TRANSITION PROBABILITIES BETWEEN ROTATIONAL STATES OF NON-AXIAL ODD NUCLEI

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DAVYDOV[1] discussed the rotational states of nuclei which, in the ground state, consist of a single outer nucleon in the state $j = 1/2$ and a spinless core having the shape of a three-axial ellipsoid. The energies of the rotational states were determined from the Schrödinger equation with the Hamiltonian

$$H_r = \hbar^2 (8B\beta^2)^{-1} \sum_\lambda (\hat{I}_\lambda - \hat{j}_\lambda)^2 / \sin^2\left(\lambda - \frac{2\pi}{3}\lambda\right).$$

The wave functions of the stationary states have the form

$$\Psi_{IM\tau} = \sum_{K\Omega} | IjK\Omega > A^\tau_{K\Omega}, \qquad (1)$$

where

$$| IjK\Omega > = \left(\frac{2I+1}{16\pi^2}\right)^{1/2} \{\varphi_\Omega D^I_{MK} + (-1)^{I-j} \varphi_{-\Omega} D^I_{M,-K}\}. \qquad (2)$$

We have calculated the coefficients $A^\tau_{K\Omega}$ for the states with $I = 3/2$ and $5/2$.

The reduced transition probability for quadrupole radiation is given by the formula[2]

$$B(E2;\ I\tau \to I'\tau') = (5/16\pi) \sum_{M'\mu} | (I'M'\tau' | \hat{Q}_{2\mu} | IM\tau) |^2,$$

where

$$\hat{Q}_{2\mu} = eQ_0 [D^2_{\mu 0} \cos\gamma + 2^{-1/2} (D^2_{\mu 2} + D^2_{\mu,-2}) \sin\gamma],$$

$$Q_0 = 3ZR^2\beta / \sqrt{5\pi}.$$

By this method we computed the reduced probabilities for electric quadrupole transitions between various levels. The results of the calculations are given in Fig. 1.

To determine the magnetic transition probabilities, we start from the expression for the magnetic moment operator (in first approximation, see references 3 and 4),

$$\mathfrak{M}(1\mu) = (-1)^\mu \sqrt{3/4\pi}\, \mu_0 (g_j - g_R) \hat{j}_\mu,$$

$$\hat{j}_\mu = \sum_k D^1_{\mu k} \hat{j}_k, \qquad k = 1, 0, -1 \qquad (3)$$

where $\mu_0 = e\hbar/2Mc$ is the nuclear magneton, g_R the gyromagnetic ratio corresponding to the collective motion of the nucleons in the nucleus, and g_j is the gyromagnetic ratio for the outer nucleon.

The effect of the operator $\hat{j}_k$ on the wave function φ is determined by the relation[5]

$$\hat{j}_k \varphi_{jm} = (-1)^{k+1} \sqrt{j(j+1)}\, (j1, m+k, -k | jm)\, \varphi_{j, m+k}. \qquad (4)$$

With the help of (3) and (4) we can compute the reduced probabilities for a magnetic dipole transition:

$$B(M1;\ I\tau \to I'\tau') = \sum_{M'\mu} | (I'M'\tau' | \mathfrak{M}(1\mu) | IM\tau) |^2. \qquad (5)$$

In this way it can be shown that

$$B(M1;\ {}^3/_2\tau \to {}^1/_2) = 0, \qquad B(M1;\ {}^3/_2 2 \to {}^3/_2 1) = 0. \qquad (6)$$

The numerical values of the reduced probabilities of the other transitions are given in Fig. 2.

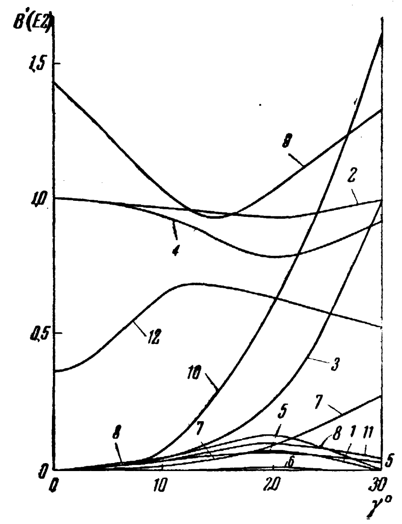

FIG. 1. Dependence of $B'(E2) \equiv B(E2)/(e^2Q_0^2/16\pi)$ on γ for 1 – ($^3/_2 2 \to {}^1/_2$), 2 – ($^3/_2 1 \to {}^1/_2$), 3 – ($^3/_2 2 \to {}^3/_2 1$), 4 – ($^5/_2 1 \to {}^1/_2$), 5 – ($^5/_2 2 \to {}^1/_2$), 6 – ($^5/_2 3 \to {}^1/_2$), 7 – ($^5/_2 2 \to {}^3/_2 1$), 8 – ($^5/_2 3 \to {}^3/_2 1$), 9 – ($^5/_2 3 \to {}^3/_2 2$), 10 – ($^5/_2 2 \to {}^5/_2 1$), 11 – ($^5/_2 3 \to {}^5/_2 1$), 12 – ($^5/_2 3 \to {}^5/_2 2$)

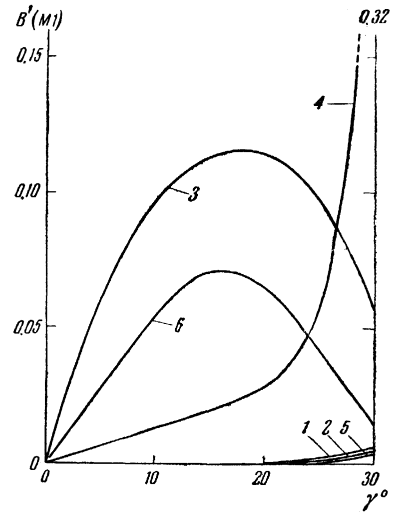

FIG. 2. Dependence of $B'(M1) \equiv B(M1)/(3/20\pi)\mu_0^2(g_j - g_R)^2$ on γ for 1 – ($^5/_2 2 \to {}^3/_2 1$), 2 – ($^5/_2 3 \to {}^3/_2 1$), 3 – ($^5/_2 3 \to {}^3/_2 2$), 4 – ($^5/_2 2 \to {}^5/_2 1$), 5 – ($^5/_2 3 \to {}^5/_2 1$), 6 – ($^5/_2 3 \to {}^5/_2 2$)

On the basis of our calculations we arrive at the following conclusions:

1. The transitions $(\frac{3}{2}1 \rightarrow \frac{1}{2})$ and $(\frac{5}{2}1 \rightarrow \frac{1}{2})$ [besides the transition $(\frac{5}{2}3 \rightarrow \frac{3}{2}2)$] have the greatest reduced probability among all the abovementioned E2 transitions. This is in agreement with the experimental results on Coulomb excitation (for W^{183}, see reference 6). If the nuclear shape deviates strongly from the axially symmetric one, the transitions $(\frac{3}{2}2 \rightarrow \frac{3}{2}1)$ and $(\frac{5}{2}2 \rightarrow \frac{5}{2}1)$ also become pronounced.

2. If the nucleus deviates little from the axially symmetric shape, all M1 transitions between the rotational levels of the nucleus have small reduced probabilities. If, however, the nuclear shape is far from being axially symmetric, the transitions $(\frac{5}{2}2 \rightarrow \frac{5}{2}1)$ and $(\frac{5}{2}3 \rightarrow \frac{3}{2}2)$ become important.

3. It follows from (5) and (6) that all magnetic transitions to the ground state should be very weak as compared to the electric transitions.

4. According to Davydov,[1] we have $\gamma \approx 27°$ for the nucleus W^{183}. From Fig. 1 we then obtain for the ratio of the reduced transition probabilities for the transitions $(\frac{1}{2} \rightarrow \frac{5}{2}2)$ and $(\frac{1}{2} \rightarrow \frac{3}{2}2)$ the value $(6/4)(4/2.3) = 2.6$. This is in good agreement with the estimate based on the experimental results of Alder et al.[6]

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ALPHA DECAY OF Th^{229}*. INTERACTION OF NUCLEAR LEVELS*

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THE α decay of Th^{229} has not been investigated heretofore. Nor is anything known concerning the structure of the levels of the daughter nucleus Ra^{225}.

We have investigated the α decay of Th^{229} with a magnetic α spectrometer;[1] the spectra of conversion electrons of Ra^{225} accompanying the α decay of Th^{229} were investigated with a high-aperture toroidal β spectrometer[2] and an α-β coincidence circuit. The measurements were carried out with the isotope Th^{229}, obtained by chemical separation of thorium from U^{233} that was aged for a long time.

The investigation of the α spectrum of Th^{229} disclosed 12 α lines. The energies of the α lines and the intensities of the corresponding transitions are listed in the table, which contains also the hindrance coefficients (ratio of the transition intensity, calculated from the formula for even-even nuclei, to the observed intensity).

In the investigation of the spectrum of the conversion electrons we observed approximately 100 conversion lines that could not yet be fully interpreted. We separated reliably the γ transitions with energies 17.2, 42.8, 69.9, 75.5, 137.2, 156.6, 193.4, and 210.5 kev. The 29.1, 31.6, 56.8, 58.9, 85.0, 132.1, 154.4, 179.6, 242.0 kev and a few other transitions are less reliable.

A comparison of the data obtained with both instruments suggests the existence of a Ra^{225} level with excitation energy ~ 3 kev. Whether this level becomes populated in α decay of Th^{229} and at what probability is still unknown. Apparently the γ transitions from high levels occur mainly at this level rather than at the ground level of Ra^{225}.

The α decay to the ground state of Ra^{225} appears to be strongly forbidden ($\eta = 330$). The more likely transition is that to the 214.5-kev level ($\eta = 1.5$). The spin of this level should therefore equal the spin of the ground state of Th^{229}, i.e., $\frac{5}{2}$. Located above this level are several others, some of which have small η. The Ra^{225} nucleus lies in a region sufficiently far from closed shells. The investigated nuclei in this region are prolate. It is natural to assume that