INFLUENCE OF THE MEDIUM DENSITY ON BREMSSTRAHLUNG IN ELECTRON-PHOTON SHOWERS IN THE ENERGY RANGE $10^{11} - 10^{13}$ ev

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Fifteen electron-photon showers in the energy range $10^{11} - 10^{13}$ ev recorded in emulsion stacks were examined. The energies of the primary γ rays initiating the showers were determined by measuring the energy spectrum of cascade electrons at the depth of $2.5 - 3$ radiation units, and from the screening effect on the first pairs. The energy spectrum of pairs produced at a depth of up to 1.5 radiation units was measured. The results are in agreement with calculations carried out taking into account the influence of multiple scattering and of the polarization of the medium on the bremmsstrahlung of high-energy electrons.

1. INTRODUCTION

IT has been stated earlier[1,2] that it is possible to test experimentally the effects of the medium on the bremsstrahlung[3,4] through a study of the energy spectrum of electrons in electromagnetic cascades of sufficiently high energies ($\gtrsim 10^{12}$ ev). The polarization effect (Ter-Mikaelyan[3]) should be felt in the emulsion for the γ-ray frequency range satisfying the condition $\hbar\omega \ll 7 \times 10^{-5}$ E, where E is the electron energy. Moreover, the expression for the radiation intensity J is then dJ $\sim \omega^2 d\omega/E^2$ instead of the Bethe-Heitler relation dJ $\sim d\omega$. The multiple scattering effect (Landau and Pomeranchuk[4]) should be felt in the emulsion for the frequency range $7 \times 10^{-5} \ll \hbar\omega/E \leq 2 \times 10^{-8}$ E/mc^2, which leads to an expression for the intensity dJ $\sim \sqrt{\omega}\, d\omega/E$.

The foregoing conditions show (see also Fig. 1 of reference 2) that, for E = $5 \times 10^{11} - 10^{12}$ ev, the influence of the medium should lead to a markedly decreased radiation probability of γ rays with energy $\hbar\omega \gtrsim 10^9$ ev. It is evident that this fact will influence the energy spectrum of cascade electrons and electron-positron pairs, this influence being the stronger the greater the cascade depth t.

Special calculations of electromagnetic cascades in nuclear emulsions have been carried out by us earlier using the non-asymptotic cross sections for the elementary electromagnetic processes.[1,2] In one variant of the calculations (B-H), we used the Bethe-Heitler formulas for the radiation processes, while, in the other variant (M), the radiation of high-energy electrons was calcu-

lated according to the formulas of Migdal,[5] which take both effects of the medium into account. The calculations have shown that, under certain conditions, the study of the energy spectra of cascade particles at small depths in electron-photon showers in the energy range $10^{11} - 10^{12}$ ev makes it possible to determine the form of the bremsstrahlung spectrum of primary electrons. Let us mention some of these conditions:

First, the energy of primary particles producing the showers should be measured with maximum possible accuracy, since the difference between the Bethe-Heitler and Ter-Mikaelyan-Landau-Pomeranchuk spectra depends very strongly on this energy.

Second, the detection efficiency for low-energy electrons and pairs should be sufficiently high, since the difference between the B-H and M spectra is especially pronounced in the soft part of the spectrum.

The experimental results should be compared with calculations carried out using the true (non-asymptotic) cross sections for the bremsstrahlung and pair production. In the interpretation of the results, one should take the large fluctuations in the number of cascade particles, leading to large statistical errors, into account.

In recent years, electron-photon showers in the energy range $10^{10} - 10^{12}$ ev have been detected in nuclear emulsions in the course of many experiments.[6-14] In several of the showers, the development in the initial stages was considerably different from the average behavior predicted by the usual cascade theories for showers produced by single γ rays. In particular, anomalies were ob-

served in the number and in the energy spectrum of cascade particles. The hypothesis that these anomalies are characteristic for high-energy processes has been proposed.[15] The most detailed study of a high-energy shower ($\sim 7 \times 10^{11}$ ev) at small depths has been carried out by Miesowicz et al.,[11] who was the first to attempt to explain the experimental results on the spectrum of cascade pairs by means of the influence of the effects of the medium on the bremsstrahlung.

However, similar anomalies have not been detected in later experiments[12-14] in which the showers were studied more systematically. Thus, the published results on the spectrum of cascade particles in high-energy showers are contradictory. The majority of the published material can, unfortunately, not be used in examining the problem of the shape of the bremsstrahlung spectrum of high-energy electrons, since the basic conditions stated above have not been satisfied.

In the present article, the results of a systematic study of 15 electron-photon showers in the energy range $10^{11} - 10^{13}$ ev are presented. Preliminary results on five showers in the energy range $3 \times 10^{11} - 2 \times 10^{12}$ ev have already been published earlier.[16]

2. GENERAL CHARACTERISTICS OF THE SHOWERS

In the course of the experiment, six emulsion stacks with a total volume of about 10 liters were used. The stacks were irradiated at $20 - 27$ km altitude. General data on the four stacks analyzed up to 1958, and on the position of the five investigated showers in the stacks, have been reported in references 16 and 17. The two new stacks (α and β) were composed of layers with dimensions $10 \times 20 \times 0.04$ cm^3, and had a volume of 1.4 and 3.1 liters respectively.

The R-NIKFI emulsion was mainly used. The grain density in relativistic electron tracks amounted to $30 - 35$ grains per 100μ, i.e., considerably greater than in the Ilford G-5 emulsion. This facilitated greatly the detection and analysis of the showers.

Showers with energy of $\sim 10^{12}$ ev were usually detected in scanning the plates with the naked eye on white background, along the well-developed part of the cascade at a depth of $2 - 3$ radiation lengths. Only those showers produced by single isolated (not connected with jets) photons, and which had a path in one layer not less than a few millimeters, were selected for analysis.

The energy E_γ of the primary photons producing the showers was determined from the number of cascade electrons with energy higher than ϵ_c = 300 Mev at the depth of $2.5 - 3 t_0$,. where t_0 = 29 mm is the radiation length in the emulsion. The method used was analogous to that described by Pinkau[13] and Miesowicz et al.[11] (For details, see references 16 and 17.) The lateral electron distribution function of Guzhavin and Ivanenko[18] was used for the calculation of the number of particles traveling within a circle with radius ρ (in contrast with experiments 11, 13, 16, and 17, in which the calculations of Eyges and Fernbach[19] were used).

Numerical integration of the results of Guzhavin and Ivanenko[18] yielded the function η (s, ρ/t_0) (where s is the "age" parameter determined by the energy of the primary electron E_0 and the values of ϵ and t), which gives the ratio of the number of electrons with energy > ϵ, whose tracks are inside a circle with radius ρ around the shower axis, to the total number of electrons N_β (ϵ, E_0, t) with energy > ϵ at a given depth t (see Table I).

Values of the energy E_γ (for 12 out of 15 showers) determined from the curves N_β (ϵ, E_0, t) for electrons, calculated by the Monte Carlo method[2] and from corresponding curves of Janossy,[20] are given in Table II. The difference between the results for the five showers[16,17] presented previously and the data in Table II is due mainly to a change in the lateral corrections inherent in the function η. In addition, those curves N_β (ϵ, E_0, t) which have been used in the present analysis are based on greater statistical material[2] and are somewhat different from the corresponding curves used earlier.[16]

In finding E_γ from the calculated curve N_β (ϵ, E_0, t), the errors in E_γ were determined

TABLE I. Fraction of the total number of electrons with energy
> ϵ falling inside a circle with radius ρ (s — cascade
parameter, x = $\epsilon\rho/E_s t_0$, E_s = 21 Mev)

x	$s = 0.4$	$s = 0.5$	$s = 0.6$	$s = 0.7$	x	$s = 0.4$	$s = 0.5$	$s = 0.6$	$s = 0.7$
0.01	0.51	0.40	0.31	0.23	0.10	0.84	0.79	0.72	0.66
0.02	0.61	0.51	0.42	0.34	0.15	0.89	0.85	0.80	0.74
0.04	0.71	0.63	0.54	0.46	0.20	0.92	0.89	0.84	0.80
0.06	0.77	0.70	0.63	0.55	0.60	0.99	0.98	0.97	0.95

TABLE II. Results of the measurement of energy E_γ
(in 10^{11} ev)

Shower	From Monte-Carlo method	According to Janossy	From Eq. (3)	From Eq. (4)
β-213			$219\ ^{+161}_{-100}$	260
β-212			$96\ ^{+110}_{-50}$	149
D-84	$16.5\ ^{+143}_{-7.0}$	$1.2\ ^{+6}_{-4.5}$	$15.7\ ^{+17.5}_{-8.2}$	24.0
O-209	$8.4\ ^{+4.8}_{-2.7}$	$6.3\ ^{+1.8}_{-1.6}$	$18.4\ ^{+18.1}_{-9.3}$	26.1
E-53	$6.8\ ^{+68}_{-3.8}$	$5.3\ ^{+2.8}_{-2.6}$	$20.2\ ^{+27.2}_{-13.1}$	31.1
α-79	$3.1\ ^{+1.1}_{-1.0}$	$2.3\ ^{+0.7}_{-0.6}$	$12.3\ ^{+12.8}_{-6.2}$	17.9
I-109	$3.1\ ^{+2.2}_{-0.9}$	$2.4\ ^{+0.7}_{-0.6}$	$0.7\ ^{+1.2}_{-0.4}$	1.5
D-44	$2.9\ ^{+2.3}_{-0.9}$	$2.2\ ^{+0.7}_{-0.5}$	$1.4\ ^{+2.2}_{-0.8}$	2.3
β-344	$2.4\ ^{+2.3}_{-0.8}$	$1.8\ ^{+0.5}_{-0.4}$	$2.3\ ^{+4.5}_{-1.8}$	8.4
β-165	$1.9\ ^{+1.1}_{-0.5}$	$1.5\ ^{+0.3}_{-0.3}$	$3.9\ ^{+6.7}_{-3.0}$	13.0
E-78	$1.5\ ^{+1.7}_{-0.5}$	$1.2\ ^{+0.4}_{-0.3}$	$1.1\ ^{+2.2}_{-0.9}$	4.2
D-89	$1.3\ ^{+1.5}_{-0.5}$	$1.0\ ^{+0.4}_{-0.3}$		
O-317	$1.3\ ^{+1.1}_{-0.5}$	$0.9\ ^{+0.3}_{-0.2}$		
E-29	$1.1\ ^{+4.8}_{-0.6}$	$0.8\ ^{+0.5}_{-0.4}$	$2.6\ ^{+5.4}_{-2.1}$	10.0
E-39			$1.5\ ^{+3.2}_{-1.2}$	5.9

from the number of shower electrons according to the calculated distribution functions[2] for showers initiated by two primary electrons with energy $E_\gamma/2$. (See reference 17 concerning the determination of errors E_γ from the curves of Janossy.)

3. MEASUREMENT OF THE ENERGY OF PRIMARY PAIRS FROM THE SCREENING EFFECT

The energies of the primary pairs for $E_\gamma > 3 \times 10^{11}$ ev were also determined from the decrease in the grain density in the pair tracks near the vertex, due to the mutual screening of the electron and positron fields. This screening effect had first been calculated by Chudakov[21] using purely classical considerations, and was then treated by other authors,[22,23] among them Burkhardt[23] using quantum mechanics. The possibility of using the screening effect for the determination of the energy of separate pairs of $E_\gamma \gtrsim 3 \times 10^{11}$ ev was confirmed experimentally by Wolter and Miesowicz,[24] by Varfolomeev et al.,[17] and by Iwadare,[25] who, in addition to the measurement of the ionization energy losses of the pairs, carried out independent measurements of the pair energy. Since the various available theoretical formulas for the ionization due to pairs, as given by different authors, differ considerably from each other in the numerical values of the constants, it is reasonable to determine these from experimental results.[17,24,25]

In a classical treatment, the ratio of the specific ionization losses of a pair I to the losses of two separate electrons $2I_{pl}$ can be written in the form

$$R = A + B \ln r^2, \tag{1}$$

where I_{pl} is the ionization at the plateau of the ionization-loss curve, r is the distance between the trajectories of the electron and the positron, and A and B are constants (at least for R < 0.9) which depend on the medium.

The distance r is determined by the initial opening angle of the pair θ and by the multiple scattering of the electron and positron over a distance x from the pair vertex. The angle θ and the deflection due to multiple scattering, and consequently also R, depend statistically on the pair energy E_γ.

For the mean value of R, we can write $\overline{R} = A + B \overline{\ln r^2}$, where the averaging, in general, should be carried out taking into account the distribution of the energy E_γ between the components of the
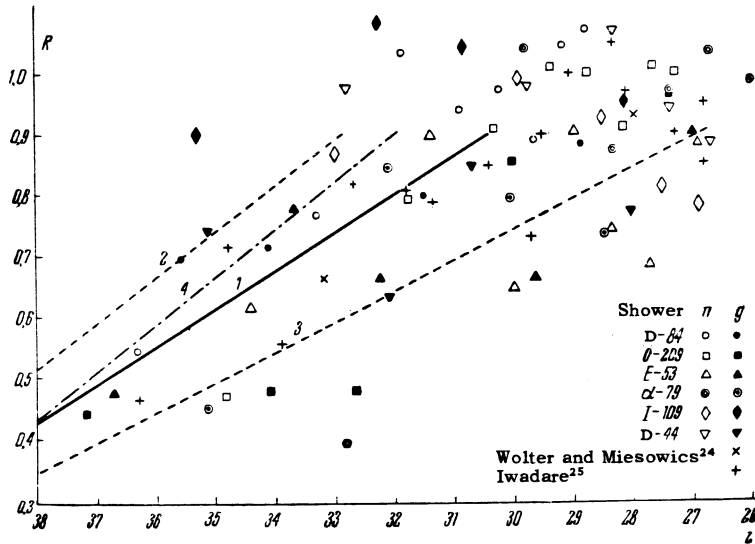
FIG. 1. Ratio of the ionization loss of a pair to that of two separate relativistic electrons R_i: 1 — curve calculated according to Eq. (3); 2 and 3 — curves corresponding to standard deviations of R from curve 1; 4 — curve calculated according to Eq. (4).

pair, the distribution of the multiple scattering deflection at a given distance x, and the distribution of the angle θ for a given energy distribution E_γ. Sacrificing some accuracy, we assume that the average value of $\ln r^2$ can be replaced by $\ln \overline{r^2}$. We furthermore write

$$r^2 = \theta^2 x^2 + \frac{k^2 x^3}{E^2}\left[\frac{1}{a^2} + \frac{2}{(1-a)^2}\right],$$

where a is the ratio of the energy of one of the electrons of the pair to the total pair energy E_γ, and k is the multiple scattering constant determining the mean-square deflection of the electron in space after traversing a path of length x from the tangent to its trajectory at the point x = 0. From the data of Rossi[26] and of Pickup and Voyvodic,[27] it can be assumed that k = 6.15 Mev rad/cm$^{1/2}$.*

If we are interested in the most probable value of R for a given value of E_γ, it is not necessary to average over all possible angles θ, but it is sufficient to consider the angle θ equal to the most probable angle $\vartheta = (4mc^2/E_\gamma) F(a)$.[28] After averaging the expression for r^2 over a in the limits of 0.5 — 0.1, using the pair distribution function with respect to the variable a for the case of full screening[26] and the values F(a) from the article by Borsellino,[28] we obtain

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*The deflection from the tangent is, by a factor of $\sqrt{2}$, smaller than the deflection determined by the second differences measured by the sagitta method (see, e.g., reference 26). Moreover, taking into account that the mean absolute value is smaller than the mean square value by a factor of $\sqrt{\pi/2}$, and that the deflection in space is greater than the deflection in a plane by a factor of $\sqrt{2}$, we find that $k = \sqrt{\pi/2}\,k'$, where k' is a constant corresponding to the mean absolute values of the second differences measured by the sagitta method in a plane. According to Pickup and Voyvodic,[24] $k' = 4.9$ Mev rad cm^{-2}.

$$\overline{r^2} = 1.6\,(2x/E_\gamma)^2(1 + 140x),$$

where x is given in centimeters, and E_γ in Mev. For the mean value of R, we can therefore write

$$\overline{R} = A' + B \ln\,[(2x/E_\gamma)^2(1 + 140x)], \qquad (2)$$

where A' and B, as before, are constants.

Experimental values of R_i are shown in Fig. 1 as a function of $z = \ln\,[\,(2x/E_\gamma)^2(1 + 140\,x)]$ (where x is given in cm, and E_γ in Mev). Data of references 17, 24, and 25, and results of measurements of the pair α-79 have been used. The constants A' and B have been determined from experimental points in the range R $\leq$ 0.9 by the least-squares method, and the following expression has been obtained as a result

$$\overline{R} = \frac{1}{16.1}\,\{45.0 + \ln\,[(2x/E_\gamma)^2(1 + 140x)]\}. \qquad (3)$$

The standard deviation of experimental points R_i from the corresponding values $\overline{R}$ given by Eq. (3) is $\sim 20\%$. It has been assumed that the statistical distribution of the ratio $R_i/\overline{R}$ is independent of $\overline{R}$. The curves for $R_{max} = 1.20\,\overline{R}$ and for $R_{min} = 0.80\,\overline{R}$, where $\overline{R}$ is given by Eq. (3), can be used for an estimate of the errors in the determination of the energy E_γ of individual pairs by means of Eq. (3).

It is evident that the errors in the determination of E_γ from the experimental values of R_{av}, averaged over an interval of x, depend also on the length of this interval x. An increase in the interval length leads to a decrease in the relative statistical fluctuations of the ionization losses of the pair, and thus to a decrease in the error of R_{av}. On the other hand, with an increase of x, the statistical spread of the values R_{av} around the curve C increases. Unfortunately, the ex-

perimental material presently available is, so far, insufficient for choosing an optimal value of x.

The pair energy E_γ found from ionization measurements is given in Table II. For the first eight pairs, the values of R_{av} were determined by measuring both grain (n) and gap (g) density.[17] For each pair, and for each variation of the ionization measurement method (n and g), two intervals were used, $0 - x_1$ and $0 - x_2$. The values of x_1 and x_2 were chosen from the conditions $\overline{R}(x_1) = 0.4 - 0.6$ and $\overline{R}(x_2) = 0.8 - 0.9$. Thus, for each of the eight pairs, four values each of $E_i(\alpha)$, of max $E_i(\alpha)$, and of min $E_i(\alpha)$, were obtained, where $\alpha = n, g$ indicates the method of measurement, and $i = 1, 2$ the interval x_1 or x_2. The results were averaged and the errors calculated in the way described below.

First, the mean logarithmic values of E_i, max E_i and min E_i, were determined separately for the first and second segments of the pair track (logarithmically averaged over the symbol α). Then, by averaging the values of E_i over i, the average values of E_γ given in Table II were obtained. The errors E_γ were determined as standard deviations of max E_i and min E_i from E_i. The described method of averaging was chosen since the results of the measurements by the various methods over the same segments are not independent. The deviations max E_i and min E_i from E_i are basically due to multiple scattering and not to errors in measuring n and g. On the other hand, the measurements over various segments can be regarded as only weakly correlated.

For the remaining five pairs given in Table II ($E_\gamma \sim 10^{11}$ ev), only the measurements of n over one segment x_2 were used.

For comparison, the values of E_γ are also given in Table II, as determined by the relation

$$\overline{R} = \frac{1}{12,9}\left\{43.6 + \ln\left[\left(\frac{2x}{E_\gamma}\right)^2(1 + 140x)\right]\right\}, \qquad (4)$$

which follows from the calculations of Burkhardt,[23] if one takes the multiple scattering into account and averages the results by the same method as described above.

Comparison of the data given in Table II shows that, in the majority of cases, a satisfactory agreement is obtained between the values of E_γ as determined from the shower development, and from the screening effect using Eq. (3).

The measurements of n and g on the first pairs of the showers β-212 and β-213 are shown in Figs. 2 and 3. The energy of these showers was determined from the screening effect only. This was, first of all, because of the lack of calculated cascade curves for energies $E_\gamma \sim 10^{13}$ ev. For
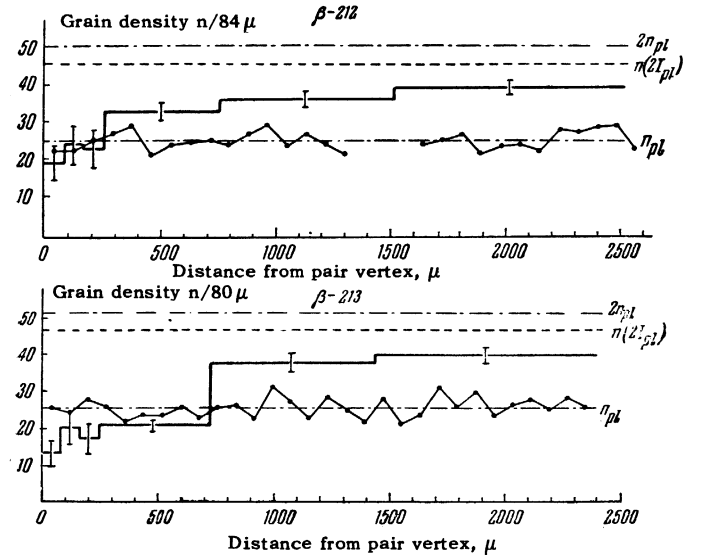

FIG. 2. Grain density n in tracks of electron-positron pairs β-212 and β-213: n_{pl} — grain density on the plateau of a track of a relativistic electron, $n(2I_{pl})$ — grain density in the track of particles with double losses $2I_{pl}$.
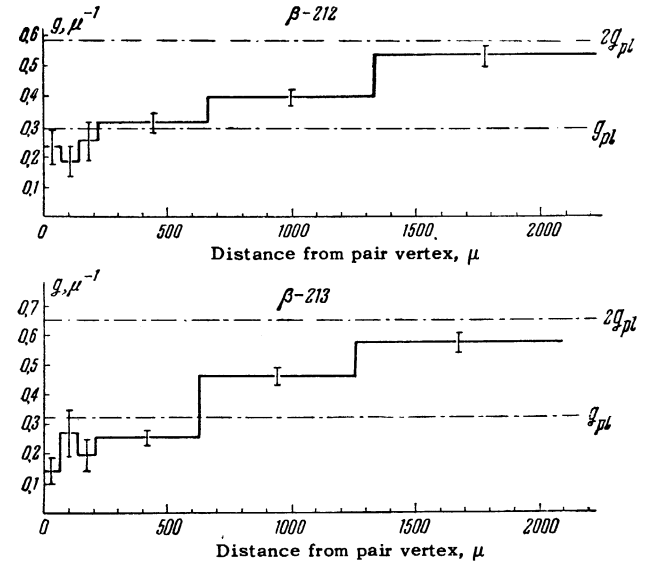

FIG. 3. Gap density g in tracks of the pairs β-212 and β-213.

such high energies, the influence of the medium can already be felt on the spectrum of cascade electrons with energies of $\sim 10^8 - 10^9$ ev (especially at small depths t). The usual cascade curves can, therefore, not be used. In addition, especially large fluctuations in the number of cascade particles at high E_γ lead to very large errors in the determination of E_γ from the shower development by the method described above.

4. MEASUREMENT OF THE ENERGY SPECTRA OF THE PAIRS

Further analysis of the showers consisted of the following: the vertices of the pairs produced

at the depth $\leq 1.5\,t_0$ were carefully searched for. The region inside the radius $\rho = 150\,\mu$ around the shower axis was investigated.* In the majority of cases, the search was repeated several times by various observers in order to reduce to a minimum the possibility of overlooking low-energy pairs.

The pairs lying outside the radius $\rho = 150\,\mu$ or at an angle greater than $10\,(mc^2/E_\gamma)$ × ln (E_γ/mc^2) to the shower axis, where E_γ is the pair energy, were regarded as part of the background and thus not related to the shower.†

After the spatial position of the shower had been reconstructed, the energy of the pair electron was measured from multiple scattering.

Measurements of the multiple scattering angle were carried out using a MBI-8M microscope with a glass guiding rail insuring low table noise. Additional head isolation of the light source made it possible to lower the thermal noise. During the measurements, the binocular attachment was rigidly connected to the base of the microscope by means of a metal frame. A guard shield was placed between the observer and the photographic plate. The microscope table was placed on rubber shock absorbers, and was loaded with a ballast of about 200 kg. As a result, the general noise in the measurements of the mean absolute values of second differences amounted to $0.13\,\mu$ over a cell of $250\,\mu$, and to $0.20\,\mu$ over a cell of $500\,\mu$. (The noise was measured on proton and electron tracks of $\sim 10^{12}$ ev.) A cell of $250\,\mu$ was mainly used. For a number of cells $n = 15 - 20$, the relative error in the determination of the electron energy is, according to Expong,[30] given by the expression

$$\sigma = \frac{1.12}{\sqrt{n}}\left[\frac{9}{16} + \frac{2}{3}\lambda^{-2} + \frac{35}{36}\lambda^{-4}\right]^{1/2},$$

where λ is the ratio of the measured second difference to the value of the general noise, amounting to $\sim 20 - 30\%$ up to an energy of $(5-7) \times 10^8$ ev, which, for the purpose of the present work, was fully satisfactory. In separate cases, either a cell of $500\,\mu$, or the relative multiple scattering method, was used for a determination of the pair energy. In several cases, because of unfavorable conditions for multiple scattering measurements, the pair energy was determined from the opening angle according to the Borsellino formula.[28] In any case, it can be assumed that the pair energy was measured with a sufficient accuracy up to 10^9 ev.

*In fact, the region inside $\rho = 400 - 500\,\mu$ was inspected, but only the region up to $\rho = 150\,\mu$ was investigated in a systematic and accurate way.

†The angle $\sim (mc^2/E_\gamma)$ ln (E_γ/mc^2) is roughly equal to the mean square angle between the γ ray producing the pair and the track of one electron of the pair.[29]

The showers β-212 and β-213 appear as solid strands of electron tracks over a relatively long path length (up to $\sim 1\,t_0$). In their analysis, our aim was to determine the number of pairs with energy > 1 Mev produced at the depth $\leq 1.0\,t_0$ and $\leq 1.5\,t_0$ respectively.

5. RESULTS AND DISCUSSION

The influence of the medium should make itself strongly felt in the total number of pairs produced at small depths. The total number of pairs N ($> \epsilon$) with total energy greater than $\epsilon = 1.5$ Mev,* produced at depths $\leq 1.0\,t_0$ and $\leq 1.5\,t_0$ is shown in Figs. 4 and 5 as a function of the primary electron energy. The calculated curves (1 and 2)
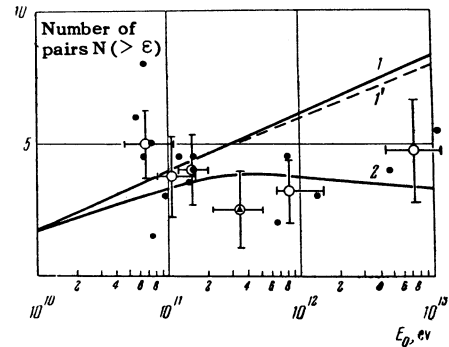
FIG. 4. Number of pairs N($> \epsilon$) with energy greater than $\epsilon = 1 - 2$ Mev produced on the average at a depth $\leq 1.0\,t_0$ per primary electron with energy E_0. 1 — calculated curve for the average of N in the B-H variant; 1' — calculated curve for the median value in the B-H variant; 2 — calculated curve for the average value of N in the M variant; ● — experimental values for showers listed in Tables II and III; ▲ — results of Miesowicz et al.[11] ○ — experimental data averaged over several showers.
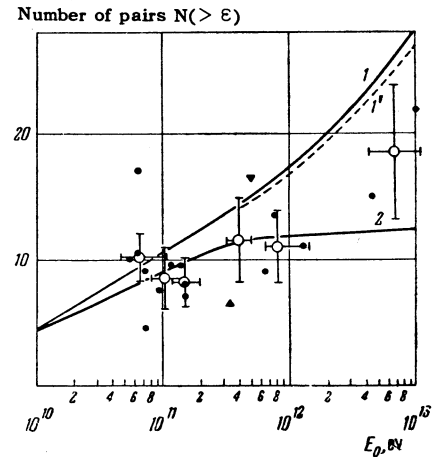
FIG. 5. Same as Fig. 4, but for pairs produced at the depth of $\leq 1.5\,t_0$. ▼ — data from the Feynves et al. shower.

*Both the experimental and the theoretical numbers of pairs with energy $1 - 5$ Mev are relatively small. The value of ϵ can, therefore, be taken as $1 - 2$ Mev.

have been obtained, respectively, with and without considering the influence of the medium on the radiation for showers produced by an electron with

energy E_0. The experimental points give the average number of pairs per primary electron.

The shower E-53 is assumed to be produced by one electron since it develops along the track of one electron. The energy of the second electron of the primary pair is $< 10^{11}$ ev.

In addition to the experimental results on 15 showers investigated in the present experiment (round points in Figs. 4 and 5), the data of Miesowicz et al.[11] on one shower with energy of $\sim 7 \times 10^{11}$ ev (Figs. 4 and 5), and data on one shower with energy of $\sim 10^{12}$ ev analyzed by the Budapest and Prague groups* (Fig. 5), have also been used.

As has already been mentioned above, the values of $E_\gamma = 2 E_0$ determined only from the screening effect (Table II) have been used for the showers β-213 and β-212. The energy of shower E-39 has been estimated only from the screening effect because of the unfavorable position of the shower in the stack.

At energies $E_\gamma \sim 10^{12}$ ev, one can consider both methods of measurements of E_γ equally accurate, and the final values of the energy of showers D-84, O-209, and E-53 were therefore determined by averaging the results from Table II.

Because of large statistical errors both in the number of pairs N and in the values of the shower energy E_γ, it was advisable to average the results over separate groups with close values of E_γ. In Figs. 4 and 5, the results for the showers with energy $E_\gamma > 1.8 \times 10^{11}$ ev, collected in six groups, are represented as large light circles. In view of the fact that, according to curves 1 and 2, the number of pairs N depends almost linearly on $\ln E_0$, the averaging has been done in coordinates N, $\ln E_0$. The errors indicated in Figs. 4 and 5 (similarly to other figures in the present article) are statistical errors.

An analysis of the calculations[1,2] showed that the distribution of showers produced by two electrons with energies E_0 with respect to the number of pairs $Q(2E_0, t)$ at a depth t can be described by Poisson's law with an average value of k if, for the independent variable, we take the value $Q(2E_0)/N(2E_0)$, where $N(2E_0) = \bar{Q}(2E_0)$ is the average number of pairs with energy $> \epsilon$. The factor k depends, in general, on E_0, ϵ, and t. For $\epsilon = 1 - 10$ Mev, and $E_0 = 10^{10} - 10^{13}$ ev, we can take $k = 3$ for the depth $1.0 t_0$ and $k = 6$ for the depth $1.5 t_0$.

The standard deviations of the number of pairs at the depth t in a shower produced by two elec-

trons can, therefore, be calculated from the relation $\sigma = N/\sqrt{k}$.

The integral energy spectrum of pairs produced at a depth $\leq 1.5 t_0$ in the three showers D-84, O-209, and E-53 is shown in Fig. 6. In the calculation, the data have been averaged for one primary electron. Theoretical curves correspond to a logarithmic mean energy of primary electrons $E_0 = 8.3 \times 10^{11}$ ev. Analogous results for three showers with energy $E_\gamma \sim 3 \times 10^{11}$ ev (D-44, I-109, and α-79) are given in Fig. 7. Theoretical curves are calculated for the energy $E_0 = 1.5 \times 10^{11}$ ev.

The comparison of experimental and theoretical results has been carried out only for the number and energy spectra of pairs, but not for electrons. This was because of the following reasons: according to the calculations carried out by us,[1,2] the difference between the two variants of the calculations
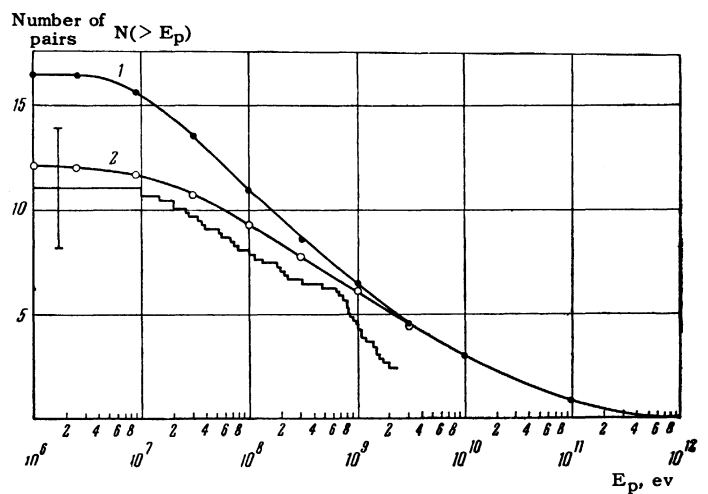

FIG. 6. Integral energy spectrum of electron-positron pairs produced on the average at a depth $\leqslant 1.5\ t_0$ in showers D-84, E-53, and 0-209 per primary electron. 1 — calculated curve in the B-H variant for $E_0 = 8.3 \times 10^{11}$ ev; 2 — the same in the M variant.
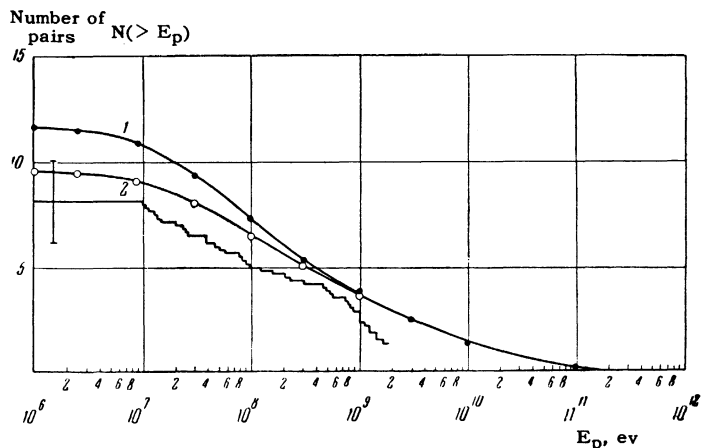

FIG. 7. Same as Fig. 6, but for showers α-79, I-109, and D-44; $E_0 = 1.5 \times 10^{11}$ ev.

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*The authors are grateful to Dr. E. Fenyves and Prof. V. Petrzilka for supplying data on this shower.[31]

B-H and M is markedly greater for the spectrum of pairs than for the spectrum of electrons, at the same finite depth.

Moreover, more reason exists for using the results of the one-dimensional calculations for pairs rather than for electrons. As a result of scattering, the length of the low-energy electron path can be substantially different from the path along the shower axis t, which has not been taken into account in the calculations.[1,2] Finally, experimental errors in detecting pairs are much smaller than for single electronic tracks in the section t_j. Slow electrons reaching a given depth can, because of scattering, be found at considerable distances from the shower axis. As a result, the tracks may be missed in scanning. The probability of such omissions is considerably lower when detecting pairs. It is also much simpler to exclude background tracks which are not related to the shower in scanning for pairs.

It follows from the study of the radial distribution of pair vertices in showers that the pairs are mainly concentrated in the region of small ρ ($\lesssim 10\mu$). The mean value of ρ decreases with increasing E_γ. Consequently, the largest experimental errors due to the omission of pairs belonging to the shower should occur in the study of showers with the smallest values of E_γ (in our case $\sim 10^{11}$ ev). However, the number of pairs satisfying an accepted selection criterion in five showers with energy of $\sim 10^{11}$ ev (see Figs. 4 and 5), are at least not smaller than the theoretically expected number. This indicates the absence of systematic negative experimental errors. One can consider that the experimental errors are at least such that they do not lead to a marked lowering of the total number of pairs as compared with statistical errors.

As can be seen from Figs. 4 − 7, the experimental points are concentrated close to curve 2, which takes the effect of the medium on radiation into account. A statistical analysis of the results shown in Figs. 4 and 5 has been carried out in order to obtain some quantitative estimate of the deviation of the experimental data from curves 1 and 2, using various significant criteria. Only the data for showers with energy $E_\gamma > 1.8 \times 10^{11}$ ev have been used. In the first type of analysis, the sign of the deviation of the experimental points from curve 1' in Figs. 4 and 5 has been taken into account. The values of the probabilities that, for a given point distribution, the curve 1' is really the median curve, is given in Table III.

The deviations have been assessed by the χ^2 criterion. The statistical distribution of the compared values around theoretical averages y should,

TABLE III. Results of a statistical analysis of the data on the number of pairs in showers (in percent)

Method of analysis and criterion	Neglecting errors in E_0		Taking errors in E_0 into account	
	B-H	M	B-H	M
Analysis of the error sign				
$1.0t_0$	32	—	—	—
$1.5t_0$	3.9	—	—	—
$1.0t_0 + 1.5t_0$	3.1	—	—	—
χ^2- criterion				
$1.0t_0$	3.5	90	4	90
$1.5t_0$	6.7	83	11	86
$1.0t_0 + 1.5t_0$	1.2	96	4	96
U- criterion				
$1.0t_0$	0.4	—	0.7	—
$1.5t_0$	1.1	—	1.3	—

in such a case, follow a normal law. In order that this condition be satisfied, it is necessary to compare the experimental values N_i averaged over groups of two to three showers (see Figs. 4 and 5).

In the beginning, let us assume that the errors of the measurements of E_γ can be neglected. The results for each of the showers are independent of each other, and we can therefore consider them as samples from a general set of theoretically possible values N for chosen (fixed) values x = ln E_0. According to the addition theorem for the χ^2 distribution for m experimental (group) values of N_i, the sum of χ^2 can be written as

$$\chi^2 = \sum_{i=1}^{m} \left(\frac{N_i - y(x_i)}{\sigma_i} \right)^2,$$

where N_i is the average experimental group value of the number of pairs, y(x) is the theoretical curve, and σ_i are the standard deviations of the theoretically possible values N_i from y(x_i). Since y(x) and σ_i are known, the sum has m degrees of freedom.

In order to take the errors in the measurements of E_0 into account, the results of the confluent analysis[32] carried out by Klepikov and Sokolov[33] have been used. The errors of E_0 and N_i can be regarded, in first approximation, as non-correlated. The theoretical curves y(x) can, with good accuracy, be approximated over each segment by a straight line, so that y" = 0. The sum of χ^2 can then be represented as

$$\chi^2 = \sum_{i=1}^{m} \frac{[N_i - y(x_i)]^2}{\sigma_i^2 + (y'\delta_i)^2},$$

where δ_i are the errors of (ln E_0)$_i$. The probability that the resulting value of the sum of χ^2 is not smaller than the observed value, assuming the

correctness of the curves 1 and 2, respectively, is given in Table III.

Finally, an analysis was carried out averaging the results at one point (U-criterion[32]). The averaging was carried out in coordinates N, ln E_0. Data on the number of pairs in 11 showers at the depth of $1.0\,t_0$ and in 12 showers at the depth of $1.5\,t_0$ ($E_\gamma > 1.8 \times 10^{11}$ ev) have been used. The relative deviations $u = [\overline{N} - y\,(\overline{E})]/\sigma_{av}$ were determined, and corresponding deviations calculated taking into account the errors of E_0

$$u = \frac{\overline{N} - y\,(\overline{E})}{\sqrt{\sigma_{av}^2 + (\delta_{av}y')^2}} \ ,$$

where σ_{av} is the statistical error of the average number of pairs $\overline{N}$, $y\,(\overline{E})$ is the theoretically expected value of the average, and δ_{av} is the statistical error of the average value of ln E_0. Data given in Table III represent the probability that, in the present experiment, the values of u will not be smaller than the observed values.

The above statistical analysis of the results, carried out using three significant criteria, points to the incorrectness of curve 1. At the same time, a contradiction with curve 2 is not discovered. Thus, one can assume that at least a qualitative proof of the influence of the medium on bremsstrahlung has been obtained. For a quantitative test of the Migdal formulas, it is necessary to increase the statistical material. The experimental possibilities, together with the results of the calculations,[2] justify considering the above-described method of analyzing electron-photon showers as one of the most suitable ones for a further study of the effects of the medium on bremsstrahlung of high-energy electrons.

In conclusion, the authors express their gratitude to D. M. Samoĭlovich and her collaborators for developing the emulsion stacks, to I. A. Svetlolobov for help in reducing the calculated data, and to laboratory assistants A. A. Kondrashina and V. S. Balova for taking part in the scanning and analysis of the showers.

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Translated by H. Kasha
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