SOVIET PHYSICS JETP VOLUME 37 (10), NUMBER 3 MARCH, 1960

GROWTH OF ELECTROMAGNETIC WAVES IN INTERPENETRATING INFINITE MOVING MEDIA

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Submitted to JETP editor April 27, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **37**, 843-846 (September, 1959)

An investigation is made of the propagation of monochromatic plane waves in interpenetrating moving media. Equations are obtained for the refractive index; these equations are used to investigate the stability of the propagating waves. The time growth (damping) factor for the wave is found for the case of motion of a plasma through a dispersionless dielectric.

As is well known, under certain conditions growing electromagnetic waves can be produced in the motion of electron streams in a plasma or any other medium. The growing plane waves (with longitudinal electric fields) produced in the motion of an electron stream in a plasma have been considered by Akhiezer and Faĭnberg[1] and by Ginzburg and Zheleznyakov.[2] Below we give a phenomenological method which can be used to analyze the building (damping) of electromagnetic waves in moving media.

We consider two infinite, uniform, isotropic, lossless and nonmagnetic media, I and II. Medium II is at rest in the laboratory Cartesian coordinate system K and has a dielectric constant ϵ_2. Medium I, in which the coordinate system K′ is fixed, moves uniformly along the x axis of system K with a velocity v and has a dielectric constant ϵ_1 in the K′ system.

If ϵ_1 and ϵ_2 are approximately equal to unity, or if we consider the motion of a plasma in a plasma, the effective electric field is equal to the average macroscopic field and the polarization vectors of medium I and medium II add.* The material equations that relate the electromagnetic field vectors **D**, **B**, **E**, and **H**, in the K coordinate system can be written as follows:†

$$\mathbf{D} = \frac{1}{(1-\varepsilon_1\beta^2)}\left\{\varepsilon_1(1-\beta^2)\mathbf{E} + (\varepsilon_1 - 1)\left(\left[\frac{\mathbf{v}}{c}\times\mathbf{H}\right] - \varepsilon_1\frac{\mathbf{v}}{c}\left(\frac{\mathbf{v}}{c}\mathbf{E}\right)\right)\right\} + (\varepsilon_2 - 1)\mathbf{E},$$
$$\mathbf{B} = \frac{1}{(1-\varepsilon_1\beta^2)}\left\{(1-\beta^2)\mathbf{H} - (\varepsilon_1 - 1)\left(\left[\frac{\mathbf{v}}{c}\times\mathbf{E}\right] + \frac{\mathbf{v}}{c}\left(\frac{\mathbf{v}}{c}\mathbf{H}\right)\right)\right\}, \quad \beta = \frac{v}{c}. \tag{1}$$

These differ from the well known electrodynamic equations for moving media in that the first equation contains the additional term $(\epsilon_2 - 1)\mathbf{E}$ which characterizes the contribution to the electric induction **D** due to the polarization of the fixed medium, II.‡

Using the material equations (1) and Maxwell's equations we solve the problem of propagation of plane electromagnetic waves in medium I and medium II. Because of the symmetry of the wave propagation pattern with respect to the x and x′ axes, which coincide with the velocity vector **v**, we will assume that the plane electromagnetic wave is propagated in the direction of the unit vector γ which lies in the xy plane and forms an angle θ with the x and x′ axes and **v**. As is well known, the Maxwell curl equations lead to the following relation[5] when the expression for the plane wave is used

*If the effective field for the combination of medium I and medium II is known, in principle all of the following results can be generalized for media with high dielectric constants.

†In the monographs by Veksler[3] and Pauli[4] in the equation for **B**(**E**, **H**) the minus sign in front of the term $\mathbf{v}(\mathbf{vH})/c^2$ is incorrect.

‡If medium I and medium II move with different velocities with respect to the laboratory coordinate system, K, these equations become considerably more complicated. They may be obtained by computing the total tensor of the moments in the K coordinate system and comparing it with the electromagnetic field tensors. In a similar manner it is possible to find the equations for the motion of an anisotropic medium through another anisotropic medium (a practical example of a system of this kind is the motion of a plasma in a plasma in the presence of an external fixed magnetic field).

$$\mathbf{D} = -n[\gamma\times\mathbf{H}], \quad \mathbf{B} = n[\gamma\times\mathbf{E}], \qquad (2)$$

where n is the "refractive index" of the medium in which wave propagation takes place. Substituting (2) in (1) and eliminating the components of the electromagnetic fields **E** and **H**, we obtain after some simple manipulation the following equations for the refractive index:

$$f_1(n^*) = n^{*2}\times\left\{\frac{(1-\varepsilon_1\beta^2)(\varepsilon_1+\varepsilon_2-1)\cos^2\theta + [\varepsilon_1(1-\varepsilon_2\beta^2)+(\varepsilon_2-1)]\sin^2\theta}{(\varepsilon_1+\varepsilon_2-1)}\right\} + 2n^*(\varepsilon_1-1)\beta\cos\theta - \frac{(1-\beta^2)[\varepsilon_1(1-\varepsilon_2\beta^2)+(\varepsilon_2-1)]-(\varepsilon_1-1)^2\beta^2}{1-\varepsilon_1\beta^2} = 0,$$

$$f_2(n^{**}) = n^{**2}[(1-\beta^2)\sin^2\theta + (1-\varepsilon_1\beta^2)\cos^2\theta] + 2n^{**}(\varepsilon_1-1)\beta\cos\theta - \frac{(1-\beta^2)[\varepsilon_1(1-\varepsilon_2\beta^2)+(\varepsilon_2-1)]-(\varepsilon_1-1)^2\beta^2}{1-\varepsilon_1\beta^2} = 0, \qquad (3)$$

These differ in the coefficient of the n^2 term. As is well known, the difference in the equations in (3) when $\theta \neq 0$ denotes the existence of two characteristic waves with different polarization and velocities. The equation $f_1(n^*) = 0$ refers to the wave with the field components E_x, E_y and H_z while the equation $f_2(n^{**}) = 0$ refers to the wave with the components H_x, H_y and E_z. When $\theta = 0$ both equations coincide and, since it can be shown that

$$n^*H_z\sin\theta = -(\varepsilon_1+\varepsilon_2-1)E_x, \qquad n^{**}E_z\sin\theta = H_x,$$

the field components E_x and H_x (parallel to the velocity of medium I) vanish.*

For a given frequency and known dielectric constants $[\epsilon_1(\omega)$ and $\epsilon_2(\omega)]$ the two equations of (3) determine the velocity of propagation of the electromagnetic waves in both media (one moving). The solutions of these equations determine the "growth factor" of the electromagnetic wave in the moving medium. In this connection we determine the growth of an electromagnetic wave in a plasma moving in vacuum, since it is of some interest.

For a plasma that moves with nonrelativistic velocity ($v \ll c$) in the absence of a magnetic field in the laboratory system (K) we have $n^2 = 1 - \omega_0^2/\omega^2$, where ω is the frequency in this reference system. Thus, the flow velocity v does not appear in the expressions for the "refractive index" and the "drag" of the wave does not appear in explicit form. On the other hand

$$n^2 = 1 - \omega_0^2(1-n\beta)^2/\omega^2(1-n\beta)^2 \approx 1 - \omega_0^2(1-2n\beta)/\omega'^2,$$
$$n\beta \ll 1, \qquad (4)$$

where $\omega' = \omega(1-n\beta)$ is the frequency of the wave in the K′ reference system, which moves with the plasma (for simplicity it is assumed that the wave is propagated in the direction of motion of the plasma). Equation (4), which is quadratic in n, coincides with Eq. (3) in the case of longitudinal propagation ($\theta = 0$) when $\beta \ll 1$ and $\epsilon_2 = 1$ if the quantity $\epsilon_1 = 1 - \omega_0^2/\omega'^2$ is the dielectric constant of the plasma in the reference system that moves with the plasma. Solving Eq. (4) for n we have

$$n \approx -(\varepsilon_1 - 1)\beta \pm \sqrt{\varepsilon_1}. \qquad (5)$$

This equation indicates that the wave is dragged. Thus a plasma is an example of a medium in which dispersion leads to the appearance (disappearance) of a drag effect, depending on the choice of the reference system* used for computing ϵ.

Equation (3) can also be used to compute growth (decay) of the electromagnetic waves [when the dispersion relations for $\epsilon_1(\omega)$ and $\epsilon_2(\omega)$ are known] by replacing n by kc/ω and solving these equations for ω for real wave numbers, k. In this case ϵ_1 must obviously be expressed in the K′ coordinate system (fixed in the moving medium) by transforming the frequency from the laboratory reference system to the K′ coordinate system.

As an example we solve the problem given above for the case in which an infinite uniform plasma moves in a nondispersive medium characterized by a dielectric constant ϵ_2. In the K′ coordinate system (fixed in the moving plasma) the dielectric constant is

$$\varepsilon_1 = 1 - \frac{\omega_0^2(1-\beta^2)}{\omega^2(1-n\beta\cos\theta)^2} = 1 - \frac{\omega_0^2(1-\beta^2)}{(\omega - kv\cos\theta)^2}, \qquad (6)$$

where ω and k are the frequency and wave number of the electromagnetic wave in the K coordi-

*In the derivation of the first equation of (3) one actually obtains a somewhat more complicated expression: $f_1(n^*)(\varepsilon_1 + \varepsilon_2 - 1)E_x/\sin\theta = 0$. The requirement $E_x \neq 0$ means that either $f_1(n^*) = 0$ or $\varepsilon_1 + \varepsilon_2 - 1 = 0$. In the case in which medium I and medium II are plasmas the last condition corresponds to a plasma wave with a longitudinal electric field. Replacing ε_1 and ε_2 by the appropriate expressions which apply in the plasma for nonrelativistic velocities ($\beta \ll 1$) it is easy to obtain the dispersion equation $\omega_0^2/\omega^2 + \Omega^2/(\omega - kv)^2 = 0$ which coincides (if the thermal-motion corrections are neglected) with the equation obtained in references 1 and 2 on the basis of a kinetic analysis. In the equation given above ω_0^2 is the square of the plasma frequency of the fixed plasma, Ω^2 is the square of the plasma frequency of the moving plasma, and **k** is the wave vector.

*This situation has been clarified in a discussion with B. N. Gershman and V. V. Zheleznyakov.

nate system. If we substitute the expressions which have been derived for ϵ_1 in Eq. (3) for $n = n^*$ and $n = n^{**}$ and take $\omega = \tilde{\omega} + \xi$ and $\tilde{\omega} - kv \cos\theta = 0$, very complicated algebraic equations are obtained for ξ. If it is assumed that the plasma is rarefied ($\omega_0^2 \rightarrow 0$) and we impose the additional condition $\omega_0^2/\xi^2 \rightarrow 0$ when $\omega_0 \rightarrow 0$ so that $\epsilon_1 \rightarrow 1$ when $\omega_0 \rightarrow 0$ and assume that $\xi \ll \tilde{\omega}$, the equations can be simplified considerably:

$$\xi^3 2\tilde{\omega}\varepsilon_2 - \xi^2\tilde{\omega}^2 \frac{1-\varepsilon_2\beta^2\cos^2\theta}{\beta^2\cos^2\theta} - \tilde{\omega}^2\omega_0^2\tan^2\theta\frac{\varepsilon_2-1}{\varepsilon_2} = 0,$$

$$\xi^3 2\tilde{\omega}\varepsilon_2 - \xi^2\tilde{\omega}^2 \frac{1-\varepsilon_2\beta^2\cos^2\theta}{\beta^2\cos^2\theta} = 0, \tag{7}$$

where $\tilde{\omega} = kv\cos\theta$ and k is real. We note that in the derivation of Eq. (7), because $\omega_0^2 \ll \xi^2 \ll \tilde{\omega}^2$ we have neglected terms which contain ξ^4 and higher powers of ξ and terms proportional to ω_0^4. The first equation in (7) corresponds to a wave whose polarization coincides with the polarization of the Cerenkov wave ($E_x \neq 0$) while the second is for a wave with $H_x \neq 0$. When $1 - \epsilon_2\beta^2\cos^2\theta = 0$, which is the Cerenkov condition for a single charged particle, the solution of the first equation in (7) is

$$\xi = [\omega_0^2\tilde{\omega}\tan^2\theta(\varepsilon_2 - 1)/2\varepsilon_2^2]^{1/3}, \tag{8}$$

while the solution of the second equation is $\xi = 0$.

In extracting the roots in Eq. (8) it is found that two values are complex; one of these corresponds to a wave whose amplitude increases in time. It is apparent from the solution to Eq. (8) that the original assumption (for which $\omega_0^2/\xi^2 \rightarrow 0$ when $\omega_0^2 \rightarrow 0$) is actually satisfied. When $\epsilon_2 \rightarrow 1$ we find $\xi \rightarrow 0$, which is to be expected. We note also that when $\theta \rightarrow \pi/2$ the quantity ξ remains finite because when $1 - \epsilon_2\beta^2\cos^2\theta = 0$ and $\cos\theta \rightarrow 0$, $\epsilon_2 \rightarrow \infty$. If $\theta \rightarrow 0$, $\xi \rightarrow 0$ and consequently a wave which propagates in the direction of motion of the plasma exhibits no growth. In this case, as has been noted above, the electric field component E_x also tends to zero. Since this component is parallel to the direction of motion of medium I, the work done by the plasma on the wave, by virtue of which the amplitude of the field increases, also tends to zero. The same interpretation applies for the absence of the growing solution in the second equation of (7), which refers to the wave for which $H_x \neq 0$ but $E_x = 0$.

Equation (8), which determines the growth of the transverse waves at an angle found from the "Cerenkov" condition (in the sense of the dependence of ξ on ω_0 and $\tilde{\omega}$) is analogous to the growth factor for longitudinal plasma waves.[1,2] The only difference is that in plasma waves the plasma frequency of the fixed plasma is used rather than $\tilde{\omega}$.

Incidentally, if we assume formally that ϵ_2 in Eq. (8) is large, the expression obtained coincides exactly with the expression for the gain in a traveling wave tube (derived in the nonrelativistic approximation) transformed for the case of propagation of the wave at the "Cerenkov angle" with respect to the motion of the electron beam.*

The author is indebted to A. V. Gaponov and V. V. Zheleznyakov for valuable advice in the execution of this work and for many useful discussions.

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Translated by H. Lashinsky
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*This has been pointed out by A. V. Gaponov.