EMISSION OF NEUTRINO PAIRS BY ELECTRONS AND THE ROLE PLAYED BY IT IN STARS

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Bremsstrahlung emission of neutrino pairs by a nondegenerate gas is investigated. In a certain range of high densities and temperatures the energy loss by bremsstrahlung emission of neutrino pairs becomes greater than the loss due to radiative thermal conductivity. The inclusion of the energy loss due to neutrino pair emission may turn out to be significant, and in some cases even of decisive importance, for the theory of white dwarfs and stellar evolution, particularly for the dynamics of supernova explosions. The process under consideration leads to even greater energy losses than the process of neutrino pair formation in reaction (1) proposed by Gamow and Schoenberg in 1941.

1. INTRODUCTION

FEYNMAN and Gell-Mann[1] have assumed that all weak interactions arise as a result of the current

$$I_\mu = (\bar{\psi}_p \gamma_\mu a \psi_n) + (\bar{\psi}_\nu \gamma_\mu a \psi_e) + (\bar{\psi}_\nu \gamma_\mu a \psi_\mu) + \dots$$

interacting with itself. Their theory has been brilliantly confirmed in β decay and in the decay of the μ meson, i.e., for the interactions $(\bar{p}n)(\bar{e}\nu)$ and $(\bar{\nu}\mu)(\bar{e}\nu)$. The theory yields the possibility of new direct interactions, in particular $(\bar{e}\nu)(\bar{\nu}e)$. Such an interaction results first of all in direct neutrino-electron scattering.[1]

B. M. Pontecorvo has drawn attention to the fact that in the case of this interaction there also exists the possibility of formation of neutrino pairs in the collision of electrons with nuclei. He noted that the bremsstrahlung "emission" of neutrino pairs might serve as an additional mechanism for energy loss from stars.[2] The probability of creation of neutrino pairs is considerably smaller than the probability of bremsstrahlung emission of photons. However, a neutrino that interacts weakly with matter has a mean free path which exceeds by a large factor the dimensions of the star, while under the same conditions a photon has a very much smaller mean free path. Because of this the losses of energy from stars as a result of the creation of neutrino pairs may, within a certain range of densities and temperatures, turn out to be comparable to, or even greater than, the losses of energy by photon emission. At large densities ρ and for large Z the creation of neutrino pairs becomes more probable, and the photon mean free path becomes smaller.

As early as 1941 Gamow and Schoenberg[3] have indicated a mechanism for the loss of energy from stars by neutrino emission

$$_ZN^A + e^- = {}_{Z-1}N^A + \nu, \qquad {}_{Z-1}N^A = {}_ZN^A + e^- + \bar{\nu}. \quad (1)$$

This process is possible when the temperature and the density in the interior of the star are both large. However, its intensity depends on the presence of nuclei with a low threshold.

In this paper we calculate the cross section for the creation of neutrino pairs in the collision of electrons with nuclei and the effective slowing down; in view of the intended astrophysical application the calculation is carried out in the nonrelativistic case.

2. CROSS SECTION FOR THE CREATION OF NEUTRINO PAIRS. EFFECTIVE SLOWING DOWN

For the evaluation of the matrix element we make use of the universal four-fermion interaction

$$\sqrt{8}\, G \left[\bar{e}\, \tfrac{1}{2} \gamma_\mu (1 - i\gamma_5) \nu\right] \left[\bar{\nu}\, \tfrac{1}{2} \gamma_\mu (1 - i\gamma_5) e\right].$$

Two Feynman diagrams for the process under consideration are shown in Fig. 1.

The matrix element for the creation of a neutrino pair in the collision of an electron with a nucleus has the form

$$M = \sqrt{8}\, eG \left\{ \left[\bar{\nu}_{\mathbf{n}} \frac{1}{2} \gamma_\mu (1 - i\gamma_5) \frac{\hat{a}_q}{\hat{p} - m} e_i\right] \left[\bar{e}_f \frac{1}{2} \gamma_\mu (1 - i\gamma_5) \nu_{\mathrm{a}}\right] + \left[\bar{e}_f \frac{\hat{a}_q}{\hat{p}' - m} \frac{1}{2} \gamma_\mu (1 - i\gamma_5) \nu_{\mathrm{a}}\right] \left[\bar{\nu}_{\mathbf{n}} \frac{1}{2} \gamma_\mu (1 - i\gamma_5) e_i\right] \right\}. \quad (2)$$

Here $p_1(p_{10}, \mathbf{p}_1)$ and $p_2(p_{20}, \mathbf{p}_2)$ are the four-momenta of the electron in the initial and final

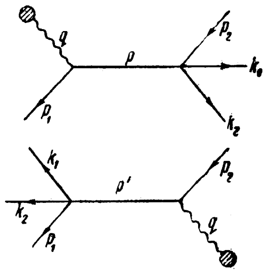

FIG. 1

stages respectively; $k_1\,(k_{10}, \mathbf{k}_1)$ and $k_2\,(k_{20}, \mathbf{k}_2)$ are the four-momenta of the neutrino and the antineutrino respectively; the subscript n denotes a neutrino, while the subscript a denotes an antineutrino;

$$a_q = \int A e^{-iqx} d^4x = 2\pi i a_q^0 \delta(E_1 - E_2 - k_{10} - k_{20}),$$

$$a_q^0 = \int A_0 e^{-i\mathbf{q}\mathbf{r}}\, d\mathbf{r}; \tag{3}$$

and q is the four-momentum transferred to the nucleus. The nucleus is taken to be at rest and the effect due to it is regarded as the action of an external static field, so that $q_0 = 0$. For the case of the Coulomb field of a nucleus of charge Ze we have:

$$a_q^0 = Ze / q^2. \tag{4}$$

From the conservation laws we have

$$p = p_2 + k_1 + k_2, \qquad p' = p_1 - k_1 - k_2,$$

$$q = p_2 + k_1 + k_2 - p_1, \qquad q = p - p_1 = p_2 - p'.$$

We substitute (3) and (4) into (2). In order to simplify the calculation we reduce the matrix element to the form $(\bar{e}O_1 e)(\bar{\nu}O_2\nu)$ (the Fierz transformation). We then obtain

$$M = 2\pi\sqrt{8}\,\frac{Ze^2}{q^2}\,G\left\{\bar{e}_f\left[\frac{1}{2}\gamma_\mu(1 - i\gamma_5)\frac{1}{\hat{p} - m}\gamma_0 + \gamma_0\frac{1}{\hat{p}' - m}\frac{1}{2}\gamma_\mu(1 - i\gamma_5)\right]e_i\right\}$$
$$\times\left[\bar{\nu}_n\frac{1}{2}\gamma_\mu(1 - i\gamma_5)\,\nu_a\right]\delta(E_1 - E_2 - k_{10} - k_{20}).$$

We introduce the notation

$$c_\mu = \left[\bar{\nu}_n\tfrac{1}{2}\gamma_\mu(1 - i\gamma_5)\,\nu_a\right]$$

and we make use of c_μ inside the figure brackets. By utilizing the Dirac equation $(\hat{p}_1 - m)e_i = 0$ and $\bar{e}_f(\hat{p}_2 - m) = 0$, we bring the matrix element into the form

$$M = 2\pi\sqrt{8}\,\frac{Ze^2}{q^2}$$
$$\times G\bar{e}_f\left[\hat{c}\,\frac{1}{2}(1 - i\gamma_5)\frac{\hat{q}\gamma_0 + 2p_{10}}{p^2 - m^2} + \frac{2p_{20} - \gamma_0\hat{q}}{p'^2 - m^2}\,\hat{c}\,\frac{1}{2}(1 - i\gamma_5)\right]e_i. \tag{5}$$

The evaluation of the square of the matrix element (5) is fairly complicated, but in the nonrelativistic approximation the form of (5) is considerably simplified. In this case we have

$$p_{10} \sim m, \quad p_{20} \sim m, \quad |\mathbf{p}| \ll m, \quad |k_0| \ll m.$$

Taking this into account, we have

$$p^2 - m^2 \sim 2m(k_{10} + k_{20}),$$

$$p'^2 - m^2 \sim -2m(k_{10} + k_{20}), \qquad \mathbf{q} = \mathbf{p}_2 - \mathbf{p}_1.$$

On averaging the square of the matrix element over the initial polarizations of the electron, and on summing over the final polarizations of the electron and of the emitted neutrino and antineutrino, we obtain

$$\Sigma|M|^2 = 4\pi\,\frac{Z^2e^4G^2}{q^4m^2(k_{10} + k_{20})^2}\left[k_{10}k_{20}\mathbf{q}^2 + (\mathbf{q}\mathbf{k}_1)(\mathbf{q}\mathbf{k}_2)\right]$$
$$\times\,\delta(E_1 - E_2 - k_{10} - k_{20}).$$

The differential cross section of the process under consideration has the form

$$d\sigma = \Sigma|M|^2(2\pi)^{-9}\,\frac{p_2^2dp_2k_1^2dk_1k_2^2dk_2}{dE_f}\,\frac{m}{E_2k_{01}k_{02}J}$$
$$\times\,\delta(E_1 - E_2 - k_{10} - k_{20})\,dE_f\,d\Omega_\nu\,d\Omega_{\bar{\nu}}\,d\Omega_e. \tag{6}$$

Here $J = p_1/m$ is the flux density of incident electrons. By integrating (6) over the energy of the final state E_f and over the direction and the momentum of the neutrino we obtain:

$$d\sigma = \frac{1}{15\pi^4}Z^2r_0^2g^2\frac{p_2}{p_1}(E_1 - E_2)^3\frac{dE_2}{(\mathbf{p}_2 - \mathbf{p}_1)^2}\,d\Omega_e, \tag{7}$$

where $g = G/mc^2(\hbar/mc)^3 = 3 \times 10^{-12}$ for $G = 1.41 \times 10^{-49}$ erg-cm^3, and r_0 is the classical electron radius. The energy and the momentum are expressed in units of mc^2 and mc respectively.

The total effective cross section for the creation of a neutrino pair is obtained by integrating (7) over the direction and the energy of the final electron

$$\sigma = (8Z^2/525\pi^3)\,r_0^2g^2E_1^3 = \sigma_0Z^2E_1^3,$$

$$\sigma_0 = 8r_0^2g^2/525\pi^3 = 3.52\cdot 10^{-52}\ \mathrm{cm}^2.$$

The energy transferred by the electrons to the neutrino pairs, i.e., the effective slowing down κ, is equal to:

$$\varkappa = \int_0^{E_1}(E_1 - E_2)\,d\sigma = \tfrac{32}{45}\,\sigma_0Z^2E_1^4.$$

In ordinary units we have

$$\varkappa = (32/45)\,\sigma_0Z^2E_1(E_1/mc^2)^3 = 2.5\cdot 10^{-52}Z^2E_1(E_1/mc^2)^3. \tag{8}$$

3. CREATION OF NEUTRINO PAIRS IN STARS

Let us evaluate the energy q_ν given up by the electrons to neutrino pairs per cm^3 per sec. If we assume that the electrons have a Maxwellian distribution we obtain

$$q_\nu = \int \varkappa v n_{\text{nuc}} \, dn_e$$
$$= 2.5 \cdot 10^{-52} 5!2 \sqrt{2/\pi} \, n_{\text{nuc}} n_e Z^2 mc^3 (T/mc^2)^{9/2}, \quad (9)$$

n_e and n_{nuc} are the numbers of electrons and of nuclei per cm^3. If the substance has a density ρ and consists of a mixture of atoms which are completely ionized, then

$$n_e = 6 \cdot 10^{23} \rho / \mu_e, \text{ where } 1/\mu_e = \sum_i C_i Z_i / A_i;$$

C_i is the concentration of the element by weight:

$$\Sigma n_{\text{nuc}\,i} Z_i^2 = 6 \cdot 10^{23} \rho / \nu, \qquad 1/\nu = \sum_i C_i Z_i^2 / A_i.$$

On substituting these expressions into (9) we obtain*

$$q_\nu = 2.75 \cdot 10^{-10} (\rho^2 / \nu\mu_e) T^{9/2} \text{erg/cm}^3\text{-sec}, \quad (10)$$

where T is measured in kev.

It can be easily shown that in the case of a hydrogen star q_ν is smaller than the rate of liberation of energy in hydrogen reactions by a factor of $\sim 10^7$. Therefore the process under consideration may play a role only for large Z, when the thermonuclear reaction proceeds much more slowly.

It is of interest to compare q_ν with the rate of liberation of energy w by means of neutrino emission in the Gamow and Schoenberg process. The latter depends in an essential manner on the threshold Q for e^--capture; for large values of Q it falls off exponentially $\sim \exp(-Q/T)$ and has a maximum at $Q \sim 2T$. If we take the matrix element for the β process equal to 1, and the concentration of the element equal to 100%, then in the characteristic range $\rho \sim 10^5$ and $T < 100$ kev the maximum value w_{max} at a given temperature exceeds q_ν by a factor of approximately $10^6/A$. Moreover, both w_{max} and q_ν are proportional to $T^{4.5}$. The isotopes which are most important for the Gamow-Schoenberg process are (cf. reference 4) the elements with a low threshold Q:

$$Cl^{35} \text{ (threshold } Q = 170 \text{ kev)}, \quad N^{14} (Q = 155 \text{ kev}),$$
$$Sc^{45} (Q = 212 \text{ kev}), Ga^{72} (Q = 300 \text{ kev}), \quad Ni^{60} (Q = 300 \text{ kev}).$$

It turns out that C^{14} which is formed from the abundant element N^{14} has an anomalously small matrix element, so that $w = 2.3 \times 10^7$ erg/cm^3 sec for $T = 50$ kev, $\rho = 10^5$ even at a concentration of 100%, while

$$q_\nu = 3.6 \cdot 10^8 \quad \text{at} \quad T = 50 \text{ kev}, \quad \rho = 10^5.$$

In the case of Cl^{35}: $w = 0.8 \times 10^{11}$ for $T = 50$ kev, but its concentration is $< 5 \times 10^{-4}$. The remaining elements (Sc^{45}, Ga^{72}, Ni^{60}) also lead to $w \ll q_\nu$ as a result of their low abundance.

It is therefore possible to assert that even though we have $w \sim g^2$ in the Gamow-Schoenberg process, while in our case we have $w \sim g^2 e^4$, nevertheless, owing to the low abundance of elements with a low threshold Q and to the anomalously long lifetime of certain nuclei, the effect under consideration is considerably larger for $T < 100$ kev.

4. COMPARISON OF THE NEUTRINO AND THE PHOTON LUMINOSITY OF STARS

Let us compare the energy carried away by neutrino pairs from the stars (the "neutrino" luminosity L_ν), with the ordinary photon luminosity L_γ. We examine the ratio of these two quantities for all possible stellar configurations, utilizing as independent variables the density ρ_c and the temperature T_c at the center of the star.

The neutrino and the photon luminosities have a different dependence on the stellar radius:

$$L_\nu \sim R^3, \qquad L_\gamma \sim R^2 \partial T^4 / \partial R \sim R.$$

However, for a given ρ_c and T_c the radius R and the mass M are determined in terms of them by the conditions of mechanical equilibrium. In what follows we consider the regime in which the electrons are nondegenerate and the radiation pressure can be neglected. For such stars the expressions for the radius and for the mass in terms of ρ_c and T_c depend neither on the mechanism of energy liberation in nuclear reactions, nor on the mechanism of energy loss.

The equation for the equilibrium of a star has the form

$$dP/dr = -\rho G M_r / r^2, \qquad P = a\rho T/\mu,$$
$$dM_r / dr = 4\pi r^2 \rho, \quad (11)$$

where $1/\mu = \Sigma_i C_i (Z_i + 1)/A_i$.

By utilizing a similarity transformation we obtain from the system (11)

$$M \sim R^3 \rho_c, \qquad R^2 \sim aT / G\mu\rho_c. \quad (12)$$

The total luminosity of the star is determined

*In the case that the electrons have a Fermi-Dirac distribution and are nearly degenerate, we have:

$q_\nu = 0.82 \cdot 10^{-7} (\rho/\nu) T^6 \ln (0.89 E_0/T)$, $E_0/mc^2 = 5.07 \cdot 10^{-5} (\rho/\mu e)^{2/3}$
(T in kev)

by

$$L_\gamma = -4\pi r^2 \frac{lc}{3} \frac{d}{dr}(\sigma T^4). \tag{13}$$

The photon mean free path l deep within the star is determined primarily by the photoeffect. According to Kramers' formula we have

$$l = bT^{3.5} / \rho^2. \tag{14}$$

On substituting (12) and (14) into (13) we obtain on the basis of considerations of similarity the following expression for the photon luminosity of the star

$$L_\gamma = \text{const} \cdot B \sqrt{\frac{a}{G}}\, T_c^8 / \rho_c^{2.5} \mu^{0.5}, \text{ where } B = 16\pi\sigma cb / 3 \cdot 7.5, \tag{15}$$

where the value of the constant can be obtained on the basis of a definite stellar model. For the model with a homogeneous distribution of energy we have: const = 9.6; for the point source model we have:[5] const = 3.37. Other models yield intermediate results.

For making further estimates we shall utilize the point source model since we are interested in the case when the star consists of heavy elements. In this case the rate of a nuclear reaction depends on a high power of the temperature, and falls off rapidly as the distance from the center increases.

On substituting the numerical coefficients into (15) we obtain

$$L_\gamma = 0.98 \cdot 10^3 b T_c^8 / \mu^{0.5} \rho_c^{2.5}. \tag{16}$$

Here T_c is in kev, L_γ is in solar units ($L_\odot = 3.78 \times 10^{33}$ erg/sec), ρ_c is in g/cm^3.

Let us obtain the expression for the neutrino luminosity:

$$L_\nu = \int q_\nu dV = 2.75 \cdot 10^{-10} \frac{1}{\mu_e \nu} 4\pi \int_0^R \rho^2 T^{9/2} r^2 dr. \tag{17}$$

We take the temperature and density distributions from the same point source model. Since in this case the convective core has a small radius ($0.169R$) we take the temperature and the density to be constant within it and equal to T_c and ρ_c. Within the bulk of the stellar mass we have

$$T = T_c \frac{R/r - 1}{1/\xi - 1}, \qquad \rho = \rho_c \left(\frac{R/r - 1}{1/\xi - 1}\right)^{3.25}$$

where[6] $\xi = 0.169$. Integration yields:

$$\int_0^R \rho^2 T^{9/2} r^2 dr = 2.07 \cdot 10^{-3} R^3 T_c^{4.5} \rho_c^2.$$

In our case we have[5] $T_c = 0.7 \times 10^{-22} \mu M/R$, $\rho_c = 37\bar{\rho} = 111\, M/4\pi R^3$. If we solve these expressions for R in terms of ρ_c and T_c and substitute it into L_ν, we shall obtain the final formula for the neutrino luminosity of the star

$$L_\nu = 0.9 \cdot 10^{-10} (T_c^6 / \mu_e \mu^{1.5} \nu) \rho_c^{0.5}. \tag{18}$$

We compare the photon and the neutrino luminosities expressed in terms of ρ_c and T_c:

$$L_\nu / L_\gamma = (0.92 \cdot 10^{-13} / b\mu\mu_e\nu) \rho_c^3 / T_c^2. \tag{19}$$

The cross section for the creation of neutrino pairs and for bremsstrahlung is proportional to Z^2/A, but $L_\gamma \sim 1/\sigma_{brems} \sim A/Z^2$, i.e.,

$$L_\nu / L_\gamma \sim Z^4 / A^2 \sim Z^2.$$

It may be easily seen from (19) that the neutrino luminosity can produce an appreciable effect only in stars of high density.

According to modern views[7] very dense stars of low luminosity consist largely of heavy elements such as magnesium. Moreover, in the central regions of the star the temperature and the density are very high, but degeneracy does not yet occur.

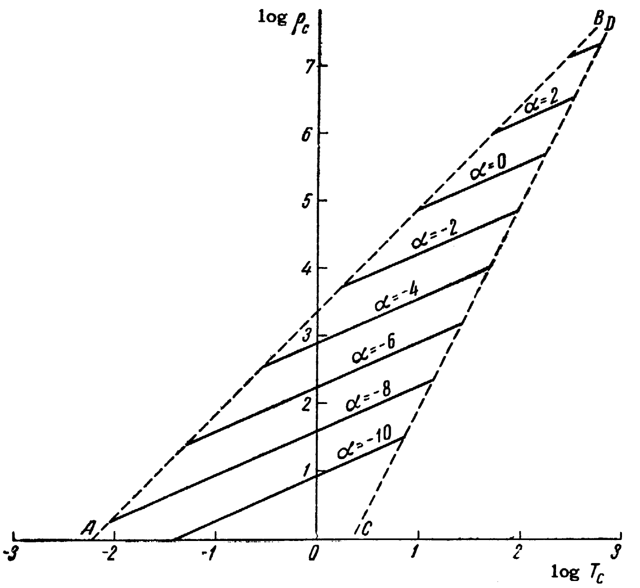

FIG. 2

Figure 2 shows the lines $L_\nu/L_\gamma = 10^\alpha$ for Mg ($\mu = \mu_e = 2$, $\nu = 2/Z = 1/6$, $b = 1$) on a logarithmic scale. The line AB in Fig. 2 is the boundary for degeneracy ($T = E_0/3$, E_0 is the limiting Fermi energy). The line CD is the boundary of the region to the right of which the radiation pressure is greater than the material pressure. We see, for example, that for $\rho = 10^5$, T = 11 kev: $L_\nu/L_\gamma = 1$; for $\rho = 2 \times 10^6$, T = 100 kev: $L_\nu/L_\gamma = 100$.

At high densities the above results must be made more precise. On the one hand the effect of the screening of nuclei by electrons, the collective interaction of nuclei will lead to a change in the cross section for creation of neutrino pairs; on the other hand the inclusion of these phenomena will also lead to a change in the photon mean free path.

We have not investigated the transition of an electron from a free state into a bound one accompanied by the creation of a neutrino pair. This is justified for $\rho > 10^4$ g/cm^3, because in this case the electron has no bound levels, since the atomic radius is of the order of the Bohr radius.

Among the observed stars the neutrino effect may play a significant role in white dwarfs which have in their central parts $\rho \sim 10^5$, and also in the process of stellar evolution.

We have not investigated the regime of the degenerate equation of state, however, from qualitative considerations we may conclude that the neutrino effect may play a role here only at still higher densities than in the nondegenerate regime. At a given temperature T degeneracy diminishes L_ν and increases L_γ.

It should be emphasized that in the development of the theory of white dwarfs and of stellar evolution, particularly in the dynamics of supernova explosions, the inclusion of the energy loss due to neutrinos may turn out to be significant, and in some cases even dominant.

In conclusion we express our sincere gratitude to Ya. B. Zel'dovich for valuable advice and discussions, and to B. M. Pontecorvo and D. A. Frank-Kamenetskiĭ for interesting discussions.

Note added in proof (September 16, 1959). Another form of radiation which interacts weakly with matter is gravitational radiation originating in the collision of particles. It can be easily evaluated by analogy with the quadrupole electromagnetic radiation [L. D. Landau and E. M. Lifshitz, Теория поля (Field Theory), p. 218]. The gravitational radiation plays its most important role in the case of Coulomb scattering of electrons, in this case calculation yields the value $q = 5.3 \times 10^{-20} \rho^2 T_{kev}^2$ erg/cm^3 sec, so that it is smaller than the neutrino radiation by a factor of 10^{10}.

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