ment which is an order of magnitude larger than the experimental value. As has been shown earlier[5] the present model of Li^7 leads to a good agreement also for the magnetic moment ($\mu_{\text{theoret}} = 3.56$; $\mu_{\exp} = 3.25$).

We finally point out that the value for the distance between the α particle and the triton (2.8×10^{-13} cm) is larger than the particle size, $\sim 1.5 \times 10^{-13}$ cm. This indicates that the employed model is not self-contradictory.

In conclusion we express our gratitude to I. Sh. Vashakidze and G. A. Chilashvili for discussions.

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ON THE THEORY OF THE NUCLEAR MOMENT OF INERTIA

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Submitted to JETP editor July 11, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **37**, 1136-1137 (October, 1959)

THE aim of the present note is to establish a connection between the two ways of determining the nuclear moments of inertia which have been proposed by Inglis[1] and Bohr and Mottelson[2] on the one hand and by Villars[3] and Hayakawa and Marumori[4] on the other hand. To begin with we have to consider the formulation of the second of these approaches. Further, we are not interested in the original abstract formulation but in the one to which we must turn when actually performing a computation.

Let φ be the collective angular variable, given by the angle of rotation of the main axes of the nucleus in a plane perpendicular to the axis of rotation, Z:*

$$\varphi = {}^1/_2 \tan^{-1}\left[\sum 2mxy \Big/ \sum m(x^2 - y^2)\right]. \tag{1}$$

The summation in (1) is over all nucleons; the indices showing the nucleon number have been omitted; m is the nucleon mass.

We note the important commutation relation: $i[M_Z, \varphi] = \hbar$ where M_Z is the projection of the angular momentum operator of the nucleus on the Z axis.

Let H_0 be a model Hamiltonian of the nucleus oriented in a given manner in the XY plane. The kinetic energy operator of such a Hamiltonian commutes with M_Z while the potential energy operator does not. We now define the quantities N_Z and I_0 by means of the relations

$$i\hbar^{-1}[H_0, \varphi] = -N_Z/I_0, \qquad -\hbar^{-2}[[H_0, \varphi], \varphi] = 1/I_0. \tag{2}$$

The quantity $L_Z = M_Z + N_Z$ is the projection of the angular momentum on the Z axis in a coordinate system fixed with respect to the nuclear axes. It commutes both with φ and M_Z, while $i[N_Z, \varphi] = -\hbar$. The quantity I_0 is the so-called hydrodynamic moment of inertia. It is a continuous function of the coordinates and commutes with φ as well as with M_Z and N_Z. As a simplification we shall take I_0 to be a c-number, but as one can easily convince oneself the final result does not depend on this assumption.

According to the references 3 and 4 the nuclear moment of inertia is roughly given by

$$I = I_0 + 2\sum_{n\neq 0} |\langle\Phi_n, L_Z\Phi_0\rangle|^2/(E_n - E_0). \tag{3}$$

Here Φ_n and E_n are the eigenfunctions and eigenvalues of the Hamiltonian H_0 respectively.

On the other hand, according to references 1 and 2 the moment of inertia is given by

$$I = 2\sum_{n\neq 0} |\langle\Phi_n, M_Z\Phi_0\rangle|^2/(E_n - E_0). \tag{4}$$

We now compare these two expressions. First we note that in deriving (3) it is implicitly assumed that in a deformed nucleus the orientation of the main axes cannot deviate appreciably from the orientation of the self-consistent field. This implies in particular that the first of the relations (2) can be replaced by

$$i\hbar^{-1}(E_n - E_0)\langle\Phi_n, \varphi\Phi_0\rangle = -I_0^{-1}\langle\Phi_n, N_Z\Phi_0\rangle. \tag{5}$$

From (5) and from the commutation relations one can easily derive the expressions

$$2\sum_{n\neq 0}\frac{\langle\Phi_0, M_Z\Phi_n\rangle\langle\Phi_n, N_Z\Phi_0\rangle}{E_n - E_0} = -I_0, \quad 2\sum_{n\neq 0}\frac{|\langle\Phi_n, N_Z\Phi_0\rangle|^2}{E_n - E_0} = I_0. \tag{6}$$

Exchanging L_Z by $M_Z + N_Z$ in (3) and applying (6) we immediately obtain (4).

Thus the expressions (3) and (4) are equivalent.

*Here and in the following we speak about a rotation of the nucleus around a fixed axis. Consideration of the rotation around a free axis will only introduce complications in the intermediate expressions and will not lead to any essential changes in the final results.

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EFFECT OF VACUUM FLUCTUATIONS ON THE POLARIZATION OF ELECTRONS MOVING IN A MAGNETIC FIELD

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Submitted to JETP editor June 12, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **37**, 1137-1139 (October, 1959)

OWING to the anomalous magnetic moment, the spin of an electron moving in a uniform magnetic field does not preserve its orientation along or opposite to the direction of motion, but precesses about the direction of the momentum. The quasi-classical interpretation of this effect was given by Mendlowitz and Case,[1] who obtained the equations of motion for the spin operator in the Heisenberg representation, from where they derived the precession. The corresponding experimental investigations have also been carried out.[2]

It seems useful to give a consistent quantum mechanical description of this effect which is valid for electrons with arbitrary energy. We start from the Dirac equation with radiative corrections taking into account the effects of the photon vacuum (see, for example, reference 3), which, in first approximation, has the form

$$(i\hat{\partial} + e\hat{A} - m)\psi(x) = -ie^2\int\gamma^\nu S^c(x, x')\gamma_\nu D^c(x - x')\psi(x')\,d^4x', \tag{1}$$

where D^c is the causal photon function, and S^c is the causal Green's function of the electron, expressed in terms of the exact solutions of the Dirac equation for an electron moving in a magnetic field. The time integration transforms (1) into

$$(E - \mathcal{H})\psi(\mathbf{r}) = \int K(\mathbf{r}, \mathbf{r}')\psi(\mathbf{r}')\,d\mathbf{r}', \tag{2}$$

where $\mathcal{H}$ is the Hamiltonian of the Dirac equation.

Each energy level is doubly degenerate with respect to the quantum number $s = \pm 1$ characterizing the projection of the spin on the direction of the momentum. The right hand side of (2) is a constant perturbation and causes periodic transitions between these states. Writing ψ as a superposition of ψ_1 and ψ_{-1}, multiplying (2) by ψ_s^+ and integrating over $\mathbf{r}$, we obtain a system of equations which determines the two energy values and the coefficients of the expansion. We introduce a time dependent wave function which satisfies the initial condition $\Psi(0) = \psi_1$, and find the following expression for the average value of the projection of the spin on the direction of the momentum:

$$\left\langle\frac{\boldsymbol{\sigma}\mathbf{k}}{k}\right\rangle = \int\Psi^+(t)\frac{\boldsymbol{\sigma}\mathbf{k}}{k}\Psi(t)\,d\mathbf{r} = \cos^2\delta t + \frac{1}{4}A^{-2}[(W_{1,1} - W_{-1,-1})^2 - 4W_{-1,1}^2]\sin^2\delta t, \tag{3}$$

where

$$A = {}^1/_2[(W_{1,1} - W_{-1,-1})^2 + 4W_{1,-1}W_{-1,1}]^{1/2}, \qquad \delta = A/\hbar,$$

$$W_{ss'} = \int\psi_s^+(\mathbf{r})K(\mathbf{r}, \mathbf{r}')\psi_s(\mathbf{r}')\,d\mathbf{r}\,d\mathbf{r}'.$$

Expression (3) has no divergencies connected with the mass of the field. Only for the calculation of the energy of the interaction with the vacuum it becomes necessary to introduce the corresponding compensating term in (1). If the electron moves in the direction of the field, we find $W_{s,-s} = 0$ and $\langle\sigma\mathbf{k}/k\rangle = 1$, i.e., the spin of the electron preserves its initial orientation. In the other limiting case — motion in the plane perpendicular to the direction of the field — we have $W_{1,1} = W_{-1,-1}$ and (3) takes the form

$$\langle\sigma\mathbf{k}/k\rangle = \cos 2\delta t, \qquad \delta = |W_{1,-1}|/\hbar. \tag{4}$$

Also $\langle\sigma_z\rangle = 0$. These relations can be interpreted as the precession of the spin in the plane