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ON THE BUILD-UP OF ELECTROMAGNETIC WAVES IN A PLASMA MOVING IN A NON-DISPERSIVE DIELECTRIC IN THE PRESENCE OF A CONSTANT MAGNETIC FIELD

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We have obtained a dispersion relation that determines the propagation of plane electromagnetic waves in a plasma beam moving in a fixed plasma along the lines of force of a constant and uniform magnetic field. We have found the damping (or build-up) coefficients of the rarefied plasma moving along the magnetic field in a non-dispersive dielectric.

RECENTLY several papers have been published on the propagation of electromagnetic waves in interpenetrating moving media, in particular in plasma beams moving in a fixed plasma or in a non-dispersive dielectric.[1-3] The range of problems for which suitable solutions have been given in sufficient completeness and for which expressions have been obtained for the damping (or build-up) coefficients of the waves is all the same very limited. We attempt in the present paper to remedy this situation partly by solving the problem of the build-up (damping) coefficient of plane electromagnetic waves propagating in a plasma moving in a non-dispersive medium along the lines of force of a constant and uniform magnetic field.

We shall determine the damping coefficients of the waves by a phenomenological method proposed earlier.[2] The main point of this method is that we first find the phenomenological equations that relate the electromagnetic fields in the interpenetrating media. By combining these equations with the Maxwell equations describing the propagation of plane electromagnetic waves, we can obtain a dispersion relation of the form $f(\omega, \mathbf{k}) = 0$, where ω is the frequency of the electromagnetic wave and $\mathbf{k}$ its wave vector. Solving the dispersion relation we find the required damping coefficients for the wave.

We shall derive the dispersion relation for the case where a non-magnetic, anisotropic, and gyrotropic medium I moves in a fixed non-magnetic medium II, which is also anisotropic and gyrotropic. (A plasma in a constant magnetic field has such properties.) The medium II is at rest in the laboratory frame of reference, and its electrical properties are characterized by its dielectric-constant tensor ϵ_{ij}. The medium I, which is related to a frame of reference K′, moves uniformly with respect to the medium II and in a straight line along the x axis of the frame K with a velocity v, so that the x and the x′ axes coincide and are parallel to the lines of force of the external magnetic field $\mathbf{H}$, while the y′ and z′ axes are parallel to the y and z axes.

The electrical properties of medium I in the frame K′ are characterized by the tensor ϵ'_{ij}.* Since we have assumed that media I and II are gyrotropic, the tensors ϵ_{ij} and ϵ'_{ij} are Hermitian and thus $\epsilon_{ij} = \epsilon^*_{ji}$ and $\epsilon'_{ij} = \epsilon'^*_{ji}$. As we have in view a plasma in a constant external magnetic field, we shall also assume[4] that $\epsilon_{12} = \epsilon_{13} = \epsilon'_{13} = \epsilon'_{12} = 0$.

Once we know the tensors ϵ_{ij} and ϵ'_{ij}, we can determine the tensors of the moments (the polarization and magnetization tensors) $M_{ij}(\mathrm{II}, K)$ and $M_{ij}(\mathrm{I}, K')$ in the fixed and the moving media. The components of the electromagnetic fields $\mathbf{D}$, $\mathbf{B}$, $\mathbf{E}$, and $\mathbf{H}$ which are defined in the frame of reference K, enter, of course, also into the tensor $M_{ij}(\mathrm{II}, K)$, and into the tensor $M_{ij}(\mathrm{I}, K')$ the components of the fields $\mathbf{D}'$, $\mathbf{B}'$, $\mathbf{E}'$, and $\mathbf{H}'$ in the frame of reference K′, which are found from the unprimed fields by a Lorentz transformation. Using a Lorentz transformation to transform the tensor of the moments $M_{ij}(\mathrm{I}, K')$ to the frame of reference K, we can find the resulting tensor of the moments in the frame K

$$M_{ij}(\mathrm{I}, \mathrm{II}, K) = M_{ij}(\mathrm{II}, K) + M_{ij}(\mathrm{I}, K) = (H_{ij} - F_{ij})/4\pi, \qquad (1)$$

*We assume that the effective electrical field in the presence of two media is equal to the average macroscopic field, since the polarizations are additive only in that case. It is well known that the effective and the average macroscopic fields are equal in a plasma and also in any other sufficiently rarefied medium.[4]

where H_{ij} and F_{ij} are the electromagnetic field tensors in the frame of reference K (refrence 5; cf. footnote on preceding page).

After substituting into the Maxwell equations which contain the curl operator the expressions for plane waves, these equations become

$$\mathbf{D} = -n[\nu\times\mathbf{H}], \qquad \mathbf{B} = n[\nu\times\mathbf{E}], \tag{2}$$

where n is the refractive index of the medium in the frame of reference K, and ν is a unit vector in the direction of propagation of the wave. The condition that there be a nontrivial simultaneous solution of the algebraic equations (1) and (2) for the 12 components of the electromagnetic field vectors means that the determinant of the set (1) and (2) must vanish. The 12-th order determinant easily reduces to a 4-th order one. Writing the latter out in detail we can get, after straightforward, though tedious transformations, the following dispersion relation

$$(\varepsilon_{11} + \varepsilon_{11}' - 1)[(\varepsilon_{22} + \overset{*}{\varepsilon}_{22}\gamma^2 - n^2\cos^2\theta)(\varepsilon_{33} + \overset{*}{\varepsilon}_{33}\gamma^2 - n^2) + (\varepsilon_{23} + \overset{*}{\varepsilon}_{23}\gamma^2)^2] - n^2\sin^2\theta\,[(\varepsilon_{22} + \overset{*}{\varepsilon}_{22} - \beta^2\varepsilon_{22}\overset{*}{\varepsilon}_{22}) \times (\varepsilon_{33} + \overset{*}{\varepsilon}_{33}\gamma^2 - n^2) + (\varepsilon_{23} + \overset{*}{\varepsilon}_{23}\gamma)^2 - \beta^2(\overset{*}{\varepsilon}_{22}\varepsilon_{23}^2 + \varepsilon_{22}\overset{*2}{\varepsilon}_{23}\gamma^2)] = 0;$$

$$\overset{*}{\varepsilon}_{ii} = (\varepsilon_{ii}' - 1)/(1-\beta^2), \qquad \overset{*}{\varepsilon}_{23} = \varepsilon_{23}'/(1-\beta^2),$$

$$\gamma = 1 - n\beta\cos\theta, \qquad \beta = v/c. \tag{3}$$

The most interesting case of propagation of electromagnetic waves, which can be studied by using Eq. (3), is the case where a plasma moves in a fixed plasma along the lines of force of a constant and uniform magnetic field. In the following, however, we study the simpler case where the plasma moves along the magnetic field in a non-dispersive dielectric. The dispersion relation (3) simplifies then considerably and one can obtain relatively simple solutions after a few additional assumptions.

Assuming that the fixed medium is a non-dispersive dielectric ($\epsilon_{23} = 0$, $\epsilon_{ij} = \epsilon$), we get by rearranging the terms in (3)*

$$\varepsilon(\varepsilon - n^2)^2 + \overset{*}{\varepsilon}_{22}(\varepsilon - n^2)(\varepsilon\gamma^2 + \chi) + \gamma^2\chi(\overset{*2}{\varepsilon}_{22} + \overset{*2}{\varepsilon}_{23}) + \overset{*}{\varepsilon}_{11}[(\varepsilon - n^2)(\varepsilon - n^2\cos^2\theta + \overset{*}{\varepsilon}_{22}\gamma^2) + \overset{*}{\varepsilon}_{22}\gamma^2(\varepsilon - n^2\cos^2\theta + \overset{*}{\varepsilon}_{22}\gamma^2) + \overset{*2}{\varepsilon}_{23}\gamma^4] = 0; \tag{4}$$

where

$$\overset{*}{\varepsilon}_{11} = \varepsilon_{11}' - 1 = -\omega_0^2(1-\beta^2)/(\omega - \tilde{\omega})^2,$$

$$\overset{*}{\varepsilon}_{22} = -\omega_0^2/\left((\omega - \tilde{\omega})^2 - \overset{*2}{\omega}_H\right),$$

$$\overset{*}{\varepsilon}_{23} = -i\omega_0^2\overset{*}{\omega}_H/(\omega - \tilde{\omega})\left[(\omega - \tilde{\omega})^2 - \overset{*2}{\omega}_H\right],$$

$$\overset{*}{\omega}_H = \omega_H\sqrt{1-\beta^2}, \qquad \chi = \varepsilon\gamma^2 - n^2\sin^2\theta(1 - \varepsilon\beta^2),$$

$$\tilde{\omega} = kv\cos\theta = \omega n\beta\cos\theta.$$

The dispersion relation (4) establishes a connection between the frequency ω of the electromagnetic wave and its wave vector **k**. In order to establish the presence of damped (or growing) waves and to find the damping coefficient it is necessary to solve Eq. (4) for ω with real wave numbers k. The presence of plane electromagnetic waves which are damped in time is indicated in this procedure by the existence of complex solutions for ω.*

We study in the following the solutions of the dispersion relation (4) for a rarefied plasma beam, i.e., under the condition that the plasma frequency $\omega_0 = (4\pi e^2N/m)^{1/2}$ is small compared with the frequency of the wave ω and compared with the characteristic frequencies $\tilde{\omega} = kv\cos\theta$ and ω_N^*. When, however, we go over to the limiting case of no magnetic field ($\omega_H^* = 0$), we shall assume that $\omega_0 \ll \omega, \tilde{\omega}$. In solving Eq. (4) we must distinguish between two cases. In the first case none of the denominators in the expressions for the ϵ_{ij}^* tends to zero as $\omega_0 \to 0$, while in the second case at least one of the denominators in the ϵ_{ij}^* tends to zero as $\omega_0 \to 0$. We study first the character of the solutions of Eq. (4) for the first case.

We look for the solution by the method of successive approximations, writing for the refractive index $n = \sqrt{\epsilon} + \Delta n$. Substituting this expression into (4) we find

$$4\varepsilon^2(\Delta n)^2 - 2\sqrt{\varepsilon}\,[\overset{*}{\varepsilon}_{22}(\varepsilon\gamma^2 + \chi) + \overset{*}{\varepsilon}_{11}\varepsilon\sin^2\theta]\,\Delta n + \gamma^2\chi(\overset{*2}{\varepsilon}_{22} + \overset{*2}{\varepsilon}_{23}) + \overset{*}{\varepsilon}_{22}\overset{*}{\varepsilon}_{11}\gamma^2\varepsilon\sin^2\theta + \overset{*2}{\varepsilon}_{23}\overset{*}{\varepsilon}_{11}\gamma^4 = 0.$$

*The expressions for ε_{11}^*, ε_{22}^*, and ε_{23}^* were obtained by taking into account the fact that in going to the system of reference K′, which moves with the plasma, we must substitute in the expression for the components of the tensor ε_{ij}' the frequency ω' transformed according to the Doppler formula $\omega' = \omega(1 - n\beta\cos\theta)/\sqrt{1-\beta^2}$. The frequency $\omega_H^* = \omega_H\sqrt{1-\beta^2}$ is the gyro frequency in the plasma beam from the point of view of an observer in the frame of reference K. The occurrence of the factor $\sqrt{1-\beta^2}$ is connected with the Lorentz transformation of the time.

*If we solve the dispersion relation for k for real values of ω, as was done in reference 3, the solutions for k can in principle also be complex. For the case of infinite media, considered here, the selection of solutions with complex k which correspond to a true build-up is difficult, since complex solutions for k can result from the "entrainment" of waves by the moving medium.

If we separate explicitly the factor containing ω_0 from the coefficients of $(\Delta n)^2$, of Δn, and of the constant term, this equation becomes

$$a_2 \Delta n^2 + 2a_1 \omega_0^2 \Delta n + a_0 \omega_0^4 = 0,$$

where the coefficients a_0, a_1, and a_2 no longer contain the parameter ω_0. The solution of this equation, which is quadratic in Δn, is such that $\Delta n \sim \omega_0^2$. Since ω, n, and the wave number k, which we fix and assume to be real, are connected through the relation $\omega n = kc$, we can assert that the correction to the frequency ω of the wave, due to the presence of a plasma beam, must also be of the order ω_0^2.

We shall show in the following that in the second case, when the denominators in the ϵ_{ij}^* tend to zero as $\omega_0 \to 0$, i.e., when some peculiar resonance takes place, we obtain a correction to the frequency of the order ω_0, or an even more appreciable one. This means that if the solutions of the first kind include complex solutions, the damping of the waves determined by them will for rarefied plasma beams be negligibly small compared with the damping of the waves as determined by solutions of the second kind. When solving the dispersion relation (4) for resonances in the ϵ_{ij}^*, we must again distinguish between two different cases.

1. The case of strong perturbations. a) Putting $\omega - \tilde{\omega} = \xi$, we assume that ξ, $\omega_0 \ll \omega$, ω_H^* and we retain in the dispersion relation (4) terms without ω_0 and terms containing ω_0^2/ξ^2, dropping all other terms, since they are small compared with the terms retained. Then Eq. (4) becomes

$$\varepsilon(\varepsilon - n^2) - \omega_0^2 \xi^{-2}(1-\beta^2)(\varepsilon - n^2\cos^2\theta) = 0. \quad (5)$$

Solving Eq. (5) for ξ and taking into account that $n\beta\cos\theta = 1$, we find*

$$\xi = \pm\omega_0 \left(\frac{(1-\beta^2)(1-\varepsilon\beta^2)}{\varepsilon(1-\varepsilon\beta^2\cos^2\theta)}\right)^{1/2}\cos\theta. \quad (6)$$

It follows from (6) that the correction to the frequency turns out to be of the order ω_0. If $\epsilon\beta^2 > 1$ and $\epsilon\beta^2\cos^2\theta < 1$, the quantity ξ turns out to be imaginary, which indicates the occurrence of waves that are damped or are building up in time. There is instability outside the cone determined by the Cerenkov condition $\epsilon\beta^2\cos^2\theta = 1$. This case and the two next cases discussed below could perhaps be called cases of strong perturbations, as the difference $\epsilon - n^2$ is generally speaking not small and does not tend to zero as $\omega_0 \to 0$. That $\epsilon - n^2$ does not tend to zero as $\omega_0 \to 0$ is, of course, a consequence of neglecting in all expressions given above the corrections necessitated by taking into account thermal motion in the plasma beam. It is physically evident that when thermal motion is taken into account the refractive index n must tend to $\sqrt{\epsilon}$ when the concentration in the plasma beam is reduced. We shall formulate the condition for the validity of the results given here, which are obtained by neglecting thermal motion, at the end of the paper.

b) In the limiting case of no magnetic field $(\omega_H^* = 0)$ we put, as before, $\omega - \tilde{\omega} = \xi$ and assume that ξ, $\omega_0 \ll \omega$. Retaining in the dispersion relation (4), as before, terms without ω_0 and terms with ω_0^2/ξ^2, and dropping all other terms which are small compared with the ones retained, we get

$$\varepsilon(\varepsilon - n^2) - \omega_0^2\xi^{-2}[(1-\beta^2)(\varepsilon - n^2\cos^2\theta) - n^2\sin^2\theta(1-\varepsilon\beta^2)] = 0. \quad (7)$$

Solving Eq. (7) for ξ under the condition $n\beta\cos\theta = 1$ we find

$$\xi = \pm\omega_0[(1-\varepsilon\beta^2)(1-\beta^2\cos^2\theta)/\varepsilon(1-\varepsilon\beta^2\cos^2\theta)]^{1/2}. \quad (8)$$

The correction to the frequency is found to be of the order ω_0, as in the preceding case, and instability occurs when $\epsilon\beta^2 > 1$ and $\epsilon\beta^2\cos^2\theta < 1$, i.e., outside the Cerenkov cone. We note also that the solution (8), as is the case for the solution (6), is not valid in the neighborhood of the Cerenkov cone $(\epsilon\beta^2\cos^2\theta \to 1)$, for then $\xi \to \infty$.

c) We assume now that the resonance occurs at frequencies near $\tilde{\omega} \pm \omega_H^*$ and we put $\omega - \tilde{\omega} \pm \omega_H^* = \xi$. Proceeding as in the preceding cases, we get from the dispersion relation (4)

$$\varepsilon(\varepsilon - n^2) - \omega_0^2(\varepsilon\gamma^2 + \chi)/(\omega - \tilde{\omega} \mp \omega_H^*)\xi = 0 \quad (9)$$

and hence

$$\xi = \omega_0^2(\varepsilon\gamma^2 + \chi)/\varepsilon(\varepsilon - n^2)(\omega - \tilde{\omega} \mp \omega_H^*). \quad (10)$$

The correction to the frequency turns out to be real and proportional to ω_0^2. There is therefore no instability in this case. The solution found here is invalid when $\epsilon - n^2 \approx 0$, i.e., when $\epsilon\beta^2 \times \cos^2\theta \approx n^2\beta^2\cos^2\theta \approx \tilde{\omega}^2/(\tilde{\omega} \mp \omega_H^*)^2$. The solution when $\epsilon - n^2 \to 0$ is studied below.

2. The case of weak perturbations combines the solutions of the dispersion relation (4) under the condition $\epsilon - n^2 \to 0$ as $\omega_0 \to 0$.

a) We put $\omega - \tilde{\omega} = \xi$ and assume as before that ξ, $\omega_0 \ll \omega$, ω_H^*. Under the condition $\epsilon\beta^2 \times \cos^2\theta = 1$, which is the same as the condition for Cerenkov radiation of a single charged particle,

$$n^2 = \frac{n^2\omega^2\beta^2\cos^2\theta}{\omega^2\beta^2\cos^2\theta} = \frac{\varepsilon\tilde{\omega}^2}{\omega^2} = \varepsilon\frac{\tilde{\omega}^2}{(\tilde{\omega}+\xi)^2}$$

*The equation $n\beta\cos\theta = 1$ follows immediately from the condition $\xi/\omega \ll 1$ since $\omega - \tilde{\omega} = \omega(1 - n\beta\cos\theta) = \xi$.

or

$$\varepsilon - n^2 = \varepsilon(1 - \tilde{\omega}^2/(\tilde{\omega} + \xi)^2) \approx \varepsilon 2\xi/\tilde{\omega}.$$

The dispersion relation (4) is then appreciably simplified

$$2\varepsilon^2\xi/\tilde{\omega} - \omega_0^2\xi^{-2}(1-\beta^2)(\varepsilon - n^2\cos^2\theta) = 0. \quad (11)$$

Solving Eq. (11) for ξ and assuming that $n^2 \approx \epsilon$ we find

$$\xi = (\omega_0^2\tilde{\omega}(1-\beta^2)\sin^2\theta/2\varepsilon)^{1/3}. \quad (12)$$

It follows from (12) that the correction to the frequency $\xi \sim \omega_0^{2/3}$ turns out to be larger than in the preceding cases. ξ tends to 0 more slowly than ω_0, as $\omega_0 \to 0$. When we evaluate the root in (12), two values turn out to be complex, one of which corresponds to a wave that builds up in time.

b) Putting the external magnetic field equal to zero ($\omega_H^* = 0$) we assume as before $\omega - \tilde{\omega} = \xi$ and $\epsilon\beta^2\cos^2\theta = 1$, so that $n^2 = \epsilon\tilde{\omega}^2/(\tilde{\omega} + \xi)^2$. Upon suitable obvious simplification, the dispersion relation (4) becomes

$$2\varepsilon^2\xi/\tilde{\omega} - \omega_0^2\xi^{-2}[(1-\beta^2)(\varepsilon - n^2\cos^2\theta) - n^2(1-\varepsilon\beta^2)\sin^2\theta] = 0. \quad (13)$$

Using the equation $n^2 \approx \epsilon$ and eliminating β^2, we can write the solution of Eq. (13) in the form

$$\xi = (\omega_0^2\tilde{\omega}(\varepsilon - 1)\tan^2\theta/2\varepsilon^2)^{1/3}, \quad (14)$$

which shows the existence of increasing waves. Equation (14) has been obtained earlier.[2] It was shown then that it determines the build-up of an electromagnetic wave, which has a component of the electrical field parallel to the velocity of the plasma beam, i.e., a wave such as occurs in the Cerenkov effect for a single charged particle.

c) As in the case of strong perturbations, we now assume that $\xi = \omega - \tilde{\omega} - \omega_H^*$ and $\xi \ll \omega$, ω_H^*. In order that $\epsilon - n^2$ tend to zero as $\omega_0 \to 0$ it is necessary, as already noted, to put $\epsilon\beta^2\cos^2\theta = \tilde{\omega}^2/(\tilde{\omega} + \omega_H^*)^2$. The result is

$$n^2 = \varepsilon(\tilde{\omega} + \omega_H^*)^2/(\tilde{\omega} + \omega_H^* + \xi)^2, \quad (\varepsilon - n)^2 \approx 2\varepsilon\xi/(\tilde{\omega} + \omega_H^*),$$

and the dispersion relation is, after the usual simplifications, of the form*

$$2\varepsilon^2\xi/(\tilde{\omega} + \omega_H^*) - \omega_0^2(\varepsilon\gamma^2 + \chi)/\xi(\omega - \tilde{\omega} + \omega_H^*) = 0. \quad (15)$$

Solving Eq. (15) for ξ we find

$$\xi = \pm\omega_0[\omega_H^*/2\varepsilon(\tilde{\omega} + \omega_H^*) - (\tilde{\omega} + \omega_H^*)(1-\varepsilon\beta^2)\sin^2\theta/4\varepsilon\omega_H^*]^{1/2} \quad (16)$$

*If the plasma beam moves along a constant magnetic field in vacuo ($\varepsilon = 1$), Eq. (15) for the case of the propagation of a wave along the direction of motion of the plasma beam ($\theta = 0$) goes over into the dispersion relation studied in a paper by Twiss.[6]

Equation (16) is obtained under the condition $\epsilon\beta^2 \times \cos^2\theta = \tilde{\omega}^2/(\tilde{\omega} + \omega_H^*)^2 < 1$. When the velocity of the plasma beam exceeds that of light, $\epsilon\beta^2 > 1$, the expression under the square root sign is positive, and the correction to the frequency is a real quantity indicating that there is no build-up in the case under consideration.

d) If $\xi = \omega - \tilde{\omega} + \omega_H^*$ and $\epsilon\beta^2\cos^2\theta = \tilde{\omega}^2/(\tilde{\omega} - \omega_H^*)^2$, the sign of ω_H^* changes in the dispersion relation. The solution of Eq. (15) then becomes

$$\xi = \pm\omega_0[-\omega_H^*/2\varepsilon(\tilde{\omega} - \omega_H^*) + (\tilde{\omega} - \omega_H^*)(1-\varepsilon\beta^2)\sin^2\theta/4\varepsilon\omega_H^*]^{1/2}. \quad (17)$$

When $\sqrt{\epsilon}\,\beta\cos\theta > 1$, $\omega_H^* < \tilde{\omega}$, we have $\epsilon\beta^2 > 1$, and the correction to the frequency turns out to be imaginary showing the existence of increasing waves. It follows from (16) and (17) that the criterion for instability is in this case the inequality $\sqrt{\epsilon}\,\beta\cos\theta > 1$. This criterion was formulated by Zheleznyakov.[1]

If the inequality $\sqrt{\epsilon}\,\beta\cos\theta > 1$ is satisfied, the component of the beam velocity along the direction of propagation of the wave turns out to be larger than the phase velocity of the wave. An interpretation of the phenomena that occur then and are connected with the build-up of the electromagnetic wave is contained in the cited paper by Zheleznyakov. We note that the instability determined by Eq. (17) occurs also for $\theta = 0$, i.e., for a wave propagated in the direction of motion of the plasma beam.

It was noted in the foregoing that we did not take into account the thermal motion in the plasma beam when deriving the dispersion relation. One can obtain a criterion for the validity of the solutions found here. Zheleznyakov[1] found in the nonrelativistic approximation a dispersion relation for the case of interpenetrating plasma beams, taking thermal motion into account. The form of this dispersion relation is such that the thermal correction remains inessential, as long as an inequality of the kind

$$(\omega - \tilde{\omega} \pm \omega_H)/kv_T \gg 1, \quad (18)$$

where v_T is the velocity of the thermal motion of the electrons in the plasma, is satisfied. The meaning of inequality (18) becomes clear if one takes into account that the required correction to the frequency $\xi = \omega - \tilde{\omega} \pm \omega_H$ and that when there is thermal motion the parameter $\tilde{\omega} = kv\cos\theta$ is spread over a range $\Delta\tilde{\omega} \approx kv_T$. It is clear that the thermal motion can be neglected if $\xi = \omega - \tilde{\omega} \pm \omega_H \gg \Delta\tilde{\omega} \approx kv_T$, i.e., just when inequality (18) is satisfied.

In the case of weak perturbations considered above, when $\epsilon - n^2 \approx \epsilon\,\xi/\widetilde{\omega}$, the inequality $\Delta\widetilde{\omega}/\xi \ll 1$ can easily be shown to lead to the inequality $|c/\sqrt{\epsilon} - c/n| \gg v_T$. (This was pointed out by V. V. Zheleznyakov.) This means that the difference between the velocity of propagation of an electromagnetic wave in a dielectric without a plasma and the velocity of a wave when a plasma beam is present must be much larger than the velocity of the thermal motion in the plasma.

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