DEPENDENCE OF THREE-DIMENSIONAL DEVELOPMENT OF A CASCADE SHOWER ON THE ENERGY OF THE PRIMARY PARTICLE

V. V. GUZHAVIN and I. P. IVANENKO

Institute of Nuclear Physics, Moscow State University

Submitted to JETP editor October 8, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **38**, 662-664 (February, 1960)

IN three-dimensional cascade theory, the functions of angular and spatial distribution of particles are usually calculated under the assumption that the energy of the primary particle is infinitely large. However, the energy of the electrons and photons near the axis of the shower is comparable with the energy E_0 of the primary particle, and therefore E_0 cannot be assumed infinite. We give here a convenient approximate method of calculating these functions, which yields a sufficiently accurate account of the influence of the finite E_0 on the form of the distribution function. Let, for example, $N_p(E_0, E, r, t)\,dr$ be the number of electrons with energy greater than E at a depth t and a distance r, $r+dr$ from the shower axis, due to a primary particle with energy E_0. If we introduce $x = Er/E_s$, where $E_s = 21$ Mev, then when $E_0 = \infty$ the function $N_p^\infty(E_0, E, r, t)$ can be represented in the form*[1]

$$N_p^\infty(E_0, E, r, t) = f_p^\infty(x, s)\, N_p(E_0, E, t)\, E^2/E_s^2, \quad (1)$$

where $N_p(E_0, E, t)$ is a function that describes the one-dimensional development of the shower. The function of lateral distribution $f_p^\infty(x, s)$ is normalized as follows:

$$\int_0^\infty f_p^\infty(x, s)\, x\,dx = 1.$$

In one-dimensional cascade theory, the quantities E_0, E, and t are related as follows

$$-\lambda_1'(\bar{s})\, t = y, \quad (2)$$

where $\bar{s}$ is the cascade parameter, $y = \ln(E_0/E)$, and $\lambda_1'(s)$ is a tabulated function. In three-dimensional theory for a finite value of E_0, the quantities E_0, E, x and t are related by

$$-\lambda_1(s)\, t = y + \ln x, \quad (3)$$

which holds when $1 > x > E/E_0$. It is seen from (3) that at a finite value of E_0 and a fixed depth t, the quantity x is a function of s, and as r decreases s increases, reaching a limiting value $s = \infty$ when $x_{min} = E/E_0$. If we let E_0 approach infinity in (3), then the dependence $x = x(s)$ becomes weaker and weaker, and in the limit $E_0 = \infty$ it disappears entirely. In this case the parameter s is constant for $0 < x < \infty$ and coincides with the parameter $\bar{s}$ of the one-dimensional theory. Taking this into account, the function of lateral distribution of particles at a depth t in a shower, induced by a primary particle of energy E_0, can be represented in the form

$$N_p(E_0, E, x(s), t(\bar{s})) = f_p^{E_0}(x, s)\, N_p(E_0, E, s, t(\bar{s}))\, E^2/E_s^2, \quad (4)$$

where the dependence $x(s)$ at constant E_0, E, and t is determined from (3), while $t(\bar{s})$ is determined from (2). To calculate $N_p[E_0, E, x(s), t(\bar{s})]$, we can use the functions $f_p^\infty(x, s)$ which were previously calculated[2] by the method of moments. Actually, the moments $\overline{\theta^n}(E_0, E, s)$ and $\overline{r^n}(E_0, E, s)$ of the functions of angular and spatial distribution depend little on E_0 if s is constant and $E_0/E > 10$, (reference 3). Consequently, one can assume approximately that at constant s the functions $f_p^{E_0}(x, s)$ depend little on $E_0/E > 10$, i.e., $f_p^{E_0}(x, s) \approx f_p^\infty(x, s)$. Therefore, if we determine the variation of $x(s)$ from (3) at constant E_0, E, and t, we get

$$N_p(E_0, E, x(s), t(\bar{s})) \approx f_p^\infty(x, s)\, N_p(E_0, E, s, t(\bar{s}))\, E^2/E_s^2. \quad (5)$$

We introduce the function of spatial distribution of electrons for a finite value of E_0, normalized to one particle:

$$f_p(E_0, E, x(s), t) = N_p(E_0, E, x(s), t(\bar{s}))\, E_s^2/N_p(E_0, E, \bar{s}, t(\bar{s}))\, E^2. \quad (6)$$

We then obtain finally

$$f_p(E_0, E, x, t) = f_p^\infty(x, s)\, N_p(E_0, E, s, t)/N_p(E_0, E, \bar{s}, t). \quad (7)$$

It is easy to calculate analogously the functions of spatial distribution of photons with energy greater than E, and also the corresponding distribution functions of particles with energies $E > 0$, with allowance for ionization losses. The method used for the calculation can be applied to the calculation of angular-distribution functions.

Let us obtain the lateral distribution function of electrons with energy greater than E for the case of equilibrium. In reference 4 it was shown that if the electrons and photons of energy E_0 are generated over the entire thickness of the substance in accordance with the law $e^{-\mu t}\delta(E_0 - E)$, then $\bar{s}$, t and y are related by

$$-\lambda_1'(\bar{s})\,[t - 1/(\lambda_1(\bar{s}) + \mu)] = y, \tag{8}$$

where μ is the coefficient of absorption of the component that generates the primary electrons or photons. The integral energy spectrum of the electrons has the form

$$N_p(E_0, E, t) = \frac{H_1(\bar{s})}{\bar{s}} \exp\left\{y\bar{s} + \lambda_1(\bar{s})\,t - \ln[\lambda_1(\bar{s}) + \mu]\right\} \times \left[2\pi\left\{\lambda_1''(\bar{s})\,t - \frac{\lambda_1''(\bar{s})\,[\lambda_1(\bar{s}) + \mu] - \lambda_1''^2(\bar{s})}{[\lambda_1(\bar{s}) + \mu]^2}\right\}\right]^{-1/2}. \tag{9}$$

It can be shown that in the case of continuous generation in depth, the quantities s, t, y, and x in three-dimensional theory are related by

$$-\lambda_1'(s)\,[t - 1/(\lambda_1(s) + \mu)] = y + \ln x. \tag{10}$$

We used the foregoing method to calculate the following functions: the laterial distribution function of electrons and photons with energy greater than E (approximation A) at $\bar{s}$ = 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, and 1.6 for various values of the ratio $E_0/E = 10^6, 10^4, 10^3, 10^2$ and 10; the functions of lateral distribution of electrons with energy $E > 0$ (approximation B) for $\bar{s}$ = 0.6, 0.8, 1.0, 1.2, 1.4, 1.6 and for values of the ratio $E_0/\beta = 10^6, 10^4, 10^3, 10^2$ and 10. We also calculated the equilibrium functions of angular and lateral distributions of electrons for several values of the parameters.

*The condition $E_0 = \infty$ is used here only for calculating the function $f_p^\infty(x, s)$.

[1] S. Z. Belen'kiĭ, Лавинные процессы в космических лучах (Cascade Processes in Cosmic Rays), Gostekhizdat, 1948.

[2] V. V. Guzhavin and I. P. Ivanenko, Dokl. Akad. Nauk SSSR **115**, 1089 (1957), Soviet Phys.-Doklady **2**, 407 (1958).

[3] L. Eyges, Phys. Rev. **74**, 1801 (1948).

[4] S. Z. Belen'kiĭ and I. P. Ivanenko, Usp. Fiz. Nauk **69**, 591 (1959), Soviet Phys.-Uspekhi **2**, 912 (1960).

Translated by J. G. Adashko
134

ON THE PRODUCTION OF AN ELECTRON-POSITRON PAIR BY A NEUTRINO IN THE FIELD OF A NUCLEUS

A. M. BADALYAN and CHOU KUANG-CHAO

Submitted to JETP editor November 26, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **38**, 664-665 (February, 1960)

PRESENT experimental possibilities have allowed a rather close approach to a measurement of the cross section for scattering of a neutrino by an electron.[1] This process is a very important one for testing the theory of the universal weak interaction.

In the laboratory system, in which the electron is at rest, and for incident neutrino energy $\omega_1 \gg m$, the cross section for scattering of a neutrino by an electron is

$$\sigma_1 = (g^2/3\pi)\,m\omega_1, \tag{1}$$

i.e., a linear function of ω_1.

There is another process, $\nu + Z \to \nu + Z + e^+ + e^-$, for which the laboratory system coincides with the center-of-mass system. On one hand, it could be expected that the cross section for this process would be smaller than that for scattering, since it contains the factor $(Ze^2)^2$, and the phase volume gives an additional numerical factor $(2\pi)^{-2}$. On the other hand, the phase volume is proportional to ω_1^3, since there are three particles in the final state.

This process is described by two second-order diagrams. The calculation of the contributions of the two diagrams to the cross section leads to extremely cumbersome formulas. We shall, however, get the right order of magnitude for the total cross section if we confine ourselves to the contribution of one diagram. The differential cross section for the process then has the form

$$d\sigma_2 = \frac{16g^2(Ze^2)^2}{\omega_1\omega_2\varepsilon_+\varepsilon_-}\,\frac{d\mathbf{p}_-\,d\mathbf{p}_+\,d\mathbf{k}_2}{q^4(2\pi)^8}\,\frac{(k_1k_2)}{m^2 - f^2} \times \left[2\varepsilon_+\varepsilon_- - (p_+p_-) + 2fp_+\frac{2\varepsilon_-^2 + m^2 - (fp_-)}{m^2 - f^2}\right] \times \delta(\omega_1 - \omega_2 - \varepsilon_+ - \varepsilon_-), \tag{2}$$

where

$$f = k_1 - k_2 - p_+, \qquad q = k_1 - k_2 - p_+ - p_-.$$

Here k_1, k_2, p_+, and p_- are four-vectors that refer respectively to the neutrino in its initial and