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SCATTERING OF GAMMA-RAY QUANTA BY NUCLEONS NEAR THE THRESHOLD FOR MESON PRODUCTION

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The elastic scattering of γ-ray quanta near the threshold for single meson production is treated by means of dispersion relations. It is shown that when one takes into account meson production in the s state there are appreciable departures from monotonic variation with energy of the scattering amplitudes, cross sections, and other observable quantities near the threshold of the reaction. On definite assumptions about the analysis of photoproduction in the range of γ-ray energies up to 220 Mev, calculations are made of the scattering amplitude and the differential and total cross sections for elastic scattering of polarized and unpolarized γ-rays by protons, and also of the polarization of the recoil protons above the photoproduction threshold.

1. The study of the scattering of γ-ray quanta by nucleons is especially interesting near the threshold for single meson production. The region near the photoproduction threshold is of interest not only for comparisons with the predictions of dispersion relations, but also in particular in connection with the studies of departures from monotonic variation with the energy of the cross-sections (and polarizations) near the threshold of the reaction.[1] From this latter point of view the scattering of γ-ray quanta by nucleons and nuclei near the threshold for meson production is of especial interest as an example of a process going with a comparatively small cross section and being strongly perturbed above threshold by the process of intense meson production. Thus marked effects can be expected in the region near the threshold. It is clear that a sufficiently accurate experimental study of the anomalies near the threshold can be useful in understanding the process of pion production near threshold.

As a more detailed examination shows, the polarization effects are especially sensitive to the parameters characterizing the photoproduction of pions. Our main purpose here is a detailed examination of the effect of meson production on the cross section, the polarization of the recoil nucleons, and the polarization of the γ rays near the photoproduction threshold.

Phenomenological analysis and dispersion relations are used to obtain formulas useful for the analysis of experimental data. The results of the numerical calculations, which are based on definite assumptions about the analysis of the photoproduction, must be regarded as preliminary. In making the numerical estimates we have completely neglected fine-structure effects associated with the mass difference of the mesons (and of the nucleons).

There are many well known papers in which the scattering of γ-ray quanta by nucleons has been treated by various methods (see literature references in our previous paper[2]). In the present paper we have tried to manage with a minimum number of assumptions, without resorting to approximate methods, whose use is hard to justify. We consider not only the scattering cross sections for unpolarized γ rays, but also the polarization effects in the scattering. In this connection we have also considered the polarization of the γ rays.

2. Let us represent the transition matrix in the form

$$\mathcal{M} = \sum_{\mu\nu} e'_\mu N_{\mu\nu} e_\nu \equiv (\mathbf{e}'\mathbf{N}\mathbf{e}).$$

Let us choose two coordinate systems x, y, z and x′, y′, z′ in which the z and z′ axes are parallel to the initial and final momenta of the photon, and the y and y′ axes are in the same direction. In these coordinates the functions for the spin eigenstates of the photon with the eigenvalues $S_z = \pm 1$ have the following form:

$$\zeta_1 = -(\mathbf{h} - i\mathbf{j})/\sqrt{2}, \qquad \zeta_{-1} = (\mathbf{h} + i\mathbf{j})/\sqrt{2},$$
$$\zeta'_1 = -(\mathbf{h}' - i\mathbf{j})/\sqrt{2}, \qquad \zeta'_{-1} = (\mathbf{h}' + i\mathbf{j})/\sqrt{2}, \tag{1}$$

where **h**, **j**, and **k** are unit basis vectors directed along these coordinate axes. In the general case the polarization state of the photon will be a linear combination, i.e.,

$$\mathbf{e} = c_1\zeta_1 + c_{-1}\zeta_{-1}, \qquad (2)$$

where c_1 and c_{-1} are the respective probabilities (sic) of the photon states with $S_z = +1$ and $S_z = -1$.

Using the spin eigenstates as the basis of the representation, we can write the transition matrix in the form

$$\mathscr{M} = \begin{pmatrix} (\zeta_1'^*\mathbf{N}\zeta_1) & 0 & (\zeta_1'^*\mathbf{N}\zeta_{-1}) \\ 0 & 0 & 0 \\ (\zeta_{-1}'^*\mathbf{N}\zeta_1) & 0 & (\zeta_{-1}'^*\mathbf{N}\zeta_{-1}) \end{pmatrix}. \qquad (3)$$

Let us further introduce the density matrix of the photon in the form

$$\rho = \begin{pmatrix} c_1c_1^* & 0 & c_1c_{-1}^* \\ 0 & 0 & 0 \\ c_{-1}c_1^* & 0 & c_{-1}c_{-1}^* \end{pmatrix}. \qquad (4)$$

The density matrix ρ_f of the final state is connected with the density matrix ρ_{in} of the initial state by the relation

$$\rho_f = \mathscr{M}\rho_{in}\mathscr{M}^+. \qquad (5)$$

Although in Eqs. (3) and (4) the transition matrix and the density matrix are written as three-rowed matrices, they have only four independent nonzero elements. Consequently we can represent them by means of two-rowed matrices and use the well known apparatus of the Pauli matrices.[3]

$$\mathscr{M} = \begin{pmatrix} (\zeta_1'^*\mathbf{N}\zeta_1) & (\zeta_1'^*\mathbf{N}\zeta_{-1}) \\ (\zeta_{-1}'^*\mathbf{N}\zeta_1) & (\zeta_{-1}'^*\mathbf{N}\zeta_{-1}) \end{pmatrix} = A + \mathbf{B}\sigma_\gamma, \qquad (6)$$

$$\rho = \begin{pmatrix} c_1c_1^* & c_1c_{-1}^* \\ c_{-1}c_1^* & c_{-1}c_{-1}^* \end{pmatrix} = \frac{1}{2}(1 + \sigma_\gamma\mathbf{P}), \qquad (7)$$

where P_x, P_y, and P_z are the Stokes parameters. Nonvanishing P_x and P_y correspond to linear polarization of the photons along the x and y axes, while $P_z \neq 0$ corresponds to circular polarization of the photon.

From Eq. (6) it is not hard to get

$$2A = (\zeta_1'^*\mathbf{N}\zeta_1) + (\zeta_{-1}'^*\mathbf{N}\zeta_{-1}) = \mathrm{Sp}\mathscr{M},$$
$$2B_z = (\zeta_1'^*\mathbf{N}\zeta_1) - (\zeta_{-1}'^*\mathbf{N}\zeta_{-1}) = \mathrm{Sp}(\sigma_\gamma^z\mathscr{M}),$$
$$2B_x = (\zeta_1'^*\mathbf{N}\zeta_{-1}) + (\zeta_{-1}'^*\mathbf{N}\zeta_1) = \mathrm{Sp}(\sigma_\gamma^x\mathscr{M}),$$
$$2iB_y = (\zeta_1'^*\mathbf{N}\zeta_{-1}) - (\zeta_{-1}'^*\mathbf{N}\zeta_1) = i\,\mathrm{Sp}(\sigma_\gamma^y\mathscr{M}) \qquad (8)$$

where the spurs (traces) are taken over the photon variables. The quantities A and B_i can be connected with the quantities $R_1, \ldots, R_6$, which were introduced in reference 2 (hereafter referred to as I) and which determine the matrix $\mathscr{M}$ [cf. Eq. (I, 15)]:

$$2A = (R_1 + R_2)(1 + \cos\theta) - i(R_3 + R_4)\sin\theta(\boldsymbol{\sigma}\mathbf{n}),$$
$$2B_z = (R_3 + R_4)\boldsymbol{\sigma}(\mathbf{k} + \mathbf{k}') + (1 + \cos\theta)(R_5 + R_6)\boldsymbol{\sigma}(\mathbf{k} + \mathbf{k}'),$$
$$2iB_y = [R_3 - R_4 - (1 - \cos\theta)(R_5 - R_6)]\boldsymbol{\sigma}(\mathbf{k} - \mathbf{k}'),$$
$$2B_x = (R_1 - R_2)(1 - \cos\theta) + i(R_3 + R_4)\sin\theta(\boldsymbol{\sigma}\mathbf{n}), \qquad (9)$$

where $\mathbf{n}\sin\theta = \mathbf{k}\times\mathbf{k}'$, $\cos\theta = \mathbf{k}\cdot\mathbf{k}'$.

It is easy to calculate the density matrix of the final state:

$$\rho_f = \tfrac{1}{2}(A + \boldsymbol{\sigma}_\gamma\mathbf{B})(1 + \boldsymbol{\sigma}_\gamma\mathbf{P})(A^+ + \boldsymbol{\sigma}_\gamma\mathbf{B}^+)$$
$$= \tfrac{1}{2}\{AA^+ + \mathbf{BB}^+ + (A\mathbf{B}^+ + \mathbf{B}A^+)\mathbf{P} - i([\mathbf{BB}^+]\mathbf{P})\}$$
$$+ \tfrac{1}{2}\boldsymbol{\sigma}_\gamma\{A\mathbf{B}^+ + \mathbf{B}A^+ + i[\mathbf{BB}^+] + (AA^+ - \mathbf{BB}^+)\mathbf{P} + [\mathbf{B}(\mathbf{PB}^+)$$
$$+ (\mathbf{BP})\mathbf{B}^+] + iA[\mathbf{PB}^+] - i[\mathbf{PB}]A^+\}. \qquad (10)$$

By means of the expression (10) one can calculate all observable quantities. For the interaction of unpolarized γ rays and nucleons the differential cross section will have the form

$$d\sigma/do \equiv I_0(\theta) = \tfrac{1}{2}\mathrm{Sp}(AA^+ + \mathbf{BB}^+), \qquad (11)$$

where the spur is taken over the nucleon variables. Substituting Eq. (9) in Eq. (11), we get

$$4I_0(\theta) = |R_1 + R_2|^2(1 + \cos^2\theta) + |R_1 - R_2|^2(1 - \cos\theta)^2$$
$$+ |R_3 + R_4|^2(3 - \cos^2\theta + 2\cos\theta)$$
$$+ |R_3 - R_4|^2(3 - \cos^2\theta - 2\cos\theta)$$
$$+ 2|R_5 + R_6|^2(1 + \cos\theta)^3 + 2|R_5 - R_6|^2(1 - \cos\theta)^3$$
$$+ 4\mathrm{Re}(R_3 + R_4)^*(R_5 + R_6)(1 + \cos\theta)^2$$
$$- 4\mathrm{Re}(R_3 - R_4)^*(R_5 - R_6)(1 - \cos\theta)^2. \qquad (12)$$

The expression for the polarization of the nucleon after the interaction of an initially unpolarized photon and nucleon can be represented in the form*

$$2I_0(\theta)\langle\boldsymbol{\sigma}\rangle_f = \sin\theta\,\mathbf{n}\,\mathrm{Im}[(R_3 + R_4)(R_1 + R_2)^*(1 + \cos\theta)$$
$$- (R_3 - R_4)(R_1 - R_2)^*(1 - \cos\theta)]$$
$$= 2i[\mathbf{k}\times\mathbf{k}']\{R_1R_4^* - R_1^*R_4 + R_2R_3^* - R_2^*R_3$$
$$+ [R_1R_3^* - R_1^*R_3 + R_2R_4^* - R_2^*R_4]\cos\theta\}. \qquad (13)$$

The well known fact that the cross section $I_0(\theta)$ does not change when one replaces electric transitions by magnetic appears in the fact that Eq. (12) is invariant under the simultaneous interchanges:

$$R_1 \rightleftarrows R_2, \quad R_3 \rightleftarrows R_4, \quad R_5 \rightleftarrows R_6. \qquad (14)$$

It can be seen from Eq. (13) that the expression for the polarization of the recoil nucleon also remains unchanged by this transformation.

3. Let us now establish the relations between the Stokes parameters and the statistical tensor moments. As is well known, the statistical tensor moments are defined by the relations

*Eqs. (19), (23), and (24) in reference 4 contain errors.

$$T_{00}=1/\sqrt{3},\quad T_{10}=S_z/\sqrt{2},\quad T_{20}=\sqrt{\tfrac{2}{3}}\left(\tfrac{3}{2}S_z^2-1\right),$$
$$T_{22}=\tfrac{1}{2}[S_x^2-S_y^2+i(S_xS_y+S_yS_x)],$$
$$T_{2-2}=\tfrac{1}{2}[S_x^2-S_y^2-i(S_xS_y+S_yS_x)]. \qquad (15)$$

They are normalized so that

$$\mathrm{Sp}\, T_{JM}T^{+}_{J'M'}=\delta_{JJ'}\delta_{MM'}. \qquad (16)$$

By means of these tensor moments the density matrix can be represented in the form

$$\rho_f=\rho_{00}T_{00}+\rho_{10}T_{10}+\rho_{20}T_{20}+\rho_{22}T_{22}+\rho_{2-2}T_{2-2}. \qquad (17)$$

Here $\rho_{00}=2^{1/2}\rho_{20}=3^{-1/2}$. The parameters ρ_{JM} are connected with the Stokes parameters:

$$\rho_{10}=\sqrt{2}P_z,\qquad \rho_{22}=P_x-iP_y,\qquad \rho_{2-2}=P_x+iP_y. \quad (18)$$

In virtue of time-reversal invariance,[5,4] the expression for the cross section $I(\theta,\varphi)$ for scattering of a polarized γ-ray beam by unpolarized protons can be put in the form

$$I(\theta,\varphi)=I_0(\theta)[1+2\langle T_{22}\rangle_i\langle T_{22}\rangle_f\cos 2\varphi], \qquad (19)$$

where

$$2I_0(\theta)\langle T_{22}\rangle_f=\sin^2\theta(|R_1|^2+|R_4|^2-|R_2|^2-|R_3|^2), \quad (20)$$

$\langle T_{22}\rangle_i$ is the initial polarization of the γ-ray beam. We note that the expression (20) changes sign under the transformation (14).

4. For practical calculations we use the results of reference 2. The deviations from monotonic variation of the cross-section for scattering of γ rays in the immediate neighborhood of the meson-production threshold are due to the production of mesons in the s state. According to the available experimental data, the cross section for production of π^+ mesons in the s state is much larger than the cross section for production of π^0 mesons in this state. Difficult experiments had to be done even to establish the fact that π^0 mesons indeed are produced in the s state.

The energy ν of a photon in the laboratory system (l.s.) and the energy ν_c in the center-of-mass system (c.m.s.) are connected by the well known relation

$$\nu_c=\nu/\sqrt{1+2\nu/M}.$$

Using the expression for the total energy of the meson in the c.m.s.

$$\omega_c=(\nu+m_\pi^2/2M)/\sqrt{1+2\nu/M}$$

we find without difficulty that the expression for the square of the momentum of the meson produced

$$q_c^2=\omega_c^2-m_\pi^2=(\nu-\nu_0)(\nu+\nu_0-m_\pi^2/2M)/(1+2\nu/M);$$
$$\nu_0=m_\pi(1+m_\pi/2M)$$

can with some accuracy (better than 7.5 percent) be replaced by

$$q_c^2\approx(\nu^2-\nu_0^2)/(1+2\nu/M). \qquad (21)$$

This then gives

$$q_c/\nu_c\doteq(\nu^2-\nu_0^2)^{1/2}/\nu.$$

Because there is a mass difference between neutron and proton and between π^+ and π^0 mesons, the effects near threshold in the scattering of γ rays by nucleons have a "fine structure." To make reliable numerical calculations one would need a much more detailed analysis of the data on photo-production than we now have available. Wishing to get an idea of the scale of size of the effects near threshold, we shall confine ourselves mainly to a consideration of π^+-meson production. The quantities E_1, E_{33}, and M_{33} are taken from the analysis of Watson and others.[6] The production of π^0 mesons is taken into account only in the resonance state (through M_{33}). The connection of the photo-production amplitudes with the π-N scattering phase shifts is well known (cf., e.g., references 6, 7).* In the range of energies where we can neglect the difference between the $\nu_0(\pi^0)$ and $\nu_0(\pi^+)$ thresholds, when we sum the contributions from π^+ and π^0 mesons the terms containing the π-N scattering phase shifts cancel each other. For example

$$|M_3^+|^2+|M_3^0|^2=6|M_{33}|^2+\tfrac{3}{4}|M_{13}^{(1)}-2\delta M_{13}^{(1)}|^2\approx 6|M_{33}|^2.$$

We have calculated the dispersion integrals by using simple expressions to interpolate the energy dependence of $|E_1|^2$ and $|M_{33}|^2$ and then integrating directly. Setting $\nu_0=150$ Mev, and hereafter measuring energies in terms of ν_0, in the range $1\le\nu\le\nu_1=2.20$ we approximate the energy dependence of $|E_1|^2$ by the following expression:

$$|E_1|^2\approx|E_1^+|^2=A\sqrt{\nu^2-1}/\nu,$$
$$A=(3.3\cdot10^{-15}\,\mathrm{cm/sr}^{1/2})^2\nu_0=0.54\ e^2/M. \qquad (22)$$

It is just the contribution E_1^2 in the dispersion integrals that leads to the nonmonotonic behavior in the energy dependence of the real parts of the amplitudes. As can be seen from (I, 42), the contribution of $|E_1|^2$ is characterized by two integrals

$$\frac{2\nu^2}{\pi}\int_1^{\nu_1}\frac{|E_1|^2}{\nu'^2-\nu^2}d\nu',\qquad \frac{2\nu^3}{\pi}\int_1^{\nu_1}\frac{|E_1|^2d\nu'}{\nu'(\nu'^2-\nu^2)}. \qquad (23)$$

Substitution of Eq. (22) in the expression (23) gives

*In the more general form of the problem[4] one requires the parametrization of a three-rowed S matrix, which describes both the photo-production and scattering of π mesons and also the scattering of γ rays by nucleons. For the scattering of γ rays effects of deviations from isotopic invariance can give additions to the scattering phase shifts (and to the mixing coefficients) that are by no means small.

$$\frac{2\nu^2}{\pi}\int_1^{\nu_1}\frac{|E_1|^2}{\nu'^2-\nu^2}d\nu' = \frac{2}{\pi}A\times\begin{cases}\tan^{-1}(\nu_1^2-1)^{1/2}-\dfrac{(\nu^2-1)^{1/2}}{2}\ln\left|\dfrac{(\nu_1^2-1)^{1/2}+(\nu^2-1)^{1/2}}{(\nu_1^2-1)^{1/2}-(\nu^2-1)^{1/2}}\right|, & \nu>1\\ \tan^{-1}(\nu_1^2-1)^{1/2}-(1-\nu^2)^{1/2}\tan^{-1}\sqrt{\dfrac{\nu_1^2-1}{1-\nu^2}}, & \nu<1\end{cases}\tag{24}$$

and

$$\frac{2}{\pi}\nu^3\int_1^{\nu_1}\frac{|E_1|^2}{\nu'(\nu'^2-\nu^2)}d\nu' = \frac{2}{\pi}A\nu\times\begin{cases}\left(\dfrac{\nu_1^2-1}{\nu_1^2}\right)^{1/2}-\dfrac{(\nu^2-1)^{1/2}}{2\nu}\ln\left|\dfrac{\nu(\nu_1^2-1)^{1/2}+\nu_1(\nu^2-1)^{1/2}}{\nu(\nu_1^2-1)^{1/2}-\nu_1(\nu^2-1)^{1/2}}\right|, & \nu>1\\ \left(\dfrac{\nu_1^2-1}{\nu_1^2}\right)^{1/2}-\left(\dfrac{1-\nu^2}{\nu^2}\right)^{1/2}\tan^{-1}\sqrt{\dfrac{\nu^2(\nu_1^2-1)}{\nu_1^2(1-\nu^2)}}, & \nu<1.\end{cases}\tag{25}$$

From Eqs. (24), (25), (22), (I, 42), and (I, 32) it can be seen that at the meson-production threshold the derivatives of the quantities R_1 and R_3 go to infinity (approaching threshold from the side $\nu > 1$), and the derivatives of the real parts of these quantities also go to infinity (on the side $\nu < 1$), whereas on the other side of threshold the derivatives are finite. This result is very general. Thus the dispersion relations turn out to contain specific effects near the reaction threshold like those discussed and analyzed without use of the dispersion relations by Wigner, Baz', Okun', Breit, Capps, Newton, and others.*

The use of dispersion relations makes it possible to examine in more detail the effect on the elastic scattering (or on the reaction) of the inelastic processes that occur in a certain energy range. Moreover, the interesting effects that occur in the immediate neighborhood of the reaction threshold ("local effects" which could be discussed when one does not use the method of analytic continuation given by the dispersion relations) are only a part of the total effect of the inelastic processes on the energy dependences of the quantities that characterize the elastic scattering.

From the example of the scattering of γ rays by protons we can see how the presence of the inelastic process of meson photoproduction in the energy range $\nu > 1$ affects the characteristics of the elastic scattering, including also effects for $\nu < 1$ (deviation from the Powell formula, or from Eq. (I.16) for $\gamma < 1$). The deviation from monotonic variation in Eqs. (24) and (25) is characterized by a sharp drop from the value of the function at $\nu = 1$ in the region $\nu < 1$ (with an infinite derivative at $\nu = 1$) and a slow drop in the region $\nu > 1$ (with a finite derivative at $\nu = 1$).

5. In the range of energies 330 — 500 Mev (2.2 < ν < 3.34) the quantity $|E_1|^2$ is represented in the form

$$|E_1|^2 = 1.27(1-0.175\nu)^2e^2/M.\tag{26}$$

The contribution from this energy range to the values of the real parts of the amplitudes is small, if for the scattering of the γ rays we consider the energy near and below the threshold.

The analysis of the photoproduction made previously, and particularly the results of Akiba and Sato, indicate that

$$|M_3|^2 = 6|M_{33}|^2 \approx |E_2|^2 \approx \mathrm{Re}(E_2^*M_3).\tag{27}$$

For our estimates we adopt Eq. (27). The polarization of the recoil nucleons is especially sensitive to this assumption. In the energy range $1 < \nu < 2$ the quantity $|M_{33}|^2$ can be approximated by the expression

$$|M_{33}|^2 = B_0\nu(\nu^2-1)^{3/2},\qquad B_0 = 0.009e^2/M.\tag{28}$$

Consequently,

$$|M_3|^2 = 6|M_{33}|^2 = B\nu(\nu^2-1)^{3/2},\qquad B = 0.054e^2/M,$$

and the contribution of this expression, which describes the production of mesons in the p state, to the dispersion relations is given by the integrals

$$\frac{2\nu^2}{\pi}\int_1^{\nu_1}\frac{|E_2|^2}{\nu'^2-\nu^2}d\nu' = \frac{2B}{\pi}\nu^2\left[\frac{1}{3}(\nu_1^2-1)^{3/2}+(\nu_1^2-1)^{1/2}(\nu^2-1)\right] + \frac{2B}{\pi}\nu^2\begin{cases}-\dfrac{1}{2}(\nu^2-1)^{3/2}\ln\left|\dfrac{(\nu_1^2-1)^{1/2}+(\nu^2-1)^{1/2}}{(\nu_1^2-1)^{1/2}-(\nu^2-1)^{1/2}}\right|, & \nu>1\\ (1-\nu^2)^{3/2}\tan^{-1}\sqrt{(\nu_1^2-1)/(1-\nu^2)}, & \nu<1\end{cases}\tag{29}$$

and

$$\frac{2\nu^3}{\pi}\int_1^{\nu_1}\frac{d\nu'}{\nu'}\frac{|E_2|^2}{\nu'^2-\nu^2} = \frac{B}{\pi}\nu^3\left[\nu_1(\nu_1^2-1)^{1/2}+(\nu^2-3/2)\ln\left|\frac{\nu_1+(\nu_1^2-1)^{1/2}}{\nu_1-(\nu_1^2-1)^{1/2}}\right|\right] - \frac{B}{\pi}\nu^3(\nu^2-1)\begin{cases}\sqrt{\dfrac{\nu^2-1}{\nu^2}}\ln\left|\dfrac{\nu(\nu_1^2-1)^{1/2}+\nu_1(\nu^2-1)^{1/2}}{\nu(\nu_1^2-1)^{1/2}-\nu_1(\nu^2-1)^{1/2}}\right|, & \nu>1\\ 2\sqrt{\dfrac{1-\nu^2}{\nu^2}}\tan^{-1}\dfrac{\sqrt{(\nu_1^2-1)/\nu_1^2}}{\sqrt{(1-\nu^2)/\nu^2}}, & \nu<1\end{cases}\tag{30}$$

which have the characteristic feature that the second derivative with respect to the energy goes to infinity (again on the side $\nu < 1$).

*The writers plan to turn to the application of dispersion relations to this problem in another paper.

In the energy range $2 < \nu < 3.34$

$$6\,|M_{33}|^2 = 2.17\,(1 - 0.244\nu)^2 e^2 / M. \tag{31}$$

The contributions of the expressions (28) and (31) are given by integrals of the forms

$$J_1(\nu) = \frac{2\nu^3}{\pi}\int_{\nu_1}^{\nu_2} \frac{\alpha + \beta\nu' + \gamma\nu'^2}{\nu'(\nu'^2 - \nu^2)}\,d\nu'$$

$$= \frac{\nu}{\pi}\ln\left\{\left(\frac{\nu_2 - \nu}{\nu_1 - \nu}\right)^{\alpha+\beta\nu+\gamma\nu^2}\left(\frac{\nu_2 + \nu}{\nu_1 + \nu}\right)^{\alpha-\beta\nu+\gamma\nu^2}\left(\frac{\nu_1}{\nu_2}\right)^{2\alpha}\right\}, \tag{32}$$

$$J_2(\nu) = \frac{2\nu^2}{\pi}\int_{\nu_1}^{\nu_2} \frac{d\nu'\,(\alpha + \beta\nu' + \gamma\nu'^2)}{\nu'^2 - \nu^2}$$

$$= \frac{\nu}{\pi}\left\{2\gamma\nu\,(\nu_2 - \nu_1) + (\alpha + \gamma\nu^2)\ln\left(\frac{\nu_2 - \nu}{\nu_1 - \nu}\,\frac{\nu_1 + \nu}{\nu_2 + \nu}\right) + \beta\nu\ln\left(\frac{\nu_2^2 - \nu^2}{\nu_1^2 - \nu^2}\right)\right\} \tag{33}$$

6. The energy dependences of the real parts of the amplitudes $R_1, \ldots, R_6$ (in the l.s.), calculated by means of dispersion relations, are shown in Fig. 1, a, b. The half-widths of Re (R_1) and Re (R_2)

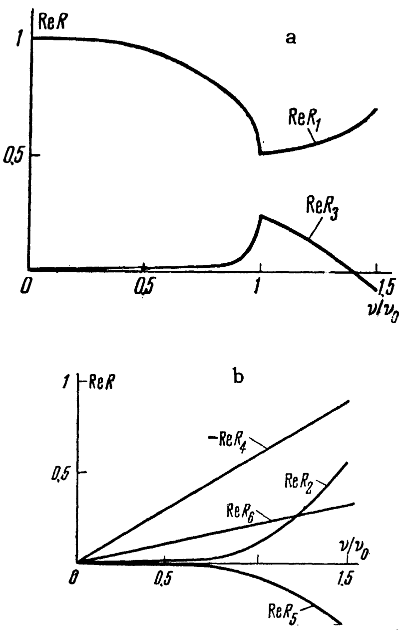


FIG. 1. Energy dependences of the real parts of the amplitudes R_1 and R_3 (a) and R_2, R_4, R_5, and R_6 (b) (the values of the functions are expressed in terms of e^2/M as a unit).

are $\nu_0/10$ and $\nu_0/20$, respectively, and are mainly due to the square of the ratio of the real part to the coefficient A in Eq. (22):

$$\varepsilon = 1 - \nu = \tfrac{1}{8}\,(\mathrm{Re}\,R)^2/A^2. \tag{34}$$

In a general analysis of the nonmonotonic behavior near the threshold A. I. Baz' has given for the width of the peak restrictions of the form $r_0(1-\nu^2)^{1/2} \ll 1$ (where r_0 is the interaction radius). The more detailed treatment of the present paper has automatically given the more accurate criterion (34). The effect of the inelastic processes on Re (R_3) is very strong, although the contribution of Re (R_3) to the observable quantities is small, so that the experimental study of the energy dependence of Re (R_3) is a difficult problem. The energy dependence of Re (R_4) and Re (R_6) is given with great accuracy by the general relation (I,18). The departure from zero of Re (R_2) and Re (R_5) is entirely due to inelastic processes, but the production of mesons in the s state does not contribute to these quantities.

The differential scattering cross section (in the c.m.s.) (12) can be written in the form

$$I_0(\theta, \nu) = A_0(\nu) + A_1(\nu)\cos\theta + A_2(\nu)\cos^2\theta + A_3(\nu)\cos^3\theta. \tag{35}$$

The results of calculations for the scattering angles 90° and 0° are shown in Figs. 2 and 3. We at once note the marked difference between the energy dependences of the cross sections at $\theta = 0$ and at 90°.

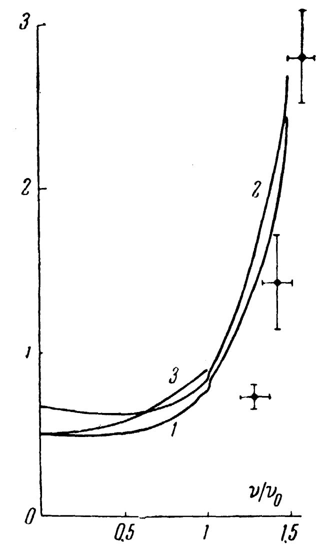


FIG. 2. Energy dependences of the differential cross section $I_0(90°)$ (curve 1), the total scattering cross section (curve 2), and the differential cross section with the dispersion part not taken into account (curve 3) (the values of the functions are expressed in terms of $(e^2/M)^2$ as a unit). Experimental data from reference 9.

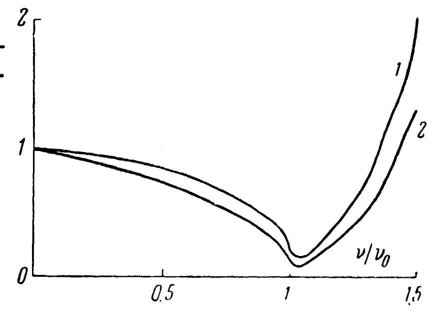


FIG. 3. Energy dependence of the differential cross section $I_0(0°)$: curve 1 is for the cross section in the l.s.; curve 2, for the c.m.s. (values of the functions in units $(e^2/M)^2$.

The function $I_0(0°, \nu)$ has been calculated earlier by Cini and Stroffolini.[8] We have improved the accuracy in the region near the threshold. Outside this region there is good agreement between the two calculations. Our results relating to $I_0(90°, \nu)$ in the energy region near 200 Mev also agree with other published calculations.[9] A new contribution is the careful treatment of the region near threshold, in which there are effects not discussed previously.

Figure 2 shows the energy dependence of the total cross section for elastic scattering, and also shows for comparison the energy dependence of the cross section calculated from Eqs. (16) and (18). The effects near threshold are practically imperceptible, but the difference between the two curves shows the general effect of inelastic processes on the elastic-scattering cross section.

The local effects are much more prominent if we calculate the difference

$$\sigma_s/4\pi - I_0(90°,\ \nu)$$

or the dependence of A_2 on the energy ν (Figs. 4 and 5). To get experimental data on A_2 one

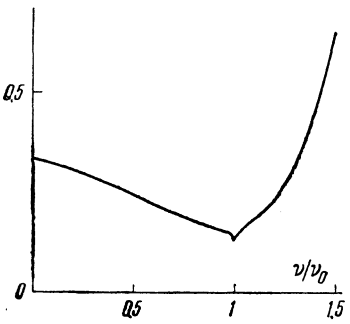


FIG. 4. Energy dependence of $2[\sigma_s/4\pi - I_0(90°)]$ (in units $(e^2/M)^2$).

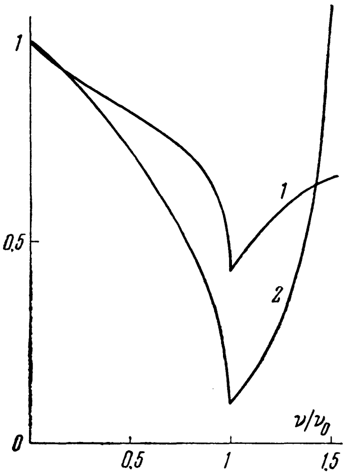


FIG. 5. Energy dependences: 1 – of the photon polarization $2\langle T_{22}(90°)\rangle$; 2 – of twice the coefficient $A_2(\theta)$ of the $\cos^2\theta$ term in the cross section (values of functions in units $(e^2/M)^2$).

needs only to study the cross sections $I_0(\theta, \nu)$ at $\theta = 45°$, 90°, and 135° with sufficient accuracy to find the energy dependence of the difference

$$I_0(45°) + I_0(135°) - I_0(90°).$$

It is interesting to note the energy dependence of the polarization of the recoil nucleon. Below the meson-production threshold the imaginary parts of the quantities $R_1, \ldots, R_6$ vanish in the e^2 approximation, the right member of Eq. (13) is zero, and there is no polarization of the recoil nucleon. Below threshold, in virtue of invariance under time reversal, the cross section for scattering by polarized protons does not differ from $I_0(\theta)$. Above the threshold for production of π mesons there is a nonvanishing polarization of the recoil nucleons. The values of the imaginary parts of the amplitudes above threshold are shown in Fig. 6. The results of calculations on the dependence of the polarization at $\theta = 90°$ (angle in c.m.s.) on the photon energy (in the l.s.) are shown in Fig. 7. It can be seen that over a rather wide range of energies, 180 — 220 Mev, the polarization reaches 20 to 25 percent.

FIG. 6. Energy dependence of the imaginary parts of the amplitudes (in units e^2/M).

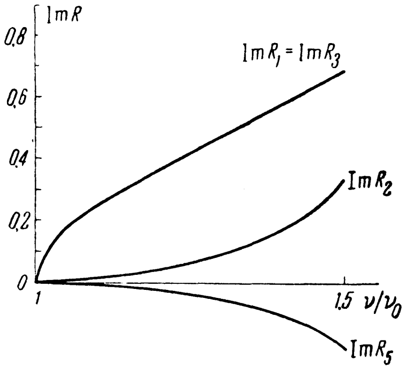
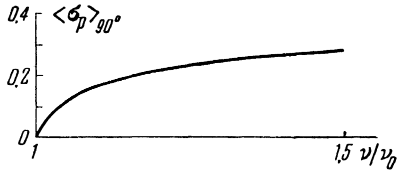


FIG. 7. Energy dependence of the polarization of the recoil protons at $\theta = 90°$.



The values of the polarization are rather sensitive to the assumptions made in the analysis of the photoproduction data, and in particular to the assumption (27). Consequently, the experimental study of the polarization of the recoil nucleons could give valuable information about the photoproduction of mesons.

In the expression (20), as compared with $I_0(\theta)$, there is a decided decrease of the contribution of $|R_4|^2$, and $|R_3|^2$ occurs with the negative sign, so that the dips near the threshold are particularly marked in the energy dependence of $\langle T_{22}(90°)\rangle$ (Fig. 5).

7. A detailed examination of the scattering of γ rays by nucleons in the region near the meson-production threshold, made by the use of dispersion relations, has made it possible to see what effect the production of mesons in the s state has on the anomalies near the threshold. The scattering of γ rays by nucleons and by nuclei is an example of the sort of process in which the energy dependence of the amplitudes is especially strongly affected by inelastic processes and the effects extend over a wide range of energies. In γ-N scattering the local effects on a number of observable

quantities are quite appreciable, but rather severe requirements are imposed on the procedures for experimental studies, expecially as regards resolution in energy, since the widths of the dips in question are of the order of 5 to 10 Mev.

The treatment given in the present paper shows that the effects near threshold are sometimes masked by the strong energy dependence of the scattering amplitudes. Therefore it seems that the most favorable conditions for the experimental study of such effects should be found at small energies, and also for the interaction of particles with small spins.

In the case of γ-N scattering, besides the contribution of the "peak" amplitudes R_1 and R_3, there are large effects from other amplitudes, particularly from R_4. The effects of these "smearing-out" factors may be smaller in the scattering of γ rays by helium nuclei (or other spinless nuclei), since in this case the transition matrix will have the form

$$M = R_1'(\mathrm{ee}') + R_2'(\mathrm{ss}').$$

A treatment of the scattering of γ-ray quanta by deuterons near the threshold for the photodisintegration of the deuteron, where local effects will evidently be large, will be presented in another paper.

From the point of view of the general effect of some processes on others it is interesting to analyze the photodisintegration of the deuteron in the energy range near and below the threshold for meson production. Noting the results of the calculations on the γ-N scattering, we can evidently suppose that the well known "resonance" energy dependence of the cross section for the photodisintegration of the deuteron is due to meson-production processes above threshold and can be treated by a method using dispersion relations.

It is commonly assumed that at quite high γ-ray energies the γ-N scattering cross sections will be almost entirely due to inelastic processes, i.e., to the imaginary parts of the amplitudes. In this connection it may be very interesting to study γ-N scattering, and especially the polarization of the recoil nucleons, near the thresholds of reactions of the production of new particles, such as

$$\gamma + N \rightarrow Y + K,$$

and a number of other processes. In this case the difficulties associated with the size of the cross section and the low energy of the recoil nucleon may very probably be smaller.

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