ON THE ANALOGY BETWEEN THE WEAK AND THE ELECTROMAGNETIC INTERACTIONS

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The analogy between the weak and the electromagnetic interactions is presented in such a way that the electric current and the charged currents in the weak interaction are obtained from a single symmetrical expression which involves the operators $\frac{1}{2} + \tau$ and $1 + \gamma_5$ after the requirements of conservation of the electric, leptonic, and baryonic charges and of vanishing of the photon mass are imposed. A definite "chirality" is ascribed to particles of half-integral spin, which is conserved in weak interactions. Doublets of "bare" Fermi particles in the weak and electromagnetic interactions are classified in terms of the values of the electric charge, the leptonic or baryonic charge, and the chirality.

THE hypothesis of Gershteĭn and Zel'dovich[1] and of Feynman and Gell-Mann,[2] that there is conservation of the vector current in weak interactions, indicates the existence of a deep analogy between the weak and the electromagnetic interactions. A concrete formulation of this analogy is proposed in the present paper. Together with the requirements of conservation of electric, leptonic, and baryonic charges and of chirality, this makes it possible to set up a complete expression for the weak current of the "bare" particles, which is in qualitative agreement with all available experimental data on the decays of ordinary and strange particles.

In what follows we use the hypothesis that there are two isotopic leptonic doublets ($\nu_1 e^-$) and ($\nu_2 \mu^-$);[3] the isospins of all baryons are assumed to be equal to $\frac{1}{2}$, in accordance with Gell-Mann's hypothesis of global symmetry.[4] When the hyperons are included in the treatment the table of the properties of fermions that was presented earlier[3] takes a new form* (see Table I). Here letters with the tilde denote antiparticles; l, n, e are the leptonic, baryonic, and electric charges,

$$Y^0 = 2^{-1/2}(\Lambda^0 - \Sigma^0), \qquad Z^0 = 2^{-1/2}(\Lambda^0 + \Sigma^0), \tag{1}$$

γ_5 is the chirality in weak interactions;[3,7] the new strangeness S′ of a particle is calculated by the formula

$$e = I_z + 1/2(l + n + S').$$

*For antiparticles the signs of all numerical entries in the table are changed.

TABLE I

	I_z	l	n	e	γ_5	S'
e	$-1/2$	-1	0	-1	$+1$	0
ν_1	$1/2$	-1	0	0	$+1$	0
μ	$-1/2$	$+1$	0	-1	$+1$	-2
ν_2	$1/2$	$+1$	0	0	$+1$	-2
p	$1/2$	0	$+1$	$+1$	$+1$	0
n	$-1/2$	0	$+1$	0	$+1$	0
$\widetilde{Y}^0$	$1/2$	0	-1	0	$+1$	0
$\widetilde{\Sigma}^+$	$-1/2$	0	-1	-1	$+1$	0
Z^0	$1/2$	0	$+1$	0	$+1$	-2
Σ^-	$-1/2$	0	$+1$	-1	$+1$	-2
$\widetilde{\Xi}^-$	$1/2$	0	-1	$+1$	$+1$	$+2$
$\widetilde{\Xi}^0$	$-1/2$	0	-1	0	$+1$	$+2$

Table I shows that definite chiralities are assigned to all fermions except the neutral hyperons Λ^0 and Σ^0; their chiralities can be ± 1, depending on whether they occur in the Z or the Y doublet:*

$$N = \begin{pmatrix} p \\ n \end{pmatrix}, \quad Y = \begin{pmatrix} \Sigma^+ \\ Y^0 \end{pmatrix}, \quad Z = \begin{pmatrix} Z^0 \\ \Sigma^- \end{pmatrix}, \quad \Xi = \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix}. \tag{2}$$

We shall take as the primary weak processes the interactions of the weak currents with charged vector mesons $X_\mu^\pm$.[2]

Then the Hamiltonians of the electromagnetic and weak interactions can be written in the form

$$H^{(0)} = e j_\mu^{(0)} A_\mu, \tag{3}$$

$$H = H^{(+)} + H^{(-)} \qquad H^{(+)} = g_1 j_\mu^{(+)} X_\mu^+,$$
$$H^{(-)} = g_1 j_\mu^{(-)} X_\mu^-, \tag{4}$$

*The chirality of a particle indicates whether the wave function of the particle appears in the weak current with the factor $1 + \gamma_5$ or $1 - \gamma_5$, and accordingly corresponds to the eigenvalues $\gamma_5 = \pm 1$.

where e is the charge of the electron; $g_1 \sim g^{1/2}$, and g is the constant of the Fermi interaction. We now assume that all three currents $j^{(0)}$, $j^{(+)}$, and $j^{(-)}$ have the same isotopic structure.* Since for all fermions the electric current $j^{(0)}$ has a structure of the form $B/2 + \tau_Z$, the corresponding structure of the weak currents must have the form

$$j_\mu^{(+)} \sim (B/2 + \tau_+) \text{ and } j_\mu^{(-)} \sim (B/2 + \tau_-).$$

Here the operator B is defined by $B = l + n + S'$.

The correct expression for the electric current of the fermions is obtained from the following more general expression:

$$i(\bar{l}_1 + \bar{l}_2 + \overline{N} + \bar{\widetilde{Y}} + \bar{Z} + \bar{\widetilde{\Xi}}) \times (B/2 + \tau_z) O_\mu (l_1 + l_2 + N + \widetilde{Y} + Z + \widetilde{\Xi}),$$

$$l_1 = \begin{pmatrix} \nu_1 \\ e^- \end{pmatrix}, \quad l_2 = \begin{pmatrix} \nu_2 \\ \mu^- \end{pmatrix}, \quad O_\mu = \gamma_\mu (1 + \gamma_5) \tag{5}$$

by imposing the requirements of conservation of the electric, leptonic, and baryonic charges, of conservation of chirality in weak interactions,[7] and of gauge invariance of the interaction.[9] It has the usual form

$$j_\mu^{(0)} = i[(\bar{l}_1 \gamma_\mu (B/2 + \tau_z) l_1) + \ldots + (\bar{\widetilde{\Xi}} \gamma_\mu (B/2 + \tau_z) \widetilde{\Xi})]$$
$$= -i[(\bar{e}\gamma_\mu e) + (\bar{\mu}\gamma_\mu \mu) - (\bar{p}\gamma_\mu p) + (\bar{\widetilde{\Sigma}}^+ \gamma_\mu \widetilde{\Sigma}^+) + (\bar{\Sigma}^- \gamma_\mu \Sigma^-) - (\bar{\widetilde{\Xi}}^- \gamma_\mu \widetilde{\Xi}^-)]. \tag{6}$$

In a similar way we get the expressions for the weak currents from general expressions of the type of Eq. (5) by using the conservation laws that have been mentioned (without gauge invariance)†

$$i(\bar{l}_1 + \bar{l}_2 + \overline{N} + \bar{\widetilde{Y}} + \bar{Z} + \bar{\widetilde{\Xi}})(B/2 + \tau_-) O_\mu (l_1 + l_2 + N + \widetilde{Y} + Z + \widetilde{\Xi}) \rightarrow j_\mu^{(-)} = i[(\bar{l}_1 O_\mu \tau_- l_1) + \ldots + (\bar{\widetilde{\Xi}} O_\mu \tau_- \widetilde{\Xi})]$$
$$\pm \frac{1}{2} i[(\bar{Z} O_\mu N) + (\bar{\widetilde{Y}} O_\mu \widetilde{\Xi})] = i[(\bar{e} O_\mu \nu_1) + (\bar{\mu} O_\mu \nu_2) + (\bar{n} O_\mu p) + (\bar{\widetilde{\Sigma}}^+ O_\mu \widetilde{Y}^0) + (\bar{\Sigma}^- O_\mu Z^0) + (\bar{\widetilde{\Xi}}^0 O_\mu \widetilde{\Xi}^-)]$$
$$+ \frac{1}{2} i[(\bar{Z}^0 O_\mu p) + (\bar{\Sigma}^- O_\mu n) + (\bar{\widetilde{Y}}^0 O_\mu \widetilde{\Xi}^-) + (\bar{\widetilde{\Sigma}}^+ O_\mu \widetilde{\Xi}^0)], \tag{7}$$

$$j_\mu^{(+)} = j_\mu^{(-)*}. \tag{8}$$

Thus the violation of isotopic invariance in the weak and electromagnetic interactions is of precisely the same character for all. We note that conservation of chirality in the expressions (5) and (7) is attained automatically, and the other requirements are imposed from outside. The requirement of gauge invariance, for example, leads to the exclusion of terms that do not conserve parity.[11,12] Furthermore the electric charge is regarded as a conserved integer quantity on an equal footing with the leptonic and baryonic charges and only "accidentally" (the isotopic structure $\frac{1}{2} + \tau_Z$ of the current) coincides with the coupling constant with the electromagnetic field.*

The expressions that have been obtained for the weak currents evidently allow us to explain all existing experimental data on the decays of ordinary and strange particles: they contain no terms with change of the strangeness S by two units; the empirical rule $\Delta S/\Delta e = +1$ is satisfied for processes with change of strangeness;[2] and there are no terms that lead to the production of pairs in the decay of hyperons.†

It is easy to see that these results are a direct consequence of our choice of the values of the chiralities of hyperons and the requirement of conservation of chirality in weak interactions.‡ This choice is not an accidental one, but is occasioned by the following considerations. It is obvious that one can carry out a classification of leptonic doublets in the weak interactions according to all possible combinations of leptonic charge, chirality, and sign of electric charge. The possible leptonic doublets are shown in Table II. In the leptonic case the primed doublets are not observed.

TABLE II

	l	γ_5	e
l_1	−1	+1	−1
l_2	+1	+1	−1
$\widetilde{l}_1$	+1	−1	+1
$\widetilde{l}_2$	−1	−1	+1
l'_1	−1	+1	+1
l'_2	+1	+1	+1
$\widetilde{l}'_1$	+1	−1	−1
$\widetilde{l}'_2$	−1	−1	−1

This is evidently connected with the fact that the mass of the neutrino is identically zero [if the primed and unprimed leptonic doublets existed simultaneously in the "world," then each of the

*The analogy between the weak and electric currents based on the idea of an intermediate vector meson has been studied by Schwinger and by Salam and Ward.[5,6]

†According to the hypothesis of the conserved vector current, in all three of these currents one must add the corresponding mesonic currents.

*This conception of the electric charge is clearly expressed in a paper by Zel'dovich[8] (cf. also reference 9).

†For discussion see a paper by Dalitz.[10]

‡It must be noted that "chirality" is not a charge, since it is conserved only in the weak currents.

two neutrinos and antineutrinos would occur in two different doublets ($\nu_1' \equiv \nu_1$, etc.)]. If in Table II we replace the leptonic charge by the baryonic charge, we get a table of possible baryonic doublets. Evidently they all actually exist (cf. Table III).

TABLE III

	n	γ_5	e
$\tilde{Y}$	−1	+1	−1
Z	+1	+1	−1
Y	+1	−1	+1
$\tilde{Z}$	−1	−1	+1
$\tilde{\Xi}$	−1	+1	+1
N	+1	+1	+1
Ξ	+1	−1	−1
$\tilde{N}$	−1	−1	−1

We note that the scheme we have given for the weak and electromagnetic interactions of "bare" particles, in which definite chiralities are ascribed to the particles, corresponds to the general feature of these interactions, that they are incapable of giving rise to a finite proper mass of particles when this mass is initially zero (this result is a direct consequence of the fact that these interactions do not change the chirality of the particles), and therefore in the absence of strong interactions all the particles involved in them can be regarded as devoid of rest mass. The strong interactions remove the degeneracy, but do not change the number of independent particles.

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