NUCLEON EMISSION BY A ROTATING NUCLEUS

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The effect of a large angular momentum on the emission of nucleons by an excited nucleus is determined in the independent particle model. The mean characteristics of neutron emission as a function of temperature and angular momentum are calculated. An estimate is made of the mean excitation energy which remains after the emission of the nucleons and which is carried off by the γ quanta.

In recent times experiments have begun to be performed in which heavy ions are used as bombarding particles.[1-3] The compound nucleus formed in these reactions usually has a relatively high excitation energy and a large angular momentum. The existence of a large moment of inertia has a great effect on the physics of the decay processes of the compound nucleus and, in particular, on the emission of nucleons.

We shall assume that the compound nucleus formed in the reaction is in thermodynamic equilibrium, which is in fact the condition for the existence of a compound nucleus. Then the excitation energy of the nucleus, E, can be written as

$$E = E(T) + E_{\rm rot} \qquad E_{\rm rot} = j^2 \hbar^2 / 2I,$$

where $E(T)$ is the thermal energy, $E_{\rm rot}$ is the energy of rotation, j is the angular momentum, I is the moment of inertia of the nucleus, and $T = \sqrt{10\,E(T)/A}$ is the temperature of the nucleus of mass A.

Generally speaking, the nuclear moment of inertia is not equal to the moment of inertia of a rigid body and depends on the formation of pairs on the Fermi surface.[4,5] However, even for relatively small thermal excitation these pairs are destroyed, and consequently the nuclear moment of inertia becomes equal to the moment of inertia of a rigid body. The effect of the rotation of the nucleus on the formation of pairs is similar to the effect of a magnetic field, i.e., the increase of the moment of inertia counteracts the formation of pairs, and starting from some angular momentum j_c, all pairs are destroyed. The quantity j_c can be estimated by comparing the energy of rotation with the change of energy connected with the formation of pairs:

$$\hbar^2 j_c^2 / 2I \sim \Delta^2 \rho / 4,$$

where Δ is the magnitude of the gap and ρ is the level density on the Fermi surface; $j_c \sim 7$ for nuclei with $A \sim 50$ and $j_c \sim 15$ for nuclei with $A \sim 250$. Even for vanishing thermal excitation energy the moment of inertia of the nucleus therefore becomes equal to the moment of inertia of a rigid body I_0 already for relatively small angular momenta (the compound nucleus usually has the maximal angular momentum $j \gtrsim 30$ for $A \sim 50$ and $j \gtrsim 70$ for $A \sim 250$). We shall assume in the following that the nucleus has spherical shape, i.e., $I_0 = \frac{2}{5}\mu A R^2$, where μ is the mass of the nucleon, and $R = r_0 A^{1/3}$ is the radius of the nucleus.

The probability for the evaporation of a nucleon from a nucleus with small angular momentum per unit time is usually determined (by using the principle of detailed balance or the reciprocity theorem) from the cross section of the reverse process, in this case from the cross section for compound nucleus formation, which is known experimentally. Since no experiments have been performed to determine the cross section for nucleon capture by a nucleus with high angular momentum, the decay probability for such a nucleus must be found directly.

The characteristics of the emission of nucleons by a nucleus with large angular momentum can be found in the independent particle model. We shall regard the nucleus as a set of spinless particles moving in a spherical square well of depth V and radius R. In the ground state of the nucleus all nucleons are distributed pairwise over the levels such that their angular momenta compensate each other and give the total angular momentum zero. If the nucleus has nonvanishing angular momentum, some of the nucleons must be placed on higher levels such that the total angular momentum of the nucleons is equal to the given one. For given angular momentum the state with the lowest energy corresponds to the state in which the energy of thermal motion is zero.

As the angular momentum increases, more and more nucleons go to higher levels, and for sufficiently high angular momentum it becomes possible that some of the nucleons find themselves above the well and can leave the nucleus. This process of emission of a nucleon from the nucleus has nothing in common with the usually considered process of evaporation and is only due to the rotation of the nucleus. Since the orbital angular momentum of the nucleons above the well is different from zero, the emission of neutrons will be hindered by the centrifugal barrier and that of the protons by the centrifugal and Coulomb barriers. The lifetime of these states will therefore be greater than the characteristic nuclear periods. To obtain a rough qualitative estimate for nucleons above the well, we can thus use the expression for the single particle level density in a spherical well of infinite depth.

In the quasi-classical approximation the number of particles within the interval $d\epsilon$ with orbital angular momentum l and projection m in a system with the total angular momentum j and temperature T is equal to[6]

$$dn_{lm} = \frac{1}{2\pi\varepsilon}\left[\exp\left\{\frac{\varepsilon - \theta - \gamma m}{T}\right\} + 1\right]^{-1} \sqrt{\frac{2\mu R^2}{\hbar^2} - (l + 1/2)^2}\, d\varepsilon, \tag{1}$$

where $\gamma = j\hbar^2/I_0$, and θ is the chemical potential of the rotating system, which up to terms of order E_{rot}/A is equal to θ_0, the chemical potential of a system with angular momentum zero. In what follows this correction will be neglected.

The width for the emission of a nucleon is of the order of magnitude of the quantity

$$\Gamma = \sum_{l,\, m} \int dn_{lm} T_l(E)\, \varepsilon,$$

where $T_l(E)$ is the penetrability of the barrier and E is the kinetic energy of the nucleons at infinity. For neutrons with $E/\epsilon \ll 1$ we may use the approximate expression for $T_l(E)$:[7]

$$T_l(E) = (4x/X)\, v_l(x);$$

$$x = \sqrt{2\mu R^2 E}/\hbar; \quad X = \sqrt{2\mu R^2 \varepsilon}/\hbar;$$

$$v_l(x) = 2/\pi x\, (J^2_{l+1/2}(x) + N^2_{l+1/2}(x)),$$

$J_{l+1/2}(x)$ and $N_{l+1/2}(x)$ are the Bessel and Neumann functions. Tables of $v_l(x)$ are given in reference 8.

In the case of protons we used the formula for the penetrability of the Coulomb barrier with account of the angular momentum of the proton obtained in the quasi-classical approximation:[6]

$$T_l(E) = \exp(-2C_l);$$

$$\frac{C_l}{g} = \frac{1}{2} t^{-1/2}\left(\frac{\pi}{2} + \arcsin\frac{1-2t}{\sqrt{1+4yt}}\right) -$$

$$-\sqrt{1+y-t} + y^{1/2} \ln \frac{1 + 2t^{1/2}(t^{1/2} + \sqrt{1+y-t})}{\sqrt{1+4yt}};$$

$$g^2 = 2\mu Z e^2 R/\hbar^2, \qquad y = l(l+1)/g^2, \qquad t = E/B_C,$$

and $B_C = Ze^2/R$ is the Coulomb barrier for a nucleus with atomic number Z.

The penetrability of the centrifugal barrier becomes close to unity for $E \geq \hbar^2 l^2/2\mu R^2$. On the other hand, the probability of finding a particle in the given interval $d\epsilon$ [formula (1)] is close to unity for $\epsilon \leq \theta + \gamma m$. Therefore, in order that the lifetime of the compound nucleus be greater than the characteristic nuclear period, the inequalities

$$V + \hbar^2 l^2/2\mu R^2 > \mu + \gamma l \geqslant \mu + \gamma m$$

must be fulfilled for all l. This leads to the condition

$$E_{rot} < \tfrac{2}{5} BA,$$

where $B = V - \theta_0$ is the binding energy of the neutron.

If the thermal energy is equal or close to zero, a nucleon can be emitted only if the change of the energy of the nucleons at the Fermi surface due to the rotation of the nucleus exceeds the binding energy:

$$B < \gamma m_f \leqslant \gamma l_f \approx \gamma \sqrt{2\mu R^2 \theta_0}/\hbar.$$

For $E_{rot} > E^k_{rot} = B^2 A/10\theta_0$ the rotating nucleus will emit neutrons. For $E_{rot} < E^k_{rot}$ there will be no emission of nucleons, and the nucleus will be in an excited state until the excitation energy is carried off by the emission of γ quanta. If this state of the nucleus is the result of the emission of several particles, the excitation energy of the nucleus will with equal probability take on any value inside some interval of excitation energies which is larger than B. The average excitation energy of a nucleus with small angular momentum is therefore equal to the known value B/2, while it is equal to $E^k_{rot} - B/2$ for a nucleus with large angular momentum (T = 0).

In heavy-ion reactions the compound nucleus may have arbitrary angular momenta — from zero up to some maximal value. For small values of the angular momentum, when the exponential in the denominator of (1) can be expanded in terms of γ after which the exact expression for the penetrability of the centrifugal barrier is replaced

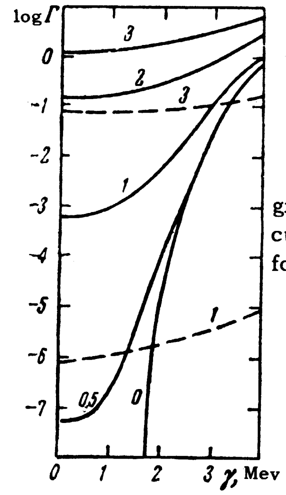

FIG. 1. The numbers at the curves give the temperature in Mev. Dotted curves – for protons, solid curves – for neutrons.

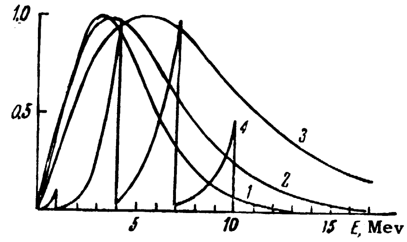

FIG. 2

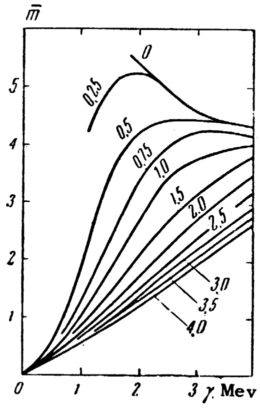

FIG. 4

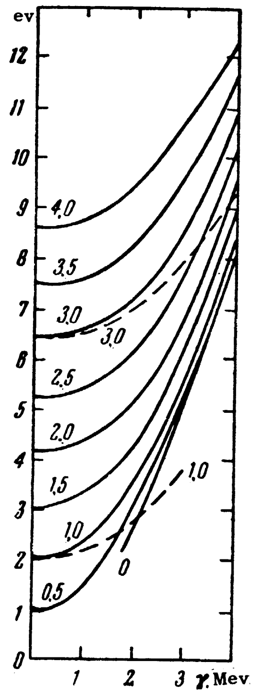

FIG. 3

FIG. 2. Curve 1: T = 1 Mev, γ = 2 Mev; 2: T = 2 Mev, γ = 1 Mev; 3: T = 3 Mev, γ = 0; 4: T = 0, γ = 3 Mev.

FIG. 3 The numbers at the curves give the temperature in Mev; the dotted curves are calculated with $T_l(E)$ taken from references 10 and 11.

FIG. 4. The numbers at the curves give the temperature in Mev.

by its classical approximation, the probability for evaporation depends on the kinetic energy of the neutron in the same way as usual.[9] The only difference is that we use the temperature of the initial nucleus instead of that of the final nucleus. This inaccuracy can be easily taken account of, however, by taking the correct temperature (this was done in the calculation of the cascade emission of neutrons). Since the replacement of the penetrability of the centrifugal barrier by its classical approximation is legitimate if the denominator of (1) changes slowly as compared to $T_l(E)$, the condition of applicability of the formulas obtained in references 10 and 11 is not determined by the smallness of the parameter E_{rot}/TA (the parameter of the thermodynamic perturbation theory) alone.

We calculated Γ numerically for the nucleus with $2\mu R^2/\hbar^2 = 1$ Mev^{-1}, V = 40 Mev, B = 8 Mev, B_C = 8 Mev, $g^2 = 8$ ($A \approx 50$, $Z \approx 25$, $r_0 = 1.23 \times 10^{-13}$ cm). Figure 1 shows the dependence of the logarithm of the neutron and proton emission widths on the temperature and angular momentum of the nucleus. It is seen from the figure that the proton emission probability is almost independent of the angular momentum, whereas the neutron emission probability depends very strongly on it, especially at small temperatures. This behavior of Γ_n and Γ_p as functions of the angular momentum has as a consequence that the number of protons emitted by a rotating nucleus is smaller than in the case where the nucleus has a small angular momentum.

Figure 2 gives the energy distribution of the neutrons. The scale of the curves is arbitrary. The existence of separate energetical groups of neutrons for T = 0 is due to the discrete values of m.

Figure 3 shows the behavior of

$$\overline{E} = \sum_{lm} \int dn_{lm} T_l(E) \varepsilon E / \Gamma$$

the average kinetic energy of the neutron as a function of temperature and angular momentum. We note that even for $(E_{rot}/TA) \ll 1$ the quantity $\overline{E}$ is quite different from that obtained in reference 10.

Figure 4 shows the behavior of

$$\overline{m} = \sum_{lm} \int dn_{lm} T_l(E) \varepsilon m / \Gamma$$

the average projection of the angular momentum as a function of γ and T. For small T, an increase of γ leads to a decrease of the admissible projection m of the emitted neutrons and hence also to a decrease of $\overline{m}$.

The average excitation energy carried off by the neutron is equal to $B + \overline{E}$. Since the neutron changes the angular momentum of the nucleus by $\overline{m}$, the rotational energy carried off is equal to

$$(\hbar^2/2I_0)[j^2 - (j-m)^2] \approx (\hbar^2/2I_0)\, 2j\overline{m} = \gamma\overline{m}.$$

Hence the change in thermal energy of the nucleus is equal to $B + \bar{E} - \gamma\overline{m}$. If the thermal excitation energy of the nucleus is zero, the neutron carries off only mechanical energy and for $T = 0$ we have $B + \bar{E} = \gamma\overline{m}$. This equation may serve as a check on the accuracy of the numerical calculations. It turns out that it is satisfied with an accuracy up to 6%.

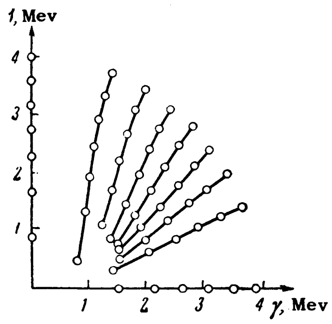

FIG. 5

Knowing the mean characteristics of the neutron emission process, we can find the approximate behavior of the nucleus during the time of the cascade process. In Fig. 5 we show an example of a cascade process with an initial total excitation energy of 80 Mev. The initial points correspond to identical values of the intervals ΔE_{rot}. It is seen from the figure that the ratio γ/T increases during the course of the cascade process. This leads to a large average excitation remaining after the emission of the neutrons and hence to a displacement of the maximum yield of a given isotope toward higher excitation energies of the nucleus. In the given case this displacement turns out to be equal to $\sim$ 8 Mev for the maxima of the yield of the isotopes corresponding to the emission of 5 to 6 neutrons. The average excitation energy remaining in the nucleus is also equal to $\sim$ 8 Mev.

In conclusion I express my gratitude to B. T. Geĭlikman, V. M. Galitskiĭ, and V. M. Strutinskiĭ for a useful discussion of this work and for valuable comments.

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