INTERNAL BREMSSTRAHLUNG IN THE β DECAY OF POLARIZED NUCLEI

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Internal bremsstrahlung in the β decay of polarized nuclei is studied. The general form of angular distributions is given. It is shown that a measurement of the correlation between the direction of emission of the internal bremsstrahlung photon and the nuclear polarization provides information on the form of the β interaction.

1. It is well known that as a consequence of parity nonconservation in β decay the emitted β electrons are longitudinally polarized. Therefore the internal bremsstrahlung of these electrons will be circularly polarized. The theory of internal bremsstrahlung was studied by a number of authors;[1-3] effects due to parity nonconservation were also discussed.[4-6] In particular the degree of circular polarization of the internal bremsstrahlung has been calculated for allowed and once forbidden β transitions.[5,6]

We wish to discuss the internal bremsstrahlung in the β decay of polarized nuclei. As a result of parity nonconservation there exist in this case correlations between the direction of emission of the photon and the polarization direction of the nucleus (of the type $\boldsymbol{\eta}\cdot\mathbf{k}$ where $\boldsymbol{\eta}$ is the polarization direction of the nucleus and $\mathbf{k}$ the momentum of the photon), and also between the direction of emission of the β electron after the internal bremsstrahlung and the nucleus polarization direction (of the type $\boldsymbol{\eta}\cdot\mathbf{p}$ where $\mathbf{p}$ is the β-electron momentum after the internal bremsstrahlung), if the direction of emission of the neutrino is not observed.

A measurement of the correlation between the direction of emission of the internal bremsstrahlung photon and the nucleus polarization provides information on the form of the β interaction. Consequently the study of internal bremsstrahlung of polarized nuclei is of well defined interest, particularly since it may turn out in a number cases that, due to considerations forced by experimental methods, an observation of this effect will be more convenient than the study of the angular distribution of β electrons in the β decay of polarized nuclei (this is due to the difficulties encountered in measurements of the β electron energy as a consequence of their scattering in the source, etc).

2. The calculation of the internal bremsstrahlung in the β decay of polarized nuclei is carried out for allowed transitions on the assumption of the V-A interaction form. The matrix element for the internal bremsstrahlung is written in the same form as in references 3 and 5.

The angular distribution of particles emitted in the β decay of polarized nuclei, accompanied by internal bremsstrahlung, may be written for the general case as follows:

$$w(\theta_{kp}, \theta_{\eta k}, \theta_{\eta p}) = A(\theta_{kp}) + B_k(\theta_{kp})\cos\theta_{\eta k} + B_p(\theta_{kp})\cos\theta_{\eta p} + \mu\left[C(\theta_{kp}) + D_k(\theta_{kp})\cos\theta_{\eta k} + D_p(\theta_{kp})\cos\theta_{\eta p}\right], \quad (1)$$

where

$$A(\theta_{kp}) = \frac{2}{\varepsilon_p p'^2 k^3}\left[\left\{|\langle 1\rangle|^2\delta_{j_2 j_1} + \frac{2j_2+1}{2j_1+1}|\langle\boldsymbol{\sigma}\rangle|^2\Lambda^2\right\}\{p^2(\varepsilon_p + k)\sin^2\theta_{kp} - k^2 p'\}\right], \quad (2)$$

$$B_k(\theta_{kp}) = \frac{2P_n}{\varepsilon_p p'^2 k^3}\left[\left\{-2\delta_{j_2 j_1}\sqrt{\frac{j_1}{j_1+1}}\Lambda\langle\boldsymbol{\sigma}\rangle\langle 1\rangle + \frac{2j_2+1}{2j_1+1}g|\langle\boldsymbol{\sigma}\rangle|^2\Lambda^2\right\}\{k^2p' - kp^2 + kp\varepsilon_p\cos\theta_{kp}\}\right], \quad (3)$$

$$B_p(\theta_{kp}) = \frac{2P_n}{\varepsilon_p p'^2 k^3}\left[\left\{-2\delta_{j_2 j_1}\sqrt{\frac{j_1}{j_1+1}}\langle\boldsymbol{\sigma}\rangle\langle 1\rangle\Lambda + \frac{2j_2+1}{2j_1+1}g|\langle\boldsymbol{\sigma}\rangle|^2\Lambda^2\right\}\{pkp' - p^3\sin^2\theta_{kp}\}\right], \quad (4)$$

$$C(\theta_{kp}) = \frac{2}{\varepsilon_p p'^2 k^3}\left[\left\{\delta_{j_2 j_1}|\langle 1\rangle|^2 + \frac{2j_2+1}{2j_1+1}|\langle\boldsymbol{\sigma}\rangle|^2\Lambda^2\right\}(kp^2\sin^2\theta_{kp} - k^2p')\right], \quad (5)$$

$$D_k(\theta_{kp}) = \frac{2P_n}{\varepsilon_p p'^2 k^3}\left[-2\delta_{j_2 j_1}\sqrt{\frac{j_1}{j_1+1}}\langle\boldsymbol{\sigma}\rangle\langle 1\rangle\Lambda\ \{-\varepsilon_p k(k - p\cos\theta_{pk}) - 2p^2k + 2k^2p'\} + \frac{2j_2+1}{2j_1+2}g|\langle\boldsymbol{\sigma}\rangle|^2\Lambda^2\{\varepsilon_p kp\cos\theta_{kp} - p^2k + k^2p'\}\right], \quad (6)$$

$$D_p(\theta_{kp}) = \frac{2P_n}{\varepsilon_p p'^2 k^3}\left[\left\{-2\delta_{j_2 j_1}\sqrt{\frac{j_1}{j_1+1}}\langle\boldsymbol{\sigma}\rangle\langle 1\rangle\Lambda + \frac{2j_2+1}{2j_1+1}g|\langle\boldsymbol{\sigma}\rangle|^2\Lambda^2\right\}kpp'\right]. \quad (7)$$

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The notation used in Eqs. (1) — (7) is as follows: $\mathbf{k}$, k — momentum and energy of bremsstrahlung photons; $\mathbf{p}$, ϵ_p — momentum and energy of β electrons after bremsstrahlung; μ — sign of circular polarization of the internal bremsstrahlung photons ($\mu = \pm 1$); θ_{kp} — angle between directions of emission of the photon and β electron; $\theta_{\eta k}$ — angle between direction of photon and nucleus polarization; $\theta_{\eta p}$ — angle between direction of emission of β electron (after bremsstrahlung) and nucleus polarization; $\langle 1 \rangle$ and $\langle \sigma \rangle$ — matrix elements of the Fermi and Gamow-Teller type; $p' = -\epsilon_p + p \cos\theta_{kp}$; j_1, j_2 — nucleus spin in initial and final state; P_n — degree of nuclear polarization; $\Lambda = C_A/C_V = 1.19 \pm 0.04$.[7]

The coefficient g depends on the change in the nucleus angular momentum in the given β transition and is equal to

$$g = \begin{cases} -1 & \text{for transitions } j_2 = j_1 + 1, \\ +1/(j_1+1) & \text{for transitions } j_2 = j_1, \\ +1 & \text{for transitions } j_2 = j_1 - 1. \end{cases}$$

3. Next we analyze expression (1). We note that for internal bremsstrahlung in the β decay of unpolarized nuclei the terms proportional to B_k, B_p and D_k, D_p vanish. Therefore the expression

$$w_0(\theta_{kp}) = A(\theta_{kp}) \tag{8}$$

represents the probability for unpolarized internal bremsstrahlung in the β decay of unpolarized nuclei and it differs from previously given expressions[1-3] only in the inclusion of parity nonconservation. On the other hand the expression

$$w_1(\theta_{kp}) = A(\theta_{kp}) + \mu C(\theta_{kp}) \tag{9}$$

represents the probability for circularly polarized internal bremsstrahlung in the β decay of unpolarized nuclei. We note that the degree of circular polarization of the internal bremsstrahlung (for the V and A interaction variants) is equal to

$$P = \mu\frac{C}{A} = \mu\,\frac{p^2k\sin^2\theta_{kp} - k^2p'}{p^2(\varepsilon_p + k)\sin^2\theta_{kp} - k^2p'}, \tag{10}$$

which agrees in magnitude with but differs in sign from the expression previously derived.[5,6] The disagreement in sign is a result of different assumptions regarding the ratio of the constants C and C′. Namely, we assume, in agreement with recent data,[8] that for the V and A interaction variants the relation C = + C′ holds; whereas Sawicki and Szymanski[5,6] assumed C = − C′.

Let us further note that the terms in expression (1) proportional to D_k and D_p are not due to parity nonconservation and represent scalars of the type $(\boldsymbol{\sigma}_\gamma\cdot\mathbf{k})(\mathbf{k}\cdot\boldsymbol{\eta})$ and $(\boldsymbol{\sigma}_\gamma\cdot\mathbf{k})(\mathbf{p}\cdot\boldsymbol{\eta})$ where $\boldsymbol{\sigma}_\gamma$ is the circular polarization vector of the photon.

Lastly, the expression

$$w_2(\theta_{kp}, \theta_{\eta p}, \theta_{\eta k}) = A(\theta_{kp}) + B_k(\theta_{kp})\cos\theta_{\eta k} + B_p(\theta_{kp})\cos\theta_{\eta p} \tag{11}$$

represents the desired angular distribution of unpolarized photons from the β decay of polarized nuclei, accompanied by internal bremsstrahlung.

4. Let us look in more detail at expression (11). After integrating it over the angle $\theta_{\eta p}$ we obtain the expression for the angular distribution of unpolarized internal bremsstrahlung photons from the β decay of polarized nuclei in the form

$$w_k \sim 1 + \alpha_k \cos\theta_{\eta k}, \tag{12}$$

where the asymmetry coefficient α_k is given by

$$\alpha_k = P_n\,\frac{[-2\delta_{j_2 j_1}\sqrt{j_1/(j_1+1)}\langle\sigma\rangle\langle 1\rangle\Lambda + [(2j_2+1)/(2j_1+1)]g|\langle\sigma\rangle|^2\Lambda^2]}{[\delta_{j_2 j_1}|\langle 1\rangle|^2 + [(2j_2+1)/(2j_1+1)]|\langle\sigma\rangle|^2\Lambda^2]} \times \Phi(\Delta, k), \tag{13}$$

and $\Phi(\Delta, k)$ is a function of the β-transition energy and the photon energy k given by

$$\Phi(\Delta, k) = \frac{6\varepsilon_p + 4k - p^{-1}[\varepsilon_p^2 + m^2 + (\varepsilon_p + k)^2]\ln[(\varepsilon_p + p)/(\varepsilon_p - p)]}{-4(\varepsilon_p + k) + p^{-1}[\varepsilon_p^2 + (\varepsilon_p + k)^2]\ln[(\varepsilon_p + p)/(\varepsilon_p - p]}. \tag{14}$$

The table shows numerical values of the function Φ integrated over the β-electron energy (all energies expressed in units of mc^2). It is seen from the table that the asymmetry coefficient is quite large and does not depend strongly on either the energy of the β transition or on the photon energy.

Values of the function $\Phi(\Delta, k)$

Δ	k		
	0.1	0.5	1.0
4.0	−0.58	−0.64	−0.78
3.5	−0.55	−0.58	−0.71
2.2	−0.53	−0.55	—

By integrating expression (11) over the angle $\theta_{\eta k}$ we get the angular distribution of β electrons, which have emitted a bremsstrahlung photon, from the β decay of polarized nuclei; this represents one of the radiative corrections to the asymmetry coefficient in the Wu effect.

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