

ON THE STRUCTURE OF THE Be⁹ NUCLEUS

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For the Be⁹ nucleus viewed as consisting of two α particles and a neutron, the equilibrium distances between the α particles and between the neutron and the centers of the α particles have been determined from the condition of minimum energy. Vibrations along the axis of symmetry and about the center of mass of the α particles are considered and the energy levels of the Be⁹ nucleus are derived. The results obtained are compared with data relating to the Λ Be⁹ hypernucleus.

1. The model of the Be⁹ nucleus in which the odd neutron moves in the centrally symmetric field of the residual Be⁸ nucleus has been effectively used by many authors^{1,2} to investigate the disintegration of this nucleus by photon and electron modes. Since the lifetime of the excited state, in the reactions investigated, was significantly shorter than the lifetime of the Be⁸ nucleus as regards decay of the latter into two α particles, it appears more or less valid to neglect the structure of the residual Be⁸ nucleus as was done by these authors. However, for the ground state or for those excitations of the Be⁹ nucleus which are not attended by emission of the odd neutron, we must evidently presume that the residual Be⁸ nucleus consists of two α particles located a certain distance apart.

Blair and Henley³ recently proposed to explain some of the Be⁹ excited levels as being rotational, viewing the nucleus as a system consisting of two α particles and a neutron. Furthermore, Suh⁴ has shown that the binding energy value of the Λ particle in the Λ Be⁹ hypernucleus can be made to agree with experiment only if it is assumed that this particle moves in the field of the two α particles.

The aim of the present paper is to examine the stability of such a system and to explain the observed energy spectrum of the Be⁹ nucleus.

2. The Hamiltonian of a system consisting of two α particles and a neutron has the form

$$H = -(\hbar^2/2\mu_\alpha)\Delta_u - (\hbar^2/2\mu)\Delta_\rho + V_{n\alpha}(|\rho - \mathbf{u}/2|) + V_{n\alpha}(|\rho + \mathbf{u}/2|) + V_{\alpha\alpha}(u) + C_{\alpha\alpha}(u), \quad (1)$$

where \mathbf{u} is the relative α -particle radius-vector; ρ is the neutron radius vector relative to the center of the two α particles; $V_{n\alpha}$ and $V_{\alpha\alpha}$ are the potential energies of the nuclear interaction of the neutron with the α particles and between

the two α particles, respectively; $C_{\alpha\alpha}$ is the potential energy of the Coulomb interaction of the α particles; $\mu_\alpha = 2M$ and $\mu = 8M/9$, where M is the nucleon mass.

In order to determine $V_{n\alpha}$, we follow Suh and proceed from the nucleon-nucleon interaction, which we take in the form

$$V_{nn} = -V_0 e^{-\beta^2 r^2}, \quad (2)$$

where, as shown by Hochberg et al.,⁵ $\beta^2 = 0.266 \times 10^{26} \text{ cm}^{-2}$; we then normalize the expression by using a distribution of the nuclear material in the α particles in the form

$$P_\alpha(r) = 4(\alpha^2/\pi)^{3/2} e^{-\alpha^2 r^2}. \quad (3)$$

For the rms radius of the α particle we use the value $\sqrt{\bar{r}^2} = \sqrt{3/2} \alpha^{-1} = 1.44$ fermis found by Hofstadter.⁶ For the n - α interaction we therefore obtain

$$V_{n\alpha}(r) = -4V_0 \left(\frac{\alpha^2}{\alpha^2 + \beta^2} \right)^{3/2} \exp\left(-\frac{\alpha^2}{\alpha^2 + \beta^2} \beta^2 r^2 \right). \quad (4)$$

Following Suh,⁴ we select the potential $V_{\alpha\alpha}$ in the form

$$V_{\alpha\alpha} = A e^{-\delta^2 u^2} - B e^{-\epsilon^2 u^2}, \quad (5)$$

where $\delta^2 = 0.406 \times 10^{26} \text{ cm}^{-2}$, $\epsilon^2 = 0.116 \times 10^{26} \text{ cm}^{-2}$, $A = 96 \text{ Mev}$, $B = 30 \text{ Mev}$. The potential (5), which shows short-range repulsion and long-range attraction, best describes the experimental results of low-energy α scattering.

To find the Coulomb potential $C_{\alpha\alpha}$, we assume that the α -particle charge distribution follows a law analogous to Eq. (3). Noting that the odd neutron in Be⁹ is in a p state, we take a zero-approximation trial function of the form

$$\Psi(\rho, \mathbf{u}) = \frac{16}{3\pi} \sqrt{2a^5 l^5} u e^{-a^2 u^2} e^{-b^2 \rho^2} \rho Y_{1m}(\mathbf{n}_\rho), \quad (6)$$

where a and b are variation parameters established by the mean-square distance between the α particles and between the neutron and the center of the α particles respectively; \mathbf{n}_ρ is the unit vector with direction ρ . We point out that the presence of the factor u guarantees a null value for the wave function describing the relative motion of the two α particles at the coordinate origin; this tallies with short-range repulsion.

On the basis of Eqs. (1), (4), (5), and (6) we finally obtain for the mean energy value of the system under investigation

$$E(x, y) = \omega x + my - \frac{V_0 M (xy)^{5/2}}{(8xy + 4x + y)^{5/2}} \left[3 + \frac{5}{2(8xy + 4x + y)} \right] + \frac{\rho x^{3/2}}{(2x + \rho)^{3/2}} - \frac{nx^{3/2}}{(2x + q)^{3/2}} + \frac{kx^{3/2}(6x + j)}{(4x + j)^{3/2}}, \quad (7)$$

where

$$\omega = 7 \hbar^2 l^2 / 6 \mu_\alpha, \quad m = 5 \hbar^2 l^2 / 2 \mu,$$

$$M = (2^{10} \sqrt{2/3}) (\alpha^2 / (\alpha^2 + \beta^2))^{5/2},$$

$$\rho = 4 \sqrt{2} A, \quad n = 4 \sqrt{2} B, \quad k = 2^6 e^2 \alpha / 3 \sqrt{2} \pi,$$

$$l^2 = \alpha^2 \beta^2 / (\alpha^2 + \beta^2), \quad x = a^2 / l^2, \quad y = b^2 / l^2,$$

$$\rho = \delta^2 / l^2, \quad q = \varepsilon^2 / l^2, \quad j = \alpha^2 / l^2.$$

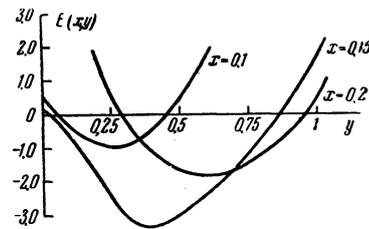
In expression (7), x and y are the variation changes; V_0 is a parameter requiring determination which enters into the expression for the Gaussian potential (2); e is the electron charge.

To determine the possible values of V_0 , we proceed from the experimental fact that there is no bound state of $\text{He}^4 + n$, i.e., that there is no binding of a neutron to a single α particle. It is easily seen that in the solution of variational problems for the system $\text{He}^4 + n$, we find the same parameters α , β , and γ that occur in our problems. If we require that the minimum energy value of the system $\text{He}^4 + n$ be positive or zero, it is possible to determine the possible values of V_0 . As a result, it turns out that $V_0 \leq 26.6$ Mev.

3. As can be seen from (7), it is impossible to find the minimum of the function $E(x, y)$ analytically. Therefore this function was tabulated within the square $0 \leq x \leq 5$, $0 \leq y \leq 5$ in steps of 0.5 for three values of V_0 ($V_0 = 26.6$, 20, and 15 Mev). We note that the interval in question for x and y corresponds to the mean square distance from ∞ to 1×10^{-13} cm.

Calculation shows that the function $E(x, y)$ has a minimum in the region of negative values only when $26.6 \text{ Mev} \geq V_0 \geq 20 \text{ Mev}$. For $V_0 < 20 \text{ Mev}$ the curve of $E(x, y)$ is positive in the square of interest, and with further decrease of V_0 the minimum gradually flattens out and finally vanishes.

The figure shows curves expressing the dependence of $E(x, y)$ on y for $x = 0.10$, 0.15, and



0.20, using $V_0 = 26.6$ Mev in every case. As can be seen from the figure, the function $E(x, y)$ has an absolute minimum for $x = x_0 = 0.15$ and $y = y_0 = 0.40$, for which the corresponding energy value is -3 Mev.

If we require that the energy at the minimum should coincide with the observed 1.6 Mev binding energy of Be^9 when considered in the form of two α particles and a neutron, then we obtain a value of 24 Mev for V_0 . It can easily be seen that for this case the minimum point coordinates x_0 and y_0 hardly move. To these values of x_0 and y_0 there correspond the distance between α particles $R_\alpha = 5.8 \times 10^{-13}$ cm and the distance between the neutron and the center of mass of the α particles $R_n = 3.6 \times 10^{-13}$ cm. The fact that on the average $R_n < R_\alpha$ gives us a basis for assuming that the Be^9 neutron moves in an axially symmetric field whose axis passes through the centers of the α particles. If it had happened that $R_n > R_\alpha$, then the existence of two centers would have been immaterial to the state of the neutron in the Be^9 nucleus. From the condition $R_n < R_\alpha$ it also ensues that the neutron in Be^9 must spend the major part of its time between the α particles, as if it were implementing the bond between them.

4. Since the Be^9 nucleus is characterized by axial symmetry, it follows that it must possess rotational and vibrational excitation levels. The rotational levels have been investigated by Blair and Henley,³ who used them in calculating certain effects related to the excitation of these levels. Upon requiring that the energy of the second excited level, which appears under the rotational assumption, should coincide with the experimentally observed level (2.4 Mev), Blair and Henley found $R_\alpha = 4.6 \times 10^{-13}$ cm, which is satisfactorily close to our value of 5.8×10^{-13} cm.

Using this value of R_α , the above-mentioned authors likewise calculated other possible values of rotational level energies, but these values are in poor agreement with experimental data. Therefore we must conclude that only the second excited level of Be^9 with 2.4 Mev energy appears to be a rotational level. As regards the remaining Be^9 levels, we can assume that all of them are vibrational, as will be shown below. For this purpose

n_2	n_1	E_{exp}	E_{theor}	n_2	n_1	E_{exp}	E_{theor}
1	0	1.7	1.7	0	4	—	12.4
0	1	3.1	3.1	1	4	—	14.1
1	1	4.8	4.8	0	5	—	15.5
0	2	6.8	6.2	1	5	17.3(17.5)	17.2
1	2	7.9	7.9	0	6	—	18.6
0	3	—	9.3	1	6	19.9	20.3
1	3	11.3	11.0	0	7	21.7	21.7

we shall consider the Be^9 nucleus as a combination of two independent oscillators with frequencies ω_1 and ω_2 , one of which corresponds to vibration along the axis of symmetry, and the other corresponds to vibration of the neutron about the center of gravity of the system. The first of these can be described as a vibration of the α particles along the axis connecting their centers, while the second can be described as a vibration of the odd neutron about the center of mass of the two α particles.

We shall designate by n_1 and n_2 the quantum numbers characterizing these vibrational modes. Then for the total excitation energy, reckoned from a given level, we can write

$$E_{n_1 n_2} = n_1 \hbar \omega_1 + n_2 \hbar \omega_2. \quad (8)$$

If we require that the first 1.7-Mev Be^9 level correspond to excitation of only the first neutron vibration ($n_1 = 0$, $n_2 = 1$), while the 3.1-Mev level correspond to excitation of only the first α -particle vibration ($n_1 = 1$, $n_2 = 0$), it is possible to determine the frequencies ω_1 and ω_2 . Assigning the values $n_1 = 0, 1, 2, 3, 4, 5, 6, \dots$ for $n_2 = 0$ and for $n_2 = 1$, it is possible to obtain the entire excitation energy spectrum for the Be^9 nucleus with the exception, as is to be expected, of the 2.4 Mev energy level, which appears to be purely rotational according to Blair and Henley.

The table shows theoretical values of the Be^9 excitation energies calculated by Eq. (8), and also shows the experimental data contained in references 7 and 3.

As can be seen from the table, the agreement of the theoretical results with experimental data appears to be quite satisfactory. It should be noted however, that (8) predicts levels of energies 9.3, 12.4, 14.1, and 15.5 Mev, which have not yet been found experimentally. If we consider that the interval between the two neighboring observed levels at 11.3 and 17.2 Mev (like the one between 7.9 and 11.3 Mev) appears to be so large that it does not correspond to the observed level density found for other energy values, then this appears to make plausible the existence of the levels predicted by theory.

In addition, experiment shows two very close levels at 17.27 and 17.47 Mev, while the theoretical value corresponding to a vibrational level is 17.2 Mev. This could lead to the assumption that one of these levels is rotational. However, calculation shows that it is impossible to obtain a rotational level of 17.2 or 17.5 Mev for any value whatsoever of the moment J for the Be^9 nucleus if we proceed from the Blair and Henley value $R_\alpha = 4.6 \times 10^{-13}$ cm. This leads to one of two choices: either these levels coincide, in which case we really have only one level to deal with, or else a level uncoupling occurs which is not covered by our Eq. (8).

It is possible to conclude from the basic data given in the table that all the Be^9 levels fall into two series, characterized by $n_2 = 0$ and $n_2 = 1$, for each of which $n_1 = 0, 1, 2, 3, 4, 5, 6, \dots$. The first spectral series corresponds to the excitation of vibrational modes along the axis of symmetry, when the odd neutron is found in the first excited state of vibrational motion.

Our assignment of the quantum number n_1 to vibration along the axis of symmetry and n_2 to vibration of the neutron about the center of mass is related to the fact, known from experiment, that the reaction for electron and photon disintegration of Be^9 with emission of a neutron is observed beginning with 1.7 Mev energy, which corresponds to the neutron binding energy. Since our investigation shows that the single-particle model as applied to the neutron is valid in several cases, it is reasonable to assume that when a Be^9 state has an excitation energy greater than 1.7 Mev, the neutron cannot receive more than 1.7 Mev of this energy. It follows that there cannot be a series with $n_2 > 1$. In fact, all the experimentally observed lines fall into series with $n_2 = 0$ or $n_2 = 1$.

5. Let us compare our deductions with results dealing with the ΛBe^9 hypernucleus. For ΛBe^9 there have been obtained⁴ the values $R'_\alpha \approx 2.5 \times 10^{-13}$ cm and $R_\Lambda \approx 2 \times 10^{-13}$ cm. We observe here that $R_\Lambda < R'_\alpha$, just as in our case before. The smaller values of R_Λ and R'_α for the ΛBe^9 hypernucleus than for the Be^9 nucleus can be explained by the fact that the Λ particle in the ΛBe^9

hypernucleus is in an s state, while the neutron in the Be^9 nucleus is in a p state; it is therefore obvious why $R_n > R_\Lambda$. Furthermore, if we consider that the interaction radius of the Λ particle with a nucleon is smaller than the nucleon-nucleon interaction radius, and that consequently the Λ - α interaction radius is smaller than the n - α interaction radius, then we can assume that $R'_\alpha < R_\alpha$, since the Λ particle does the binding in the ΛBe^9 hypernucleus while the neutron does the binding in the Be^9 nucleus. This conclusion seems all the more plausible, because the binding energy of the Λ particle in the ΛBe^9 hypernucleus is greater than the binding energy of the neutron in the Be^9 nucleus.

In conclusion we wish to express our thanks to our associates at the Computer Center of the Academy of Sciences of the Armenian S.S.R., and especially to F. M. Ter-Mikaelyan and R. A. Aleksandryan for preparing the table of functions on the "Erevan" computer.

Added in proof (February 12, 1960): A recent paper [H. H. Thies and B. M. Spicer, *J. Astr. Phys.* **12**, 293 (1959)] shows the nuclear spectrum

of Be^9 with the following new levels: 6.2, 9.2, and 13.3 Mev. As can be seen from our table, the first two are in good agreement with our values of 6.2 and 9.3 Mev, while the 13.3-Mev level falls between our theoretical values 12.4 and 14.1 Mev. The 6.8-Mev level, also observed experimentally, does not fit into our scheme and evidently has some other origin.

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