ON THE MASSES OF LEPTONS

Yu. A. GOL' FAND

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

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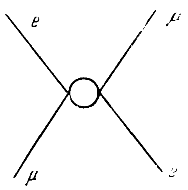
THERE is a far-reaching analogy between the properties of the electron and the muon; on the other hand, the masses of these particles differ by a factor of about two hundred. To explain this fact various authors[1,2] have proposed schemes based on the idea that the masses of "noninteracting" e and μ particles are the same, and that the difference of the observed masses arises as the result of interaction with some hypothetical particle.

In the present note we consider a different possibility.

The "anomalously" small value of the electron mass in comparison with the muon mass leads to the thought that the entire observed mass of the electron owes its origin to some interaction. The mass of the "noninteracting" electron, like the mass of the neutrino, is assumed to be zero. From experiment it is known to high accuracy that the electron and the muon enter symmetrically into all interactions. It is natural to assume that the hypothetical interaction that leads to the appearance of the mass of the electron also possesses this symmetry. The simplest possibility of this sort is the introduction of some anomalous μ-e interaction.

This reaction must be of the exchange type, i.e., of the type represented by the diagram. Furthermore we assume that the anomalous μ-e interaction conserves both the number of electrons



and the number of muons, and consequently does not lead to charge transfer processes of the type

$$\mu^+ + e^- \to \mu^- + e^+.$$

For rough qualitative calculations the interaction in question can be approximated by a contact interaction of the form

$$(G/m^2_\mu)\,(\bar{\psi}_\mu \hat{O} \psi_e)\,(\bar{\psi}_e \hat{O} \psi_\mu). \qquad (1)$$

Qualitatively we would get the same result at not too high energies (as long as the muon is nonrelativistic in the center-of-mass system; for μ-e collisions this condition is satisfied up to muon energies of several Bev in the laboratory system) for the case of an interaction (see diagram) that is the result of the exchange of a quantum of zero mass. Owing to the exchange character of the interaction the transfer of a quantum of zero mass leads to short-range forces. Of course, if such a (nonelectromagnetic) quantum exists, it must have specific properties that forbid its emission as an actual particle, as is the case for the longitudinal and scalar components of the electromagnetic field.

In order to get the observed mass of the electron $m_e \sim m_\mu/200$, we must take the reaction (1) to be sufficiently strong. In other words the coupling constant must be of the order of $e^2 = 1/137$. Therefore the interaction (1) could make an appreciable contribution to observable effects. We shall show that in most cases the correction caused by the interaction (1) is vanishingly small as compared with that from the electromagnetic interaction. For the scattering of negative muons by electrons the correction to the differential cross section in the center-of-mass system is given by the factor

$$1 + \frac{G}{e^2}\left(\frac{2m_e v}{m_\mu}\sin\frac{\theta}{2}\right)^2 \text{ — nonrelativistic electron,}$$

$$1 + \frac{G}{e^2}\left(\frac{2E}{m_\mu}\sin\frac{\theta}{2}\right)^2 \text{ — ultrarelativistic electron, nonrelativistic muon.}$$

Here v is the speed of the electron, and E its energy ($\hbar = c = 1$). Up to a muon energy of 2 Bev in the laboratory system the correction does not exceed a few percent, and it falls off very rapidly at smaller energies. The situation is similar for the scattering of positive muons by electrons.

An estimate of the contribution of the interaction (1) to the anomalous magnetic moment of the electron and the shift of energy levels leads to a quantity of the order of $(m_e/m_\mu)^2 = 0.25 \times 10^4$ of the main effect, i.e., a change that lies within the limits of experimental error. For the muon, however, the contribution to such quantities can be of the order of the effect itself. Obviously the importance of the anomalous μ-e interaction increases sharply at very high energies. For example, it can decidedly change the picture of the process $e^- + e^+ \to \mu^- + \mu^+$, which has been considered by Nikishov.[3] The writer thanks Ya. B. Zel'dovich for valuable discussions.

[1] J. Schwinger, Ann. Phys. **2**, 407 (1957).
[2] I. Saavedra, Nuclear Phys. **11**, 569 (1959).

[3] A. I. Nikishov, JETP **36**, 1323 (1959), Soviet Phys. JETP **9**, 937 (1959).

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ON THE INFLUENCE OF THE EXCHANGE INTERACTION ON THE TRANSITION TEMPERATURE OF SUPERCONDUCTORS

S. V. VONSOVSKIĬ and M. S. SVIRSKIĬ

Institute for Metal Physics, Academy of Sciences, U.S.S.R.

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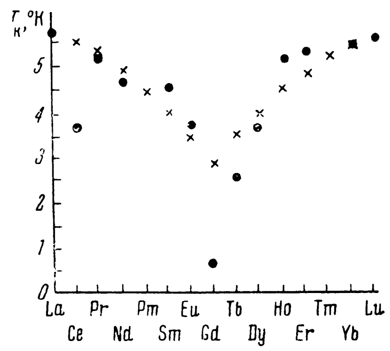
IT was shown in reference 1 that the polarization of the conduction electrons, caused by the s-d exchange interaction, prevents the establishment of a superconducting state in typical ferromagnets with high Curie points (for instance, Fe, Co, and Ni). At the same time it was shown that superconductivity could in principle occur in metals of the transition groups, if the s-d exchange were sufficiently weak. This may, apparently, occur in the rare earths where the exchange interaction between the conduction electrons and the electrons of the incomplete 4f shell is, generally speaking, weaker than the interaction in the transition metals of the iron group. However, even in the rare earth metals the effective repulsion between the conduction electrons[2] induced by the s-f exchange (which counteracts the attraction caused by the longitudinal phonons) leads to a lowering of the critical temperature T_c of the transition into the superconducting state, while this lowering must depend on the magnitude of the spin S_f of the 4f shell. Such a dependence $T_c(S_f)$ has, indeed, been found recently by Matthias et al.[3] in one-percent solid solutions of rare-earth elements in lanthanum (see figure).

To ascertain whether we can explain the experimentally observed dependence $T_c(S_f)$ by an effective repulsion caused by s-f exchange, we consider the well known expression for T_c:[4]

$$T_c = 1.14\,(\hbar\omega/k)\exp\{-1/N_0V\}. \qquad (1)$$

The matrix element V of reference 1 has now the form

$$V = V_{ph} - V_c - V_{sf} = V_0 - V_{sf}, \qquad (2)$$



Transition temperature for superconducting one percent (atomic) solid solutions of rare earth elements in La: ● – experimental points according to ref. 3, × – theoretical values evaluated using Eq. (2) of this paper.

where V_{ph}, V_c, and V_{sf} are respectively the matrix elements of the interelectronic interaction induced by the phonons, the quasi-Coulomb interaction, and the s-f exchange interaction.

We shall, moreover, take the estimates given in reference 2 for gadolinium:

$$V_{ph} = 4.2\cdot 10^{-12}N^{-1}\ \text{erg},$$

$$V_c = 1.1\cdot 10^{-12}N^{-1}\ \text{erg and } V_{sf} = 5.5\cdot 10^{-11}N^{-1}\ \text{erg}.$$

We note now that Gd and La have the same inner electron shell structure and the same crystal lattice structure. To estimate the magnitude of V_{ph} and V_c for La we can thus take the same values as for Gd.

Moreover, since $V_{sf} \equiv 0$ in pure La with a completely empty 4f shell, for a one-percent solution of Gd in La we must substitute in Eq. (2) the quantity $0.01 \times 5.5 \times 10^{-11}\,N^{-1}\,\text{erg} = 5.5 \times 10^{-13}\,N^{-1}$ erg instead of V_{sf} for 100% Gd. We find then from (1) and (2)

$$T_c = T_c^{(0)}\exp\{-0.22/N_0V_0\}, \qquad (3)$$

where $T_c^{(0)}$ is the critical temperature when there is no s-f interaction. The value of N_0V_0 was determined by Pines[5] for La to be 0.37 at $T_c^{(0)} = 5°$K. However, if we take into account that $T_c^{(0)} = 5.7°$K was obtained for La in the experiments of Matthias et al.[3] we obtain easily by the method indicated by Pines[5] the close value $N_0V_0 \approx 0.39$. Substituting now into (3) the values $T_c^{(0)} = 5.7°$K and $N_0V_0 = 0.39$, we get $T_c \approx 2.8°$K, whereas according to the figure the value of T_c for La with 1% Gd in solution is considerably smaller, $\approx 0.6°$K. This discrepancy shows that for the case of a one-percent solution of Gd in La the total lowering of T_c can apparently not be explained solely by the occurrence of an effective repulsion induced by the s-f exchange interaction.

To estimate in how far this repulsion is responsible for the decrease of T_c in solutions of rare earths other than Gd in La, we take for the one percent solution of Gd in La the value $T_c \approx 2.8°$K found above. Taking into account that according to