of the orbit. However, in view of the fact that only the projections of the spin on the direction of the momentum are integrals of the motion (± 1), it would be more consistent to avoid the term "precession" and to speak of the transition time between these states or of the transition probability per unit time.

With regard to the calculation of $W_{1,-1}$, the following should be noted. It is impossible to expand the Green's function in powers of the potential, since the potential of the uniform field is not a perturbation. Indeed, the vector potential depends on a coordinate which can become very large in the relativistic case (for example, in the relativistic case, $\langle e^2\hat{A}^2\rangle \sim e^2 \times H^2 \langle r^2\rangle \sim E^2$). Similarly, with any other method of expansion, one must guard against the appearance in the neglected terms of expressions which depend on coordinates which after integration could lead to large values. In our case the expansion in terms of H/H_0 ($H_0 = m^2c^3/e\hbar \sim 10^{13}$ oe) was introduced in the last phase of the calculations, after the integration over space and the summation over the virtual states. As a result we obtained in first approximation in H/H_0 the following value for $W_{1,-1}$, which is valid both in the relativistic and nonrelativistic regions:

$$W_{1,-1} = -(\alpha/2\pi)\mu H. \tag{5}$$

This result could have been derived from the operator $(\alpha/2\pi)(\sigma H)\mu$, but the use of this operator in the relativistic region would, according to the considerations above, require a special justification.

The time for the spin-flip caused by the interaction of the electron with the photon vacuum is, therefore, equal to $\pi/2\delta = 2\pi^2 mc/\alpha eH$. The ratio of this over the period of rotation of the electron is equal to $(\pi/\alpha)mc^2/E \sim 450\ mc^2/E$. The last quantity decreases as the energy becomes larger, and reaches the value 1 at energies of ~ 200 Mev.

The authors thank Prof. A. A. Sokolov for a discussion of this work.

[1] H. Mendlowitz and K. M. Case, Phys. Rev. **97**, 33 (1955).

[2] Louisell, Pidd, and Crane, Phys. Rev. **94**, 7 (1954).

[3] N. N. Bogolyubov and D. V. Shirkov, Введение в теорию квантованных полей (Introduction to the Theory of Quantized Fields), Gostekhizdat, Sec. 38, 1957.

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222

MINIMAL ERROR IN THE EXPERIMENTAL OBSERVATION OF ASYMMETRY

N. P. KLEPIKOV

Joint Institute for Nuclear Research; Ain Shams University, Cairo, UAR

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Many experiments with elementary particles, aimed at proving or disproving the conservation of spatial parity or proving the existence of spin in a particle, reduce to the observation of a definite asymmetry in the distribution of the particles, produced in a certain reaction. It becomes useful to estimate the probability of the error committed when conclusions concerning the presence or absence of asymmetry are drawn from such an experiment.

In the observation of asymmetry, all particles are separated (during the course of the experiment or during the data reduction) into two groups, such that in the absence of asymmetry of the observed process a particle can belong to either group with equal probability. Usually the probability of registration of each particle in one of the groups is independent of the number of particles already accumulated in these groups. Therefore, if the data are corrected for possible systematic errors, the number of particles in the two groups, n_+ and n_-, have Poisson distributions with mean values $\frac{1}{2}n(1 \pm F)$, where $n = n_+ + n_-$, and F is a constant that characterizes the force of the interaction that leads to violation of the symmetry.

It can be shown that for $n_+ > n_- \gg 1$ the relation

$$t = (n_+ - n_- - 1)/\sqrt{n} \tag{1}$$

has a Student's t-distribution with f degrees of freedom, where

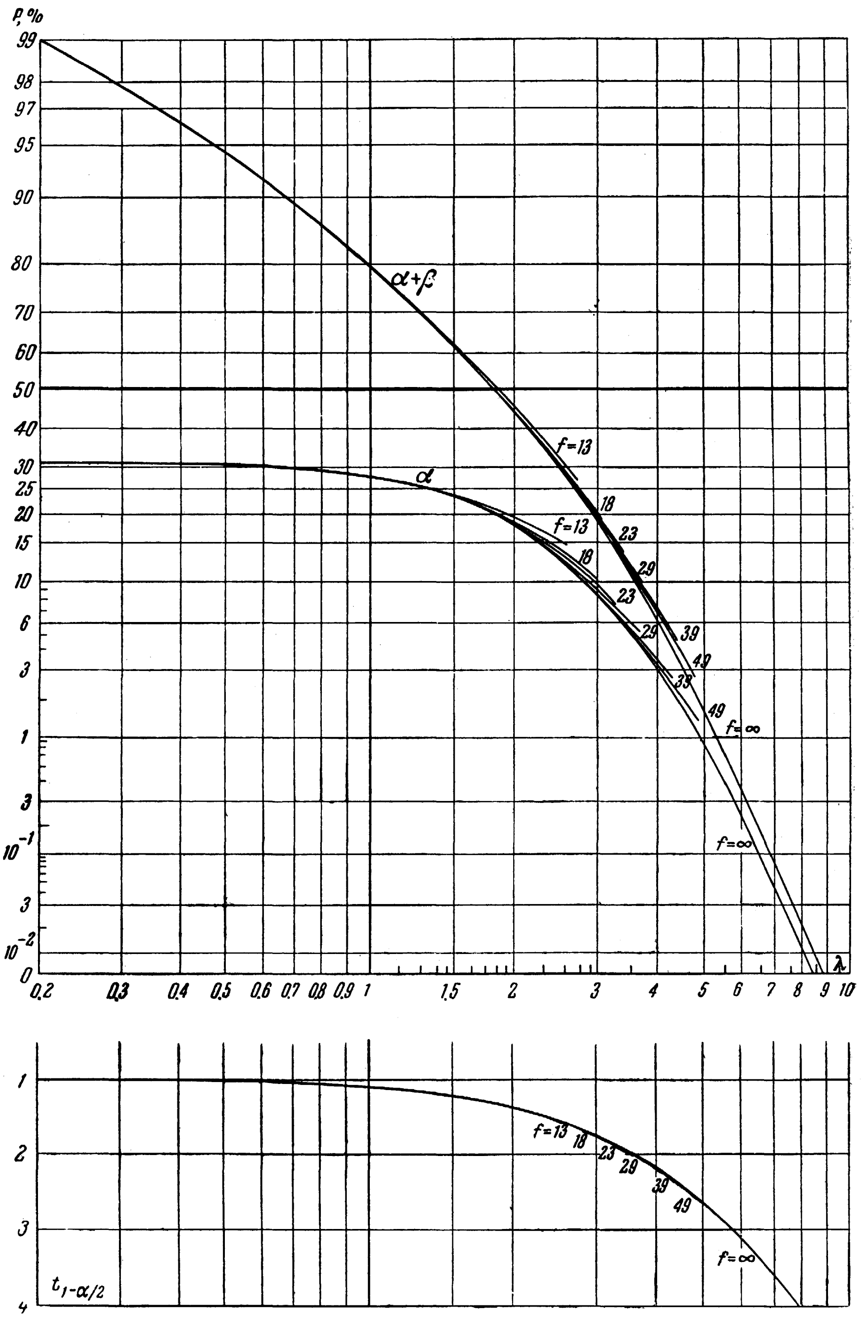
$$\frac{1}{f} = \left(\frac{n_+}{n}\right)^2 \frac{1}{n_+ - 1} + \left(\frac{n_-}{n}\right)^2 \frac{1}{n_- - 1} \approx \frac{1 + 2(n_+ - n_-)^2 n^{-3}}{n-2}. \tag{2}$$

When $n \gg 1$ the value of t tends to $F\sqrt{n}$.

When $F = 0$, relation (1) satisfies, with probability $1 - \alpha$, the inequality

$$t < t_{1-\alpha/2}(f), \tag{3}$$

where $t_P(f)$ is the number that has the probability P of satisfying the inequality $t < t_P$. If the value obtained for t does not satisfy inequality



(3), the deviation of F from 0 is conceded to be significant and the hypothesis that the asymmetry is random is rejected.

The question arises, however, of the choice of the level of significance of α, equal to the probability of first-order error, when the random deviation of t from zero is assumed to be the deviation due to the fact that $F \neq 0$. To solve this problem it is necessary to take into account the probability of an error β of the second kind, when $F \neq 0$ but the small value of t is accepted as an indication that $F = 0$. The value of β is given in special tables.[1] When $f \gg 1$

$$\beta = 1 - \Phi\left[u_{\alpha/2} + \lambda\left(1 - \frac{1 + u^2_{\alpha/2}}{4f}\right)\right] - \Phi\left[u_{\alpha/2} - \lambda\left(1 - \frac{1 + u^2_{\alpha/2}}{4f}\right)\right], \qquad (4)$$

where

$$\lambda = F\sqrt{n}, \qquad \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left\{-\frac{1}{2}u^2\right\} du.$$

We now choose α such that the sum of probabilities of errors of both kinds, $\alpha + \beta$, is a minimum. It can be readily verified that, for the approximation (4),

$$(\alpha+\beta)_{\min}=1-\Phi\left[u_{\alpha/2}+\lambda\left(1-\frac{1+u_{\alpha/2}^2}{4f}\right)\right]$$
$$-\Phi\left[u_{\alpha/2}-\lambda\left(1-\frac{1+u_{\alpha/2}^2}{4f}\right)\right]+2\Phi(u_{\alpha/2}), \quad (5)$$

where

$$u_{\alpha/2}=u+\frac{1}{4f}\left[u^3-u+\lambda(1+u^2)(1-e^{-\lambda^2})^{-1/2}\right],$$
$$u=-\frac{1}{\lambda}\cosh^{-1}e^{1/2\lambda^2}. \quad (6)$$

The last term in (5) equals the optimum value of α.

The upper half of the diagram shows the dependence of α and of $(\alpha+\beta)_{\min}$ on $\lambda = F\sqrt{n}$ and f, while the lower half shows the corresponding values of $t_{1-\alpha/2} = -t_{\alpha/2}$. With the aid of these curves we can determine the minimal value of the probability of first and second kind errors, provided n_+ and n_- are known. This probability is found to be a function of that value of F, which the experimenter undertakes to distinguish from the value F = 0. To the contrary, if a certain value of F is specified along with an upper limit of probable error, it is possible to find the number of observations $n = (\lambda/F)^2$ necessary to establish a deviation of F from 0.

Example: At n = 100 the value F = 0.1 is considered to be present when t > 1.098 and absent when t < 1.098, and the probability of error is 82%; at n = 6400, a value F = 0.1 is rejected when t < 4.087 and the probability of error is 0.018%. Another example: in order to clarify whether an asymmetrical interaction with intensity F = 0.01 exists, and in order to insure that the probability of the erroneous decision is less than 1%, it is necessary to carry out $n = (5.30/0.01)^2 = 280{,}000$ observations. Third example: an experiment yielded $n_+ = 5080$ and $n_- = 4920$; we then obtain t = 1.59 and f = 9998 $\gg 1$, from which we conclude that the values $F > 0.026$ are rejected, and the probability of error in stating the presence of F = 0.02 is 45%, that for the presence of F = 0.002 is 99.0%, and for the absence of F = 0.05 is 1.5%. If, on the other hand, $n_+ = 5200$ and $n_- = 4800$, then t = 3.99 and f = 9998; the values F > 0.078 are rejected, and the probable error in assuming that F = 0.07 is present, is 0.08%, while that of confirming the presence of F = 0.01 is 80%.

The author is grateful to R. M. Ryndin who called his attention to the usefulness of solving this problem.

[1] G. J. Resnikov and G. J. Lieberman, Tables of the Non-central t-distribution, Stanford, 1957.

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223

ENERGY LOST TO RADIATION IN A GAS-DISCHARGE PLASMA

V. D. KIRILLOV

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In all known experiments on the heating of a hydrogen plasma by Joule heat, only a small fraction of this heat serves to raise the plasma temperature.[1] It can be assumed that the energy is either carried away by the heated particles or is radiated. The present investigation was undertaken to clarify this problem.

The measurements were made with a cylindrical porcelain gas-discharge chamber (length L = 70 cm, diameter 22 cm) terminated on each end by copper electrodes 4 cm in diameter. The apparatus was evacuated to 10^{-5} mm mercury. The experiments were carried out at discharge currents of amplitude J_{max} = 13 to 45 kiloamp and half-period approximately 500 μsec. The initial deuterium pressures were 0.01—0.02 mm mercury and the intensity of the longitudinal magnetic field was H = 0 — 24,000 oe.

Under conditions satisfying the Shafranov stability criterion, we observed a plasma column with diameter $a \sim 6$ cm along the axis of the chamber.[2] We first describe briefly the probe measurements with the ionization chamber,* which have led us to attribute an important role to the radiation losses.

To count the charged particles that reached the wall of the discharge chamber, we used an instrument (Fig. 1) that combined an ordinary plane double probe (with electrodes A and B) and an ionization chamber B. From 20 to 70 volts were applied to the electrodes of the probe. The current in the probe circuit, a measure of the plasma den-