\; pq) = K_{p'q'pq}\Lambda_+(p')\Lambda_+(t)\Lambda_+(q')\Lambda_+(t)\mathfrak{R}(p'q';\; pq)$$
$$\times\Lambda_+(t)\Lambda_+(p)\Lambda_+(t)\Lambda_+(q), \tag{36a}$$

$$\mathfrak{S}(p'q';\; pq) = K_{p'q'pq}\Lambda_-(p)\Lambda_-(t)\Lambda_-(q)\Lambda_-(t)\mathfrak{R}(pq;\; p'q')$$
$$\times\Lambda_-(t)\Lambda_-(p')\Lambda_-(t)\Lambda_-(q'), \tag{36b}$$

$$\mathfrak{S}(p'q';\; pq) = K_{p'q'pq}\Lambda_+(p')\Lambda_+(t)\Lambda_-(q)\Lambda_-(t)\mathfrak{R}(p'q;\; pq')$$
$$\times\Lambda_+(t)\Lambda_+(p)\Lambda_-(t)\Lambda_-(q), \tag{36c}$$

$$\mathfrak{S}(p'q';\; pq) = K_{p'q'pq}\Lambda_-(q)\Lambda_-(t)\Lambda_+(p')\Lambda_+(t)\mathfrak{R}(qp';\; pq')$$
$$\times\Lambda_+(t)\Lambda_+(p)\Lambda_-(t)\Lambda_-(q), \tag{36c'}$$

if all the particles have spin $\frac{1}{2}$; the matrix $\mathfrak{R}$ is obtained from the matrix M [see Eqs. (31), (32)] by replacing the three-dimensional vectors $\mathbf{k}$, $\mathbf{k}'$, $\mathbf{n}$, $\mathbf{e}$, $\mathbf{s}$ by the four-vectors k, k′, n, e, s and the two-rowed matrices $I_{\pm\pm}$, $I_{\pm\mp}$, $\sigma_{\pm\pm}$, $\sigma_{\pm\mp}$ by the four-rowed matrices I, $-\gamma_5$, $\pm i\gamma_5\gamma_\mu$, $\pm i\gamma_\mu$.*

We shall present the form of the matrices $\mathfrak{R}$ for the reactions $a + b \rightarrow a' + b'$, $\gamma + b \rightarrow a' + b'$, $\gamma + b \rightarrow \gamma' + b'$, $a + b \rightarrow 2\gamma$, in which the spins of the particles do not exceed $\frac{1}{2}$. The matrices for reactions with conservation and with change of the intrinsic parity of the system are denoted respectively by $\mathfrak{R}^{(\pm)}$. We have also introduced the notation $\hat{a}^{1,2} = \gamma_\mu^{1,2} a_\mu$.

The Reaction $a + b \rightarrow a' + b'$

1) If the spins of all particles are zero, $s_a = s_b = s_{a'} = s_{b'} = 0$, then

$$\mathfrak{R} = A(\varepsilon,\; x); \tag{37a, b, c}$$

2) $s_a = s_{a'} = \frac{1}{2}$, $s_b = s_{b'} = 0$,

$$\mathfrak{R}^{(+)} = A_1 - \gamma_5\hat{n}A_2, \tag{38a, b, c}$$

$$\mathfrak{R}^{(-)} = i\gamma_5\hat{k}'A_1 + i\gamma_5\hat{k}A_2; \tag{39a, b, c}$$

3) $s_a = s_b = \frac{1}{2}$, $s_{a'} = s_{b'} = 0$,

$$\mathfrak{R}^{(+)} = -\frac{1}{\sqrt{2}}\gamma_5A_1 + \frac{1}{\sqrt{2}}\hat{n}B_1, \tag{40c'}$$

$$\mathfrak{R}^{(-)} = -\frac{1}{\sqrt{2}}i\hat{k}'B_1 - \frac{1}{\sqrt{2}}i\hat{k}B_2, \tag{41c'}$$

4) $s_a = s_b = s_{a'} = s_{b'} = \frac{1}{2}$,

*We note that in Eqs. (35), (36) we may strike out the operators $\Lambda_\pm(t)$ on the left (or on the right) of $\mathfrak{R}$ if we replace γ_μ in $\mathfrak{R}$ by $\gamma_\mu \pm it_\mu^0$, which is equivalent to the replacement

$(\gamma^1\gamma^2) \rightarrow (\gamma^1\gamma^2) - 1$ in cases a, b,

$(\gamma^1\gamma^2) \rightarrow (\gamma^1\gamma^2) + 1$ in cases c, c′.

$$\mathfrak{R}^{(+)} = \tfrac{1}{4}(1 + \gamma_5^1\gamma_5^2(\gamma^1\gamma^2))A_1 - \tfrac{1}{4}(\gamma_5^1\hat{n}^1 - \gamma_5^2\hat{n}^2)(B_1 + C_1) - \tfrac{1}{4}i\gamma_5^1\gamma_5^2\varepsilon_{\mu\nu\lambda\sigma}n_\mu\gamma_\nu^1\gamma_\lambda^2 t_\sigma^0(B_1 - C_1) + \tfrac{1}{4}(3 - \gamma_5^1\gamma_5^2(\gamma^1\gamma^2))D_1$$
$$- \tfrac{1}{2}(\gamma_5^1\hat{n}^1 + \gamma_5^2\hat{n}^2)D_2 - \gamma_5^1\gamma_5^2[\hat{k}'^1\hat{k}^2 + \hat{k}^1\hat{k}'^2 - \tfrac{2}{3}(\gamma^1\gamma^2)(k'k)]D_3 - \gamma_5^1\gamma_5^2[\hat{k}'^1\hat{k}'^2 - \tfrac{1}{3}(\gamma^1\gamma^2)k'^2]D_4$$
$$- \gamma_5^1\gamma_5^2[\hat{k}^1\hat{k}^2 - \tfrac{1}{3}(\gamma^1\gamma^2)k^2]D_5 \qquad (42a, b, c)$$

or

$$\mathfrak{R}^{(+)} = \tfrac{1}{2}\gamma_5^1\gamma_5^2 A_1 - \tfrac{1}{2}\hat{n}^1\gamma_5^2 B_1 + \tfrac{1}{2}\gamma_5^1\hat{n}^2 C_1 + \tfrac{1}{2}(\gamma^1\gamma^2)D_1 + \tfrac{1}{2}i\varepsilon_{\mu\nu\lambda\sigma}n_\mu\gamma_\nu^1\gamma_\lambda^2 t_\sigma^0 D_2 - [\hat{k}'^1\hat{k}^2 + \hat{k}^1\hat{k}'^2 - \tfrac{2}{3}(\gamma^1\gamma^2)(k'k)]D_3$$
$$- [\hat{k}'^1\hat{k}'^2 - \tfrac{1}{3}(\gamma^1\gamma^2)k'^2]D_4 - [\hat{k}^1\hat{k}^2 - \tfrac{1}{3}(\gamma^1\gamma^2)k^2]D_5 \qquad (43c')$$

(for elastic reactions it follows from the invariance of the interaction under time reversal that $B_1 = C_1$, $D_4 = D_5$), and

$$\mathfrak{R}^{(-)} = \tfrac{1}{4}i(\gamma_5^1\hat{k}'^1 - \gamma_5^2\hat{k}'^2)(B_1 + C_1) + \tfrac{1}{4}i(\gamma_5^1\hat{k}^1 - \gamma_5^2\hat{k}^2)(B_2 + C_2) - \tfrac{1}{4}\gamma_5^1\gamma_5^2\varepsilon_{\mu\nu\lambda\sigma}k'_\mu\gamma_\nu^1\gamma_\lambda^2 t_\sigma^0(B_1 - C_1) - \tfrac{1}{4}\gamma_5^1\gamma_5^2\varepsilon_{\mu\nu\lambda\sigma}k_\mu\gamma_\nu^1\gamma_\lambda^2 t_\sigma^0(B_2 - C_2)$$
$$+ \tfrac{1}{2}i(\gamma_5^1\hat{k}'^1 + \gamma_5^2\hat{k}'^2)D_1 + \tfrac{1}{2}i(\gamma_5^1\hat{k}^1 + \gamma_5^2\hat{k}^2)D_2 - \gamma_5^1\gamma_5^2(\hat{k}'^1\hat{n}^2 + \hat{n}^1\hat{k}'^2)D_3 - \gamma_5^1\gamma_5^2(\hat{k}^1\hat{n}^2 + \hat{n}^1\hat{k}^2)D_4 \qquad (44a, b, c)$$

or

$$\mathfrak{R}^{(-)} = -\tfrac{1}{2}i\hat{k}'^1 B_1 - \tfrac{1}{2}i\hat{k}^1 B_2 + \tfrac{1}{2}i\hat{k}'^2 C_1 + \tfrac{1}{2}i\hat{k}^2 C_2 + \tfrac{1}{2}\varepsilon_{\mu\nu\lambda\sigma}\gamma_\nu^1\gamma_\lambda^2 t_\sigma^0(k'_\mu D_1 + k_\mu D_2) - (\hat{k}'^1\hat{n}^2 + \hat{n}^1\hat{k}'^2)D_3 - (\hat{k}^1\hat{n}^2 + \hat{n}^1\hat{k}^2)D_4. \qquad (45c')$$

The Reaction $\gamma + b \rightarrow a' + b'$

We present the matrices $\mathfrak{R}^{(+)}$ that correspond to reactions in which the intrinsic parities of the particle b and the system a' + b' are the same, since the matrices $\mathfrak{R}^{(-)}$ that correspond to reactions in which these parities are opposite are obtained from those presented here by the interchange $e \rightleftarrows -is$.

1) For $s_b = s_{a'} = s_{b'} = 0$

$$\mathfrak{R}^{(+)} = (k'e)A_1; \qquad (46a, b, c)$$

2) $s_b = s_{b'} = \frac{1}{2}$, $s_{a'} = 0$

$$\mathfrak{R}^{(+)} = (ns)A_1 + \gamma_5\hat{s}A_2 + \gamma_5\hat{k}'(k's)A_3 + \gamma_5\hat{k}(k's)A_4; \qquad (47a, b, c)$$

3) $s_b = 0$, $s_{a'} = s_{b'} = \frac{1}{2}$

$$\mathfrak{R}^{(+)} = -\frac{1}{\sqrt{2}}\{\gamma_5(k'e)A_1 - i\varepsilon_{\mu\nu\lambda\sigma}\gamma_\mu k'_\nu e_\lambda t_\sigma^0 C_1 + \hat{s}C_2 + \hat{n}(k'e)C_3\}. \qquad (48c')$$

The Reaction $\gamma + b \rightarrow \gamma' + b'$

We present the matrices $\mathfrak{R}^{(+)}$ that correspond to reactions in which the intrinsic parities of the particles b and b' are the same, since the matrices $\mathfrak{R}^{(-)}$ that correspond to reactions in which these parities are opposite are obtained from those given here by the interchange $e \rightleftarrows -is$ or $e' \rightleftarrows -is'$.

1) For $s_b = s_{b'} = 0$

$$\mathfrak{R}^{(+)} = (e'e)A_1 + (s's)A_2; \qquad (49a, b, c)$$

2) $s_b = s_{b'} = \frac{1}{2}$

$$\mathfrak{R}^{(+)} = (e'e)A_1 + (s's)A_2 + i\gamma_5\varepsilon_{\mu\nu\lambda\sigma}\gamma_\mu e'_\nu e_\lambda t_\sigma^0 A_3 + i\gamma_5\varepsilon_{\mu\nu\lambda\sigma}\gamma_\mu s'_\nu s_\lambda t_\sigma^0 A_4 - \gamma_5\hat{k}'(e's)A_5 - \gamma_5\hat{k}(s'e)A_6 - \gamma_5\hat{k}'(s'e)A_7 - \gamma_5\hat{k}(e's)A_8. \qquad (50a, b, c)$$

For elastic reactions it follows from time-reversal invariance that $A_6 = -A_5$, $A_8 = -A_7$.*

The Reaction $a + b \rightarrow \gamma_1 + \gamma_2$

1) In the case $s_a = s_b = 0$

$$\mathfrak{R}^{(+)} = (e_1e_2)A_1 + (ke_1)(ke_2)A_2, \qquad (51a, b, c)$$

$$\mathfrak{R}^{(-)} = \varepsilon_{\mu\nu\lambda\sigma}k'_\mu e_{1\nu}e_{2\lambda}t_\sigma^0 A_1 + i[(ks_1)(ke_2) + (ks_2)(ke_1)]A_2; \qquad (52a, b, c)$$

2) $s_a = s_b = \frac{1}{2}$

$$\mathfrak{R}^{(+)} = -\frac{1}{\sqrt{2}}\{\gamma_5(e_1e_2)A_1 + \gamma_5(ke_1)(ke_2)A_2 + \hat{k}'(s_1e_2)C_1 + \hat{k}(s_1e_2)C_2 + [\hat{s}_1(ke_2) + \hat{s}_2(ke_1)]C_3 + \hat{k}'[(ks_1)(ke_2) + (ks_2)(ke_1)]C_4 + \hat{k}[(ks_1)(ke_2) + (ks_2)(ke_1)]C_5\}, \qquad (53c')$$

$$\mathfrak{R}^{(-)} = i\frac{1}{\sqrt{2}}\{-\gamma_5(s_1e_2)A_1 - \gamma_5[(ks_1)(ke_2) + (ks_2)(ke_1)]A_2 + \hat{k}'(e_1e_2)C_1 + \hat{k}(e_1e_2)C_2 + [\hat{e}_2(ke_1) + \hat{e}_1(ke_2)]C_3 + \hat{k}'(ke_1)(ke_2)C_4 + \hat{k}(ke_1)(ke_2)C_5 + \hat{n}(s_1e_2)C_6\}. \qquad (54c')$$

Thus the relativistically-invariant spin structure of the operator $\mathfrak{S}(p'q'; pq)$ defined by Eqs. (35) – (54) is closely connected with the spin structure of the matrix $S(\mathbf{p}'\mathbf{q}'; \mathbf{p}\mathbf{q})$ in the c.m.s. The functions A_i, B_i, C_i, D_i are scalar functions of the variables x and ϵ, which in the c.m.s. are the cosine of the scattering angle and the total energy of the system. In the general case the number of such scalar functions that characterize the operator $\mathfrak{S}(p'q'; pq)$ is given by (cf. references 3 and 4)

$$r = \sum_{S', S} r_{S'S}, \qquad r^0 = \frac{1}{2}\sum_{S' \neq S} r_{S'S} + \sum_S r_{SS}^0 \qquad (55)$$

*We note that the ten spin invariants for this reaction presented in reference 2 are actually not independent, and reduce to eight invariants.

for inelastic and elastic reactions, respectively, where $r_{S'S}$ is the number of scalar functions that correspond in the c.m.s. to a transition from a state in which the spin of the system is S to one in which it is S′. We have as the formulas for the numbers $r_{S'S}$ and r^0_{SS}:

1) for the reaction a + b → a′ + b′ (S, S′ and i, i′ are the spins and intrinsic parities of the systems a + b, a′ + b′):

$$r_{S'S} = \tfrac{1}{2}(2S'+1)(2S+1) + \tfrac{1}{2}i'i(-1)^{S'-S},$$

$$r^0_{SS} = (S+1)^2 \quad (S, S'\text{—integral}), \qquad (56)$$

$$r_{S'S} = \tfrac{1}{2}(2S'+1)(2S+1),$$

$$r^0_{SS} = (S+1)^2 - {}^1/_4 \quad (S, S'\text{— half-integral}),$$

2) for the reaction γ + b → a′ + b′ (S, S′ are the spins of particle b and system a′ + b′):

$$r_{S'S} = (2S'+1)(2S+1); \qquad (57)$$

3) for the reaction γ + b → γ′ + b′ (S, S′ are the spins of particles b, b′):

$$r_{S'S} = 2(2S'+1)(2S+1), \qquad r^0_{SS} = 2(2S+1)(S+1); \qquad (58)$$

4) for the reaction a + b → 2γ (S is the spin of the system a + b):

$$r_S = 2(2S+1). \qquad (59)$$

The operator $\mathfrak{S}$(p′q′; pq) and the matrix M (**p′q′**; **pq**) discussed in this paper can be used in the relativistic theory of reactions with polarized particles, in the construction of dispersion relations, and in other problems.

[1] H. P. Stapp, Phys. Rev. **103**, 425 (1956).

[2] M. Kawaguchi and N. Mugibayashi, Progr. Theoret. Phys. **8**, 212 (1952).

[3] V. I. Ritus, JETP **35**, 1485 (1958), Soviet Phys. JETP **8**, 1035 (1959).

[4] V. I. Ritus, JETP **33**, 1264 (1957), Soviet Phys. JETP **6**, 972 (1958).

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