ANOMALOUS MAGNETIC MOMENTS OF NUCLEONS IN THE CHEW METHOD

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Corrected values of the anomalous magnetic moments of nucleons caused by virtual π-meson and nucleon currents are obtained by Chew's method. Contributions of strange particles and of the hypothetical ρ^0 meson are also taken into account, on various assumptions about their intrinsic parities.

1. INTRODUCTION

FROM the point of view of present quantum field theory, the main contribution to the anomalous magnetic moments μ_p and μ_n of nucleons must come from the virtual currents that arise on account of the coupling of the nucleon and π-meson fields — a coupling which is evidently the "strongest" of all known couplings. It can be expected that an appreciable contribution to μ_p and μ_n is also made by the strange particles, since the field of these particles is also coupled rather strongly with the nucleon field. In addition it is interesting to estimate the contribution to μ_p and μ_n that arises from the coupling of the nucleon field with the field of the hypothetical ρ^0 meson allowed by the Gell-Mann scheme;[1] there is lively discussion in the literature as to whether this particle exists.

Recently Gupta[2] has taken account of the contributions to μ_p and μ_n from both the strange particles and the ρ^0 meson, in the framework of ordinary perturbation theory. Owing to the fact, however, that he did not get agreement with experiment on any assumption about the intrinsic parities of the K and ρ^0 mesons, and also owing to the fact that perturbation theory is probably not applicable to these problems,[3] it is interesting to make analogous calculations by the more acceptable method of Chew.

It must be emphasized, however, that in the Chew method, based on the idea of a fixed nucleon source, the treatment we shall give of transitions from nucleon to hyperon and vice-versa is, strictly speaking, not consistent. Besides the fact that in this method recoil is neglected, in such a treatment we have to introduce a mixed nucleon-hyperon source, and the question at once arises as to its "size." Fortunately, the not very large difference of the masses and the equality of several other characteristics of nucleons and hyperons allows us to hope that also the size of such a mixed source will be close to the "size" of the nucleon source. In what follows we shall use the standard "cutoff" — $\omega_{max} = 5.6\, m_\pi$. We remark, however, that a change of this "cutoff" within reasonable limits makes only a slight change in our numerical results.

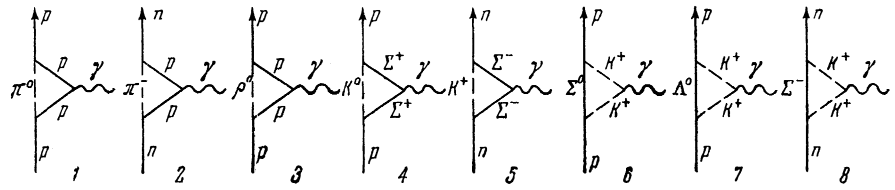
2. THE CONTRIBUTION OF THE π-MESON AND NUCLEON CURRENTS

In reference 4 calculations by the Chew method have been made on the magnetic moments of nucleons, with inclusion of only the meson current in the virtual state, and with the coupling constant $f^2 = 0.058$ the values found were* $\mu_p = 1.44$, $\mu_n = -1.44$.

In reference 5 the contribution of the virtual nucleon current has been taken into account, and an addition of -0.15 to the neutron magnetic moment was found. This treatment considered diagram 2 and its modifications in Chew's sense; these contribute only to the neutron magnetic moment (see diagram).

There is, however, one other diagram, namely diagram 1, that involves a nucleon current in the virtual state and gives a contribution to the magnetic moment of the proton. The contribution of diagram 1, corresponding to the emission of a π^0 meson, is smaller by a factor 2 than the contribution of diagram 2, which corresponds to the emission of a π^- meson. This follows from the fact that in the symmetrical theory the coupling of the charged meson field with the nucleon field contains an addition factor $2^{1/2}$. This same result can also easily be obtained formally, if we write

*Throughout this paper all values of magnetic moments are given in nuclear magnetons.



the Lagrangian for the interaction of the nucleon and electromagnetic fields (in the relativistic case) in the form

$$L = -ie : \bar{\Psi}\, {}^1/_2 (1 + \tau_3)\, \gamma_\nu \Psi A_\nu :. \qquad (1)$$

Here Ψ is the usual eight-component operator of the nucleon field, composed of the wave functions of proton and neutron, and $\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

Owing to this the value calculated in reference 5 for the contribution to μ_n caused by diagram 2 must be doubled. The contribution to μ_p caused by diagram 1 is -0.15.

Thus with the coupling constant indicated above the values of the anomalous magnetic moments of the nucleons turn out to be $\mu_p = 1.29$ and $\mu_n = -1.74$.

Recently, however, a more logical analysis of the experimental data on the basis of dispersion relations has led to a decided increase of the coupling constant of the nucleon and π-meson fields. At present we must evidently take as the most acceptable value of this constant $f^2 = 0.1$.[6] The use of this constant leads to the following values of the anomalous magnetic moments of the nucleons

$$\mu_p = 2{,}51, \qquad \mu_n = -3{,}46. \qquad (2)$$

3. THE CONTRIBUTION OF STRANGE PARTICLES AND THE ρ^0 MESON

Since the intrinsic parities of the K and ρ^0 mesons are as yet unknown, we have made calculations both for scalar and for pseudoscalar K and ρ^0 mesons. On this basis the Hamiltonian now generally accepted for the interaction of the K and ρ^0 meson fields with the baryon field[7] can be written in the Chew approximation in the form

$$H = H_{N\Sigma K} + H_{N\Lambda^0 K} + H_{NN\rho^0}, \quad H_{N\Sigma K} = \sqrt{4\pi} \times \int dr\, \rho_{N\Sigma}(r)\, \boldsymbol{\tau}\eta_K \varphi_K(r),$$

$$H_{N\Lambda^0 K} = \sqrt{4\pi} \int dr \rho_{N\Lambda^0}(r)\, \eta_K \varphi_K(r) + \text{Herm. adj.}$$

$$H_{NN\rho^0} = \sqrt{4\pi} \int dr \rho_{NN}(r)\, \eta_\rho \varphi_\rho(r), \qquad (3)$$

where $\eta_K = g'_K$ for scalar K mesons, $\eta_K = (f_K/m_K)\,\boldsymbol{\sigma}\nabla$ for pseudoscalar K mesons, and similarly with η_ρ. In Eq. (3) it is assumed that the relative intrinsic parities of the nucleons and hyperons are the same.

In the calculations we have confined ourselves to the first nonvanishing approximation of the Chew method, since, as our calculations show and as could of course be foreseen, the next approximation gives a much smaller contribution.

For the process with which we are concerned there are six diagrams, 3 – 8. Using the Hamiltonian (3), we easily get the following matrix elements: for diagram 3 in the case of scalar ρ^0 mesons

$$M_3 = -\frac{e}{2m} \sigma \mathbf{H} \frac{g_\rho'^2}{\pi} \int \frac{dk k^2 v^2(k)}{\omega_k^3}, \qquad (4)$$

for diagrams 4 – 8 in the case of scalar K mesons

$$M_4 = -\frac{e}{2m} \sigma \mathbf{H} \frac{2m}{M} \frac{g_K'^2}{\pi} \int \frac{dk k^2 v^2(k)}{\omega_k^3},$$

$$M_5 = -M_4, \quad M_6 = M_7 = M_8 = 0, \qquad (5)$$

for diagram 3 in the case of pseudoscalar ρ^0 mesons

$$M_3 = \frac{e}{2m} \sigma \mathbf{H} \frac{1}{3\pi} \frac{f_\rho^2}{m_\rho^2} \int \frac{dk k^4 v^2(k)}{\omega_k^3}, \qquad (6)$$

and for diagrams 4 – 8 in the case of pseudoscalar K mesons

$$M_4 = \frac{e}{2m} \sigma \mathbf{H} \frac{2}{3\pi} \frac{m}{M} \frac{f_K^2}{m_K^2} \int \frac{dk k^4 v^2(k)}{\omega_k^3}, \quad M_5 = -M_4,$$

$$M_6 = M_7 = -\frac{e}{2m} \sigma \mathbf{H} \frac{4}{3\pi} \frac{m}{m_K} \frac{f_K^2}{m_K} \int \frac{dk k^4 v^2(k)}{\omega_k^4}, \quad M_8 = 2M_6. \qquad (7)$$

On the basis of reference 8 the coupling constant of the K-meson and baryon fields is taken to be $g_K'^2 = 0.7$ for scalar K mesons and $g_K^2 = (2m/m_\pi)^2 \times f_K^2 = 2.6$ for pseudoscalar K mesons. To simplify the calculations we have neglected the difference of the hyperon masses, taking $M = 2300\, m_e$ for all the hyperons. Since at present there are no definite arguments regarding the value of the coupling constant of the ρ^0-meson and nucleon fields, we have taken for definiteness $g_\rho'^2 = 1$ for the scalar ρ^0 meson and $f_\rho^2 = 0.1$ for the pseudoscalar ρ^0 meson. As for the rest mass of the ρ^0 meson, we have considered two cases: $m_\rho = m_\pi$ and $m_\rho = 2m_\pi$.

With these assumptions we get the following values for the contributions to the magnetic moments of the nucleons from strange particles and ρ^0 mesons: from the hyperon and scalar K-meson currents +0.09 and −0.09; from the hyperon and pseudoscalar K-meson currents +0.002 and +0.003; from the coupling with the scalar ρ^0 meson +0.46 and 0 (for $m_\rho = m_\pi$) and +0.24 and 0 (for $m_\rho = 2m_\pi$); from the coupling with the pseudoscalar ρ^0 meson −0.17 and 0 (for $m_\rho = m_\pi$) and −0.02 and 0 (for $m_\rho = 2m_\pi$); the pairs of values are for μ_p and μ_n respectively.

4. DISCUSSION OF RESULTS

We note first of all that the so-called vector part of the magnetic moments of nucleons, caused by the couplings of the baryon field with the π and K meson fields, has been calculated recently by the method of dispersion relations.[3] The numerical results obtained in the present paper are in the main close to the results of reference 3, except for the case of the pseudoscalar K mesons, where the difference amounts to an order of magnitude. This last fact can be easily understood if we take note of the experimental fact that there is a large s-wave interaction between K mesons and nucleons (see reference 9, for example), which is automatically excluded for pseudoscalar K mesons in a calculation by the Chew method, which neglects the recoil effects. For scalar K mesons, on the other hand, this neglect evidently causes no large error.[10] An important point is that both the analysis on the basis of dispersion relations and the results we have obtained indicate that the strange particles make an extremely small contribution to the magnetic moments of nucleons.

The theoretical values of the anomalous magnetic moments of the nucleons given in Sec. 2 are in absolute value much larger than the experimental values, and, as has already been noted, inclusion of the contribution of strange particles cannot appreciably change this situation.

As for the ρ^0 mesons, our results show that only the pseudoscalar ρ^0 meson improves the theoretical value of μ_p (and only slightly, at least with the coupling constant we have chosen), but it leaves the value of μ_n unchanged.* In this respec our results are in contradiction with those of Gupta who is inclined to the idea of a scalar character foı the ρ^0 meson.

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*We note that the hypothesis that there exists a parity doublet of π mesons,[11] which provides a satisfactory explanation of the experiments on N–N scattering in the energy range 100–600 Mev, leads to additional contributions of +0.46 and +0.92 to μ_p and μ_n, respectively. This improves the value for μ_n, but makes the value for μ_p somewhat worse.