EXCITATION OF FISSION FRAGMENTS AND THEIR MASS DISTRIBUTION

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The influence of shell effects on elastic constants and mass coefficients of vibrational degrees of freedom of a nucleus, on the mass distribution of fission fragments, and on the magnitude of their excitation energy is considered.

1. The fission asymmetry and the excitation of the fragments have previously been considered.[1,2] It was shown that the degree of asymmetry and the magnitude of the excitation energy of the fragments depend on the values of the elastic constants for vibrational degrees of freedom of the fragments, and the excitation energy also depends on the mass coefficients of these degrees of freedom.

In references 1 and 2 it was noted that experiments on the Coulomb excitation of nuclei give evidence of a considerable deviation of the true values of the elastic constants and mass coefficients from their theoretical values calculated on the basis of the liquid-drop model. However, in references 1 and 2 elastic constants and mass coefficients calculated on the basis of the liquid-drop model were used in estimating the degree of asymmetry and the excitation energies of the fragments. Recently, Belyaev[3] showed a method for calculating the elastic constants and mass coefficients for vibrational quadrupole degrees of freedom of the nucleus, taking account of shell effects and of nucleon-nucleon interaction. According to Belyaev,[3] the potential energy of the nucleus, which depends on the deformation, has the form (a simpler case, when only one type of nucleon is present, was considered in reference 3)

$$U_d = \frac{1}{2}(k-\varkappa)Q_c^2 + \frac{1}{2}\varkappa\gamma_p Q_p^2 + \frac{1}{2}\varkappa\gamma_n Q_n^2 - \varkappa(Q_cQ_p + Q_cQ_n + Q_pQ_n), \tag{1}$$

where Q_c, Q_p, Q_n are the (volume) quadrupole moments of the filled shell, and of the protons and nucleons outside the filled shell, respectively.

For small Q_p and Q_n

$$\gamma_p = \frac{\Theta_{p0}}{\Theta_p(1-\varkappa/k)} - 1, \qquad \gamma_n = \frac{\Theta_{n0}}{\Theta_n(1-\varkappa/k)} - 1;$$

Θ_{p0} and Θ_{n0} are numerical parameters characteristic of the given shell, and Θ_p and Θ_n are the occupation factors:

$$\Theta_p = \frac{4N_p}{N_{p\,max}}\left(1 - \frac{N_p}{N_{p\,max}}\right), \tag{2}$$

where N_p is the number of protons in the unfilled shell, and $N_{p\,max}$ is the maximum number of protons possible in this shell. Θ_n has an analogous form. The notation is the same as in reference 3. The values of the elastic constants k and $\varkappa$ must be found from experiment ($\varkappa \approx k/2$, cf. reference 3).

The kinetic energy T is of the form

$$T = \frac{1}{2}B_c\dot{Q}_c^2 + \frac{1}{2}B_p\dot{Q}_p^2 + \frac{1}{2}B_n\dot{Q}_n^2;$$

B_c has the same meaning as in the liquid drop model: $B_c = 5m/24AR^2$; B_p and B_n are considerably larger:

$$B_p = \frac{\hbar^2}{(GN_{p\,max}/2)^2}\frac{\Theta_{p0}}{\Theta_p}\frac{\varkappa k}{k-\varkappa};$$

for B_n we have an analogous expression. Here m is the nucleon mass, A — the atomic weight, R — the nuclear radius, and G — the mean matrix element of the interaction energy; G is connected with the magnitude of the energy gap. The normal coordinates Q_1, Q_2, Q_3, and the frequencies of the normal vibrations for the Hamiltonian $H = T + U_d$ are easily found. The main role is played by the lowest frequency ω_1

$$\omega_1^2 \approx \frac{\varkappa k}{2(k-\varkappa)}\left\{\frac{1}{B_p}\left(\frac{\Theta_{p0}}{\Theta_p}-1\right) + \frac{1}{B_n}\left(\frac{\Theta_{n0}}{\Theta_n}-1\right) - \left[\left(\frac{1}{B_p}\left(\frac{\Theta_{p0}}{\Theta_p}-1\right) - \frac{1}{B_n}\left(\frac{\Theta_{n0}}{\Theta_n}-1\right)\right)^2 + \frac{4}{B_nB_p}\right]^{1/2}\right\}; \tag{3}$$

if ω_1 is real, the nucleus is spherical.[3]

The probability of Coulomb excitation is determined by the matrix element of the operator $Z_cQ_c/A_c + Q_p$ of the oscillator ψ functions for Q_1. Comparing the theoretical values of the frequencies ω_1 thus found and the probabilities of Coulomb excitation with the experimental data on the values of the vibrational levels and on the probability of Coulomb excitation (cf. reference 4), it is possible to find Θ_{n0}, Θ_{p0}, κ, and k (S. T. Belyaev, private communication).

The potential energy of the fissioning nucleus before scission is determined by the deformation energy of the future fragments; this deformation energy depends on the total deformation of the fragments, i.e., on the total quadrupole moment $Q = Q_c + Q_p + Q_n$ (cf. reference 1). Since the deformation process of the fissioning nucleus is quasistatic, it can be assumed that Q_p and Q_n take on values which correspond to a minimum of the energy U_d. Thus, to find the deformation energy of a fragment before scission, the potential energy U_d in (1) must be expressed in terms of Q, Q_p, and Q_n, and minimized with respect to Q_p and Q_n for a given Q. Here $U_d = C_Q Q^2/2$, where

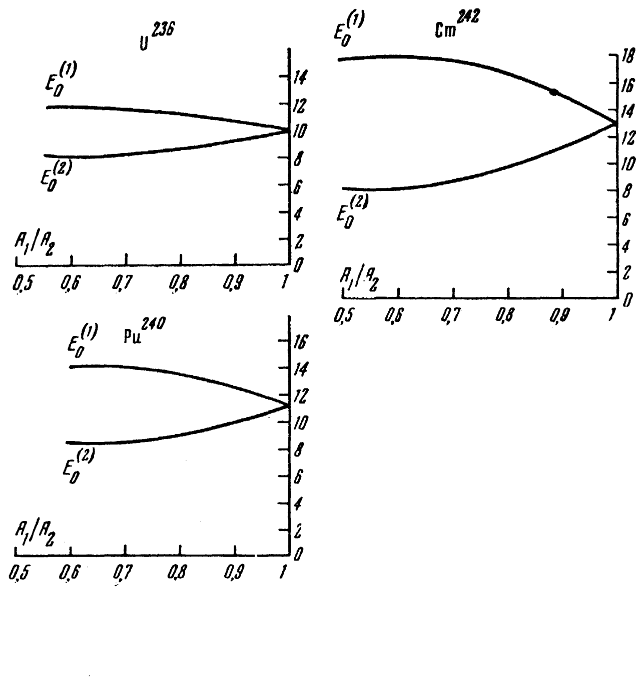
$$C_Q = \varkappa \frac{1 - \Theta_p / \Theta_{p0} - \Theta_n / \Theta_{n0}}{\Theta_p / \Theta_{p0} + \Theta_n / \Theta_{n0} + \varkappa / (k - \varkappa)}. \qquad (4)$$

It is easily seen that C_Q has maxima for filled shells. However, these maxima are not as sharp as the maxima of the frequency ω_1, of the elastic constant, and the mass coefficient for the normal coordinate Q_1 with a frequency ω_1. These sharp maxima are observed on the experimental curves of the elastic constants and mass coefficients in references 4 and 5. In C_Q values of k, κ, Θ_{n0}, Θ_{p0}, found from a comparison with experimental data should be substituted (see above).

Since C_Q has a maximum for filled shells, the sum of the deformation energy of both fragments will, for the same parameters of the fragment deformation Q, be larger for magic than for non-magic fragments. This effect will counteract the shell effects as far as the binding energy of spherical nuclei is concerned[1] and, consequently, will not favor asymmetric fission. However, an estimate of C_Q according to (4) shows that with the new values of C_Q the energy minimum of the fissioning nucleus is increased only by 1 — 2 Mev compared with calculations based on the liquid drop model,[1] and corresponds, as before, to asymmetric fission. Numerical calculations of the energy of the fissioning nucleus with new values of C_Q will be published in another paper.

2. Taking account of shell effects on the elastic constants and mass coefficients of the nucleus also changes the excitation energies of the fragments. The fragment excitation energy E_∞ consists of the energy of the inner degrees of freedom of the fragments at the instant of scission E_0, and of the additional excitation energy after scission $E_\infty - E_0$ (reference 2). In reference 2 the author showed that $E_\infty - E_0 \ll E_0$; however, there the calculations were made on the basis of the liquid-drop model. In accordance with the new model, instead of a single parameter of quadrupole fragment deformation $\alpha_2^{(i)}$, i.e., $Q^{(i)}$ $(i = 1, 2)$, three normal coordinates $Q_1^{(i)}$, $Q_2^{(i)}$, $Q_3^{(i)}$, have to be introduced, and the deformation energy U_d and the Coulomb interaction energy U_{int}, depending on $\alpha_2^{(i)}(Q^{(i)})$ (cf. reference 2), have to be expressed in terms of these normal coordinates. However, at the same time, only those terms depending on the coordinate $Q_1^{(i)}$, which corresponds to the lowest frequency $\omega_1^{(i)}$, can be retained in U_d and U_{int}, since higher excited frequencies will be considerably weaker. An estimate shows that here too, as before, $E_\infty - E_0 \ll E_0$. Therefore, for an approximate estimate of the excitation energy E_∞ it is sufficient to calculate E_0 corresponding to the most probable fragment mass ratio $A_1/A_2 \equiv A_{light}/A_{heavy}$. The value of E_0 depends only on $\alpha_2^{(i)}$ (i.e., on $Q^{(i)}$) and on $\alpha_3^{(i)}$ (cf. reference 2). Numerical calculations of E_0 and E_∞ as functions of Z and A of the fissioning nucleus will be published elsewhere.

Of great interest is also the calculation of the excitation energy of each fragment separately for different mass ratios A_1/A_2. The figure gives the values of the total energy $E_0^{(i)} = U_d^{(i)} + T^{(i)}$ (in units of e^2/r_0; for $r_0 = 1.22 \times 10^{-13}$ cm, $e^2/r_0 \approx 1.18$ Mev) at the instant of scission, calculated for three nuclei on the basis of the liquid drop model. In calculating $T^{(i)}$ in the expression for $(\alpha_2)_{eff}$ the term $\Delta U(A_1/A_2)$ [cf. formula (7) in reference 2] was, for simplicity, not taken into account. The fragment deformation energy $U_d^{(i)}$,



unlike the energy of the vibrational degrees of freedom $T^{(i)}$, changes little with A_1/A_2; only when $A_1/A_2 \ll 1$ is the energy $U_d^{(1)}$ noticeably smaller than $U_d^{(2)}$. The ratio $T^{(1)}/T^{(2)}$, however, as was to be expected,[2] increases sharply when A_1/A_2 decreases. On account of this E_0 becomes larger for a lighter fragment. In the new model, which takes into account shell effects on the elastic constants $C_Q^{(i)}$, $U_d^{(1)}$ and $U_d^{(2)}$ will, obviously, no longer remain practically constant while A_1/A_2 changes, but will increase from A_1/A_2 equal to unity up to A_1/A_2 corresponding to magic or nearly-magic fragments (that is near the most probable value of A_1/A_2); then, upon further change of A_1/A_2 they will decrease. $E_0^{(1)}$ and $E_0^{(2)}$ should prove to be more sensitive functions of A_1/A_2. Taking account of $\Delta U\ (A_1/A_2)$ leads to the same results, since $|\Delta U|$ (and, consequently, also $T^{(1)}+T^{(2)}$) has a maximum for the most probable value of A_1/A_2.

For such calculations of $E_0^{(1)}$ and $E_0^{(2)}$ we assume that scission takes place at the neck thickness d_n which is close to zero, for instance for $d_n \approx (0.1 \text{ to } 0.15)R$ (R is the radius of the fissioning nucleus) (cf. reference 2), and at that, in the most narrow place of the neck. However, as O. Bohr noted (private communication), if the fluctuations of the scission location are taken into account, an even stronger dependence of $E_0^{(1)}/E_0^{(2)}$ on A_1/A_2 may be obtained in the region where A_1/A_2 is close to unity. Indeed, if for a given position of the narrowest part of the neck the scission point is moved towards the heavier fragment, the energy of the lighter fragment is increased, since its deformation becomes larger. Inasmuch as a very strong dependence of the excitation energy ratio $E_0^{(1)}/E_0^{(2)}$ is observed[6] in the region of $A_1/A_2 = 1$, the effect of the fluctuation of the fracture location evidently plays an important role.

In conclusion, I wish to express my gratitude to S. T. Belyaev for an interesting discussion, and to I. G. Krutikova for carrying out the numerical calculations.

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