SCATTERING OF A PHOTON BY A NUCLEON IN THE ONE-MESON APPROXIMATION

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Photon-nucleon scattering due to strong interactions is treated taking into account the exchange of a single π meson. The scattering matrix is computed for values of angular momentum up to $J = 7/2$. The angular distributions for reactions involving polarized particles are presented.

1. SCATTERING MATRIX IN THE ONE-MESON APPROXIMATION

As was shown by Okun' and Pomeranchuk[1] in peripheral interactions, when in effect the particles exchange the smallest possible number of pions, it is possible to use contemporary meson theory to calculate processes involving large orbital angular momenta.

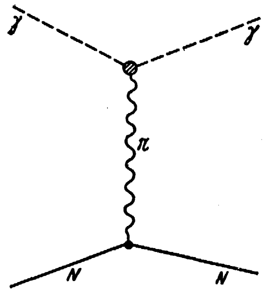
Photon-nucleon scattering in the one-meson approximation is described by the diagram shown in the figure. The matrix element for this process may be written as

$$M = \frac{\Gamma g\sqrt{\pi}}{(2\pi)^2\,\omega}\,\frac{(\bar{u}_2\gamma_5 u_1)\,m}{[(k_2-k_1)^2-\mu^2]\,E_1}\times\{\xi_2\cdot[(\mathbf{n}_1-\mathbf{n}_2)\times\xi_1]\}\,\delta^{(4)}(k_1+p_1-k_2-p_2), \qquad (1)$$

where ξ is the photon unit polarization vector, $\mathbf{k}$ and $\mathbf{p}$ are the photon and nucleon momenta, ω and E are the photon and nucleon energies, $\mathbf{n}_1$ and $\mathbf{n}_2$ are unit vectors in the direction of motion of the initial and final photon, μ is the pion mass, Γ is the vertex function $(\gamma\gamma\pi^0)$, the subscripts 1 and 2 refer to initial and final particles and units are used such that $\hbar = c = 1$. The expression is written in the center of mass coordinate system, hence $\mathbf{k}_1 = -\mathbf{p}_1 = \omega\mathbf{n}_1$, $\mathbf{k}_2 = -\mathbf{p}_2 = \omega\mathbf{n}_2$; the z axis is chosen in the direction of $\mathbf{n}_1$ and u_1, u_2 are unit spinor amplitudes.

The scattering amplitude which determines the probability for the transition from the initial state (when the photon has momentum $\mathbf{k}_1$ and prescribed polarization and the nucleon has momentum $\mathbf{p}_1$ and prescribed spin projection onto the z axis) to the final state (when the photon has momentum $\mathbf{k}_2$ and prescribed polarization and the nucleon has momentum $\mathbf{p}_2$ and prescribed spin projection onto the z axis) is given by the expression

$$f(\theta,\varphi) = C\{v_2^*\sigma(\mathbf{n}_1-\mathbf{n}_2)v_1\}\{\xi_2\cdot[(\mathbf{n}_1-\mathbf{n}_2)\times\xi_1]\}/(x-\cos\theta),$$
$$C = \Gamma g\sqrt{\pi}/8\pi\omega(E_1+\omega), \qquad x = 1+\mu^2/2\omega^2, \qquad (2)$$



Here θ is the angle between $\mathbf{n}_1$ and $\mathbf{n}_2$ in the center-of-mass system, and v_1, v_2 are unit spinors.

To separate out large angular momenta we go over to the angular momentum representation. In this representation the state of a system consisting of a photon and a nucleon is specified by the total angular momentum L and parity λ of the photon, by the spin of the nucleon, and by the conserved quantities J, M_J and Π. Here J is the total angular momentum of the entire system, M_J is its projection on the z axis, and $\Pi = (-)^{L_1+\lambda_1} = (-)^{L_2+\lambda_2}$ is the parity of the entire system. The scattering amplitude in this representation $R(L_2\lambda_2;\ L_1\lambda_1;\ J\Pi)$ is related to the scattering amplitude $f(\theta,\varphi)$ by:[2]

$$f(\theta,\varphi) = C\sqrt{\frac{\pi}{2}}\sum_{JL_2\lambda_2L_1\lambda_1}R(L_2\lambda_2;\ L_1\lambda_1;\ J\Pi)\,i^{L_1+\lambda_1-L_2-\lambda_2}\times\sqrt{2L_1+1}\times\sum_{M_1\mu_1}(-M_1)^{1+\lambda_1}C^{JM_J}_{L_1M_1{}^1\!/_2\mu_1}(-1)^{\lambda_1+\lambda_2}(\chi^*_{M_1}\xi_1)\times(v^*_{\mu_1}v_1)(v_2^*\xi_2\Psi_2(JM_JL_2\lambda_2)), \qquad (3)$$

where $\Psi_2(JM_JL_2\lambda_2)$ is the function describing the state of the system with total angular momentum J and z component M_J. Since J is determined by the vector addition $\mathbf{J} = \mathbf{L}_2 + \mathbf{S}_2$ where $\mathbf{S}_2$ is the spin of the final nucleon we have

$$\Psi_2(JM_JL_2\lambda_2) = \sum_{\mu_2}C^{JM_J}_{L_2M_2{}^1\!/_2\mu_2}\,\mathbf{Y}^{\lambda_2}_{L_2M_2}(\mathbf{n}_2)\,v_{\mu_2}. \qquad (4)$$

In (3) and (4), $C^{\cdot\cdot}_{\cdot\cdot\cdot\cdot}$ are Clebsch-Gordan coefficients and $\mathbf{Y}^{\lambda}_{LM}(\mathbf{n})$ are vector spherical harmonics (defined, for example, in reference 3).

Using the orthogonality properties of the spherical harmonics and the Clebsch-Gordan coefficients the following expression for the scattering matrix may be obtained from (2) and (3) after some simple calculations:

$$R(L_2\lambda_2;\ L_1\lambda_1;\ J\Pi) = -\frac{2}{2J+1} i^{L_2+\lambda_2-L_1-\lambda_1}(-1)^{\lambda_1+\lambda_2} \times \sum_{\mu_1\mu_2M_1} C^{JM_J}_{L_2M_2{}^1\!/_2\mu_2} C^{JM_J}_{L_1M_1{}^1\!/_2\mu_1} \times \int \frac{\{v^*_{\mu_2}\boldsymbol{\sigma}\cdot(\mathbf{n}_1-\mathbf{n}_2)v_{\mu_1}\}\{\mathbf{Y}^{\lambda_2}_{L_2M_2}(\mathbf{n}_2)\cdot[(\mathbf{n}_1-\mathbf{n}_2)\times\mathbf{Y}^{\lambda_1}_{L_1M_1}(\mathbf{n}_1)]\}}{x-\cos\theta}\,do \tag{5}$$

In order to obtain the final expressions we use the formulas

$$[\mathbf{n}\times\mathbf{Y}^{\lambda}_{LM}(\mathbf{n})] = i\,\mathbf{Y}^{1-\lambda}_{LM}(\mathbf{n}), \qquad (\boldsymbol{\sigma}\cdot\mathbf{n})v_\mu = -\sqrt{4\pi}\,\Omega_{1/2 1\mu}(\mathbf{n}), \tag{6}$$

where $\Omega_{1/2 1\mu}(\mathbf{n})$ is a spherical spinor (defined in reference 3). After some simple but tedious calculations we obtain the following expression for the scattering matrix:

$$\begin{aligned} R(L_2\lambda_2;\ L_1\lambda_1;\ J\Pi) = {} & -i\left(4\sqrt{2L_1+1}/(2J+1)\right) i^{L_2+\lambda_2-L_1-\lambda_1} \\ & \times \Big\{ \sum_{l_2} C^{l_20}_{L_2-111}\,\delta(l_2\lambda_2)\sqrt{2l_2+1}\Big[C^{L_21}_{l_2011}Q_{l_2}(x)\,b(JL_2L_1) \\ & - \sum_f C^{f0}_{l_2010}Q_f(x)\left(C^{f0}_{l_2010}C^{L_21}_{l_2011}b(JL_2L_1)\right. \\ & \left. + \sqrt{2}C^{f0}_{l_211-1}d_{l_2}(JL_2L_1)\right)\Big] - \sum_{l_2} C^{l_20}_{L_2-111}\,\delta(l_21-\lambda_2) \\ & \times\sqrt{2l_2+1}\Big[C^{L_21}_{l_2011}Q_{l_2}(x)\,b(JL_2L_1) - \sum_f C^{f0}_{l_2010}Q_f(x) \\ & \times\left(C^{f0}_{l_2010}C^{L_21}_{l_2011}b(JL_2L_1) + \sqrt{2}C^{f0}_{l_211-1}d_{l_2}(JL_2L_1)\right)\Big]\Big\}. \end{aligned} \tag{7}$$

Here we have introduced the following abbreviations:

$$b(JL_2L_1) = C^{J1/2}_{L_21{}^1\!/_2-{}^1\!/_2}C^{J1/2}_{L_11{}^1\!/_2-{}^1\!/_2} - C^{J3/2}_{L_21{}^1\!/_2{}^1\!/_2}C^{J3/2}_{L_11{}^1\!/_2{}^1\!/_2},$$

$$d_{l_2}(JL_2L_1) = C^{J1/2}_{L_11{}^1\!/_2-{}^1\!/_2}C^{J1/2}_{L_20{}^1\!/_2{}^1\!/_2}C^{L_20}_{l_2-111} - C^{J3/2}_{L_11{}^1\!/_2{}^1\!/_2}C^{J3/2}_{L_22{}^1\!/_2-{}^1\!/_2}C^{L_22}_{l_2111};$$

$\delta(l_2\lambda_2)$ indicates that in the sum over l_2 only the term with $l_2 = L_2$ enters if $\lambda_2 = 0$, and only the terms with $l_2 = L_2 \pm 1$ if $\lambda_2 = 1$; $Q_l(x)$ are the Legendre functions of the second kind (for an analytic expression for $Q_l(x)$ see reference 4; for a table of values see reference 5).

Values of $R(L_2\lambda_2;\ L_1\lambda_1;\ J\Pi)$ up to $J = 7/2$ are given in the table.

2. ANGULAR DISTRIBUTIONS

The angular distribution may be expressed in the form

$$d\sigma/do = \mathrm{Sp}\,[\rho_2 f \rho_1 f^+], \tag{8}$$

where

$$\rho_1 = \tfrac{1}{4}(1+\boldsymbol{\xi}_1\boldsymbol{\omega})(1+\boldsymbol{\zeta}_1\boldsymbol{\sigma}), \qquad \rho_2 = \tfrac{1}{4}(1+\boldsymbol{\xi}_2\boldsymbol{\omega})(1+\boldsymbol{\zeta}_2\boldsymbol{\sigma}) \tag{9}$$

are density matrices describing the initial and final states respectively of the nucleon-photon system. Here $\boldsymbol{\xi}_1$ and $\boldsymbol{\xi}_2$ are photon polarization vectors, $\boldsymbol{\zeta}_1$ and $\boldsymbol{\zeta}_2$ are nucleon polarization vectors, $\boldsymbol{\sigma}$ are the Pauli matrices and $\omega_1 = \sigma_z$, $\omega_2 = \sigma_x$, $\omega_3 = \sigma_y$. We follow here the definitions given by Tolhoek.[6]

Let us write the angular distribution as a sum of sixteen terms:

$$\begin{aligned} d\sigma/do = {} & \Phi_0 + \Phi_1(\xi_1) + \Phi_1(\xi_2) + \Phi_1(\zeta_1) + \Phi_1(\zeta_2) \\ & + \Phi_2(\xi_1, \xi_2) + \Phi_2(\zeta_1, \zeta_2) + \Phi_2(\xi_1, \zeta_1) + \Phi_2(\xi_1, \zeta_2) \\ & + \Phi_2(\xi_2, \zeta_1) + \Phi_2(\xi_2, \zeta_2) + \Phi_3(\xi_1, \xi_2, \zeta_1) + \Phi_3(\xi_1, \xi_2, \zeta_2) \\ & + \Phi_3(\xi_1, \zeta_1, \zeta_2) + \Phi_3(\xi_2, \zeta_1, \zeta_2) + \Phi_4(\xi_1, \xi_2, \zeta_1, \zeta_2). \end{aligned} \tag{10}$$

To average over the polarization states of an initial particle we remove from (10) the terms containing the corresponding polarization vector and multiply the cross section by 2; to sum over the polarization states of a final particle we simply remove the corresponding terms from the total cross section (10).

In Eq. (8) we write f in the form of a matrix whose elements represent the scattering amplitude with prescribed polarizations for the photon and nucleon

$$f = \frac{C}{x-\cos\theta}\,a(\alpha,\beta)\,b(\lambda,\mu), \tag{11}$$

where $a(\alpha, \beta)$, $b(\lambda, \mu)$ depend on the polarizations of the nucleon and photon and are given by (see Goertzel)[7]

$$a(1/2;\ 1/2) = (1-\cos\theta), \qquad a(-1/2;\ -1/2) = -(1-\cos\theta),$$
$$a(1/2;\ -1/2) = -\sin\theta e^{-i\varphi}, \qquad a(-1/2;\ 1/2) = -\sin\theta e^{i\varphi}; \tag{12}$$

$$b(1;1) = b(-1;-1) = 0, \qquad b(1;-1) = ie^{-i\varphi}(1-\cos\theta),$$
$$b(-1;1) = -ie^{i\varphi}(1-\cos\theta). \tag{13}$$

From (11), (12), and (13) we find.

$$\Phi_0 = \tfrac{1}{2}C^2(1-\cos\theta)^3/(x-\cos\theta)^2,$$

$$\Phi_1(\xi_1) = \Phi_1(\zeta_1) = \Phi_1(\xi_2) = \Phi_1(\zeta_2) = 0, \tag{14}$$

$$\Phi_2(\xi_1, \xi_2) = \frac{1}{2}C^2\frac{(1-\cos\theta)^3}{(x-\cos\theta)^2}\{-\xi_{21}\xi_{11}$$

$$+ (\xi_{23}\xi_{13} - \xi_{22}\xi_{12})\cos 2\varphi - (\xi_{23}\xi_{12} + \xi_{22}\xi_{13})\sin 2\varphi\}, \tag{15}$$

Values of the coefficients $D(L_2\lambda_2;\ L_1\lambda_1;\ J) = -iR(L_2\lambda_2;\ L_1\lambda_1;\ J\Pi)$

D^* \ ω, Mev	68.5	97	116	137	154	217	308
$D(1,0;\ 1,0;\ ^1/_2)$	0.7726	1.099	1.262	1.402	1.487	1.696	1.830
$D(1,1;\ 1,1;\ ^3/_2)$	0.3863	0.5493	0.6307	0.7012	0.7433	0.8480	0.9152
$D(1,0;\ 2,1;\ ^3/_2)$	−0.3022	−0.4065	−0.4522	−0.4877	−0.5069	−0.5470	−0.5657
$D(2,0;\ 2,0;\ ^3/_2)$	$3.739\cdot10^{-2}$	$7.986\cdot10^{-2}$	$1.085\cdot10^{-1}$	0.1380	0.1580	0.2164	0.2620
$D(2,1;\ 2,1;\ ^5/_2)$	$2.493\cdot10^{-2}$	$5.324\cdot10^{-2}$	$7.237\cdot10^{-2}$	$9.203\cdot10^{-2}$	0.1054	0.1443	0.1747
$D(2,0;\ 3,1;\ ^5/_2)$	$-3.079\cdot10^{-2}$	$-6.072\cdot10^{-2}$	$-7.863\cdot10^{-2}$	$-9.527\cdot10^{-2}$	−0.1056	−0.1309	−0.1457
$D(3,0;\ 3,0;\ ^5/_2)$	$3.157\cdot10^{-3}$	$1.031\cdot10^{-2}$	$1.677\cdot10^{-2}$	$2.467\cdot10^{-2}$	$3.072\cdot10^{-2}$	$5.168\cdot10^{-2}$	$7.170\cdot10^{-2}$
$D(3,1;\ 3,1;\ ^7/_2)$	$2.368\cdot10^{-3}$	$7.732\cdot10^{-3}$	$1.258\cdot10^{-2}$	$1.850\cdot10^{-2}$	$2.304\cdot10^{-2}$	$3.876\cdot10^{-2}$	$5.377\cdot10^{-2}$
$D(3,0;\ 4,1;\ ^7/_2)$	$-3.942\cdot10^{-3}$	$-1.174\cdot10^{-2}$	$-1.802\cdot10^{-2}$	$-2.500\cdot10^{-2}$	$-2.989\cdot10^{-2}$	$-4.424\cdot10^{-2}$	$-5.454\cdot10^{-2}$
$D(4,0;\ 4,0;\ ^7/_2)$	$0.3320\cdot10^{-3}$	$1.671\cdot10^{-3}$	$0.3270\cdot10^{-2}$	$0.5592\cdot10^{-2}$	$0.7610\cdot10^{-2}$	$1.592\cdot10^{-2}$	$2.561\cdot10^{-2}$

*The $D(L_2,\lambda_2;\ L_1,\lambda_1;\ J)$ satisfy the following relations: $D(1,1;\ 1,1;\ ^1/_2) = -D(1,0;\ 1,0;\ ^1/_2)$; $D(1,0;\ 1,0;\ ^3/_2) = -D(1,1;\ 1,1;\ ^3/_2)$; $D(2,1;\ 2,1;\ ^3/_2) = -D(2,0;\ 2,0;\ ^3/_2)$; $D(2,0;\ 2,0;\ ^5/_2) = -D(2,1;\ 2,1;\ ^5/_2)$; $D(3,1;\ 3,1;\ ^5/_2) = -D(3,0;\ 3,0;\ ^5/_2)$; $D(3,0;\ 3,0;\ ^7/_2) = -D(3,1;\ 3,1;\ ^7/_2)$; $D(4,1;\ 4,1;\ ^7/_2) = -D(4,0;\ 4,0;\ ^7/_2)$; $D(a,\alpha;\ b,\beta;\ J) = D(b,\alpha;\ a,\beta;\ J) = D(a,\beta;\ b,\alpha;\ J) = D(b,\beta;\ a,\alpha;\ J)$, where a and b are values of the angular momenta L_2 and L_1; α, β are values of the photon parities λ_1 and λ_2 respectively, and $a = b \pm 1$; $\alpha = 1 - \beta$.

$$\Phi_2(\xi_1,\zeta_1) = \Phi_2(\xi_1,\zeta_2) = \Phi_2(\xi_2,\zeta_1) = \Phi_2(\xi_2,\zeta_2) = 0,$$

$$\Phi_2(\zeta_1,\zeta_2) = \frac{1}{2}C^2\frac{(1-\cos\theta)^2}{(x-\cos\theta)^2}$$

$$\times\{(\zeta_2(\mathbf{n}_1-\mathbf{n}_2))(\zeta_1(\mathbf{n}_1-\mathbf{n}_2)) - (\zeta_1\zeta_2)(1-\cos\theta)\},$$

$$\Phi_3(\xi_1,\xi_2,\zeta_1) = \Phi_3(\xi_1,\xi_2,\zeta_2)$$

$$= \Phi_3(\xi_1,\zeta_1,\zeta_2) = \Phi_3(\xi_2,\zeta_1,\zeta_2) = 0, \tag{16}$$

$$\Phi_4(\xi_1,\xi_2,\zeta_1,\zeta_2) = \frac{1}{2}C^2\frac{(1-\cos\theta)^2}{(x-\cos\theta)^2}$$

$$\times\{-\xi_{21}\xi_{11}+(\xi_{23}\xi_{13}-\xi_{22}\xi_{12})\cos 2\varphi-(\xi_{23}\xi_{12}+\xi_{22}\xi_{13})\sin 2\varphi\}$$

$$\times\{(\zeta_1(\mathbf{n}_1-\mathbf{n}_2))(\zeta_2(\mathbf{n}_1-\mathbf{n}_2)) - (\zeta_1\zeta_2)(1-\cos\theta)\}. \tag{17}$$

To separate out large angular momenta it is necessary to express the amplitude in terms of the scattering matrix. Making use of (3), we obtain the desired expressions for the angular distributions.

a) Angular distribution for unpolarized particles. Using the formulas[8,9]

$$\sum_\beta C^{l\alpha+\beta}_{a\alpha b\beta}C^{c\gamma}_{l\alpha+\beta d\gamma-\alpha-\beta}C^{f\gamma-\alpha}_{b\beta d\gamma-\alpha-\beta}$$

$$= \sqrt{(2l+1)(2f+1)}\,C^{c\gamma}_{a\alpha f\gamma-\alpha}W(abcdef)$$

and

$$Y_{lm}(\mathbf{n})Y^*_{l'm'}(\mathbf{n}) = (-1)^{m'}\sum_\nu\sqrt{\frac{(2l+1)(2l'+1)}{4\pi(2\nu+1)}}$$

$$\times C^{\nu0}_{l'0l0}C^{\nu\sigma}_{l'-m'lm}Y_{\nu\sigma}(\mathbf{n}),$$

we obtain (see Morita et al.[2])

$$\frac{d\sigma_0}{do} = 4\Phi_0 = (C^2/16)\sum R(L_2\lambda_2;\ L_1\lambda_1;\ J\Pi)$$

$$\times R^*(L_2'\lambda_2';\ L_1'\lambda_1';\ J'\Pi')\,i^{L_1+\lambda_1-L_2-\lambda_2+L_2'+\lambda_2'-L_1'-\lambda_1'}$$

$$\times a_\nu(JL_2J'L_2')\,a_\nu(JL_1J'L_1')\,P_\nu(\cos\theta), \tag{18}$$

where

$$a_\nu(JLJ'L') = \sqrt{(2L+1)(2J+1)(2L'+1)(2J'+1)}$$

$$\times C^{\nu0}_{L1L'-1}W(LJL'J'\,{}^1/_2\,\nu). \tag{19}$$

The summation is over all values of the angular momenta, their projections and ν, $W(a,b,c,d,e,f)$ is the Racah coefficient.

b) Photon polarized before and after the reaction, nucleon unpolarized

$$d\sigma(\xi_1,\xi_2)/do = \{1-\xi_{21}\xi_{11}+(\xi_{23}\xi_{13}-\xi_{22}\xi_{12})\cos 2\varphi$$

$$-(\xi_{23}\xi_{12}+\xi_{22}\xi_{13})\sin 2\varphi\}\quad(C^2/32)\sum R(L_2\lambda_2;\ L_1\lambda_1;\ J\Pi)$$

$$\times\ R^*(L_2'\lambda_2';\ L_1'\lambda_1';\ J'\Pi')\,i^{L_1+\lambda_1-L_2-\lambda_2+L_2'+\lambda_2'-L_1'-\lambda_1'}$$

$$\times\ a_\nu(JL_2J'L_2')\,a_\nu(JL_1J'L_1')\,P_\nu(\cos\theta). \tag{20}$$

c) Nucleon polarized before and after the reaction, photon unpolarized

$$d\sigma(\zeta_1, \zeta_2)/do = \Big\{1 + \frac{(\zeta_2(\mathbf{n}_1 - \mathbf{n}_2))/(\zeta_1(\mathbf{n}_1 - \mathbf{n}_2))}{1 - \cos\theta} - (\zeta_1\zeta_2)\Big\}(C^2/32)\sum R(L_2\lambda_2; L_1\lambda_1; J\Pi)$$
$$\times R^*(L_2'\lambda_2'; L_1'\lambda_1'; J'\Pi')\, i^{L_1+\lambda_1-L_2-\lambda_2+L_2'+\lambda_2'-L_1'-\lambda_1'}$$
$$\times a_\nu(JL_2J'L_2')\, a_\nu(JL_1J'L_1')\, P_\nu(\cos\theta). \qquad (21)$$

The expressions for $R(L_2\lambda_2; L_1\lambda_1; J\Pi)$ calculated in the first section are valid only for sufficiently large angular momenta. Therefore, as was done for nucleons by Grashin,[10] the terms corresponding to small values of angular momenta may be separated from the sum over angular momenta in (18), (20), and (21) and treated as adjustable parameters to be determined by experiment or by a future exact theory.

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