

*NONRESONANCE ABSORPTION OF OSCILLATING MAGNETIC FIELD ENERGY BY A  
FERROMAGNETIC DIELECTRIC*

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Spin-wave theory is used to calculate the imaginary part of the longitudinal magnetic susceptibility of a ferromagnetic dielectric.

A ferromagnet can absorb the energy of a weak alternating magnetic field  $H' \sim e^{-i\omega t}$  that is polarized either perpendicular or parallel to its equilibrium magnetic moment. In the first case the magnetic field "rotates" the magnetic moment without changing its magnitude, while in the second case a process of magnetization changes the absolute value of the magnetic moment. In both cases dissipative processes occur that are associated with interactions of spin waves with each other and with phonons.<sup>1-3</sup> We assume saturation magnetization of the ferromagnetic dielectric at a given temperature so that our sample consists of a single domain, and that it is pure enough so that we may disregard effects due to impurities.

In the present paper we confine ourselves to the case in which the magnetic field of frequency  $\omega$  is polarized parallel to the equilibrium magnetic moment of the ferromagnetic dielectric. The imaginary part of the longitudinal magnetic susceptibility  $\mu_{\parallel} = \mu'_{\parallel} + i\mu''_{\parallel}$  must therefore be determined in order to calculate the absorption. The complicated character of relaxation processes in ferromagnetic dielectrics<sup>4</sup> leads to a complicated frequency dependence of  $\mu''_{\parallel}(\omega)$ .

Our calculation will be based on the simplest model of a ferroelectric as a body with a single magnetic moment whose oscillations are propagated as spin waves. This is valid at sufficiently low temperatures; we then have no vibrations of one magnetic sublattice with respect to the other, which correspond to high frequency limits ( $\sim \Theta_C/\hbar$ ). The real bodies to which the present form of the theory of spin waves can be applied are evidently ferrites, the semiconductor properties of which are unimportant at low temperatures (the number of free electrons vanishes exponentially as the temperature is reduced).

### 1. LOW FREQUENCIES

When the magnetic field frequency is considerably smaller than the reciprocal of the spin-spin relaxation time  $\tau_{SS}$ , i.e.,

$$\omega\tau_{SS} \ll 1, \quad (1)$$

this field can be regarded classically as the cause of spin-wave nonequilibrium. This condition (1) enables us to assume that the spin-wave system has quasi-equilibrium energy levels, i.e.,

$$E = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} n_{\mathbf{k}}, \quad \epsilon_{\mathbf{k}} = \Theta_c (ak)^2 + \mu H(t). \quad (2)$$

Here  $\epsilon_{\mathbf{k}}$  is the energy of a spin wave.\* We shall make use of (2), which is valid for relatively large values of the wave vector number  $\mathbf{k}$ , so that the results of this section are correct only when  $T \gg 2\pi\mu M_0 \sim 1^\circ \text{K}$ . The mean occupation numbers  $n_{\mathbf{k}}$  are determined by means of a kinetic equation, written schematically as

$$\partial n_{\mathbf{k}}/\partial t = (\partial n_{\mathbf{k}}/\partial t)_{SS} + (\partial n_{\mathbf{k}}/\partial t)_{st}. \quad (3)$$

The first term on the right is the part of the collision integral which describes the interaction between spin waves, while the second term pertains to the interaction between spin waves and phonons. The specific form of the collision operator has been given in references 2 and 3. It is very important in what follows that at temperatures  $T$  considerably above  $T_0 = \mu M_0 (\Theta_C/\mu M_0)^{3/7} \sim 10^\circ$  the principal part in the interaction of spin waves is played by the exchange interaction and for  $T \ll T_0$  by the relativistic interaction. This difference results in an essential distinction between the frequency dependence of the imaginary part of the magnetic sus-

\*The notation of references 2 and 3 is used.

ceptibility at low ( $T \ll T_0$ ) and high ( $T \gg T_0$ ) temperatures.

For the low-temperature region ( $T \ll T_0$ ) we have, from reference 2\*

$$\frac{1}{\tau_{ss}} \approx \frac{\mu M_0}{\hbar} \frac{\mu M_0}{\Theta_c} \left(\frac{T}{\Theta_c}\right)^{1/2} \approx 10^6 \left(\frac{T}{T_0}\right)^{1/2} (\text{sec}^{-1}) (\mu M_0 \ll T \ll T_0),$$

and (3) will be solved in the form

$$n_k = n_0(\epsilon_k/T) + n'_k, \quad |n'_k| \ll n_0, \quad (4)$$

where  $n_0$  is the equilibrium Bose function. It must also be remembered that  $\epsilon_k$  and  $T$  depend on the time. This form of the solution denotes that in the spin-wave system a quasi-equilibrium state can be established with its own temperature, different from the lattice temperature. Dissipation is here associated with two basically different mechanisms, the transfer of energy to phonons and the departure of the distribution from equilibrium. We therefore write  $\mu'' = \mu_1'' + \mu_2''$ . The first mechanism is naturally characterized by the time  $\tau_{sl}$  required for temperature equalization<sup>3</sup> and the second mechanism by the spin-spin relaxation time  $\tau_{SS}$ . Since  $\tau_{SS} \ll \tau_{sl}$ , at frequencies  $\omega\tau_{sl} \ll 1$  the principal part in dissipative processes will be played by energy transfer to phonons and at frequencies  $1/\tau_{sl} \ll \omega \ll 1/\tau_{SS}$  by the departure of the distribution from equilibrium.

To calculate the time dependence of the temperature  $T$  we must use the heat-balance equation

$$\dot{T} + (T - T_l)/\tau_{sl} = q\mu\dot{H}, \quad \tau_{sl} = C_s/\alpha, \quad q \sim 0.4, \quad (5)$$

which is easily obtained from (3) and (4). Here  $\alpha$  is the coefficient of heat transfer between spin waves and the lattice which was calculated in reference 3. An analogous equation can be derived for the phonon temperature  $T_l$ , although the problem of the interaction between phonons and the external medium now arises. On the other hand, if we assume excellent thermal contact, so that all heat entering the lattice can be removed by the external medium (unlimited heat elimination) the lattice temperature  $T_l$  can be regarded as constant. We shall hereinafter limit ourselves to this case.†

From (5) we have

$$T = T_l + T', \quad T' = \frac{qC_s\mu\dot{H}}{\alpha - i\omega C_s} = -\frac{i\omega qC_s\mu H'}{\alpha - i\omega C_s}. \quad (6)$$

As already stated, the dissipation is described

\*We have assumed  $\Theta_c \sim 10^{-13}$  erg,  $T_0 \sim 10^\circ\text{K}$ ; the unimportant logarithmic dependence on temperature has been disregarded.

†The adiabatic temperature-equalization time  $\tau_{sl}$  was calculated in reference 3.

by the imaginary part of the magnetic susceptibility, which is obtained from the imaginary part of the coefficient relating the spin temperature to the magnetic field; we have

$$M = M_0 - \mu \sum_k n_0(\epsilon_k/T).$$

From this equation and (6) we obtain

$$\mu_1 = \chi \frac{\mu^2}{a^3\Theta_c} \left(\frac{T}{\Theta_c}\right)^{1/2} \frac{\omega\tau_{sl}}{1 + \omega^2\tau_{sl}^2},$$

$$\chi = \frac{3}{40\pi^{3/2}} \frac{\zeta^2(3/2)}{\zeta(5/2)} \approx 10^{-2}. \quad (7)$$

It is shown in reference 3 that for  $T \ll T_0$  the coefficient of heat transfer from spin waves to the lattice depends exponentially on the temperature (heat is transferred through the "Cerenkov" production of phonons by spin waves); also

$$C_s = (15\zeta(5/2)/32\pi^{3/2}) (T/\Theta_c)^{1/2}.$$

Therefore

$$\frac{1}{\tau_{sl}} \approx \frac{64\beta_1^2}{5\pi^{3/2}\zeta(5/2)} \frac{\Theta_c}{\hbar} \frac{T^{1/2}}{ms_l^2\Theta_c^{1/2}} \exp\{-\Theta_l^2/4\Theta_c T\}, \quad m = \rho a^3; \quad (8)$$

$$\mu_1 \approx 5 \cdot 10^{-3} \frac{\mu^2 ms_l^2 h\omega}{a^3 T^3} \exp\left(\frac{\Theta_l^2}{4\Theta_c T}\right), \quad \omega\tau_{sl} \ll 1. \quad (8')$$

We shall now calculate  $\mu_2''$ , the part of  $\mu''$  that is associated with the departure of the distribution from equilibrium. This dissipation mechanism is important at relatively high frequencies ( $\omega\tau_{sl} \gg 1$ ), when we can still make use of (2) [in virtue of (1)] and of the kinetic equation (3). In the latter the second term of the right member may be neglected; when  $\omega \gg 1/\tau_{sl}$  energy "cannot" be transferred to phonons. Since  $(\partial n_k^0/\partial t)_{SS} = 0$  we have

$$\partial n_k^0/\partial t + \partial n'_k/\partial t = (\partial n'_k/\partial t)_{ss}. \quad (9)$$

The neglect of the term  $(\partial n_k/\partial t)_{sl}$  in the kinetic equation corresponds to the neglect of heat elimination in the thermal balance equation (5). Therefore when  $\omega\tau_{sl} \gg 1$  we have  $\dot{T} = q\mu\dot{H}$ , and (9) can be written as follows:

$$\left(1 - q\frac{\epsilon}{T}\right) \frac{\partial n_0}{\partial(\epsilon/T)} \frac{\mu\dot{H}}{T} = \frac{1}{\tau_{ss}} \hat{L}\{n'\}. \quad (10)$$

Here

$$1/\tau_{ss} = (\mu M_0/\hbar) (\mu^2/a^3\Theta_c) (T/\Theta_c)^{1/2}, \quad (11)$$

while  $\hat{L}$  is a dimensionless collision operator which is easily derived from (34) of reference 2 by changing from summation to integration over the wave vectors  $k$  and introducing the dimensionless wave vector  $\mathbf{x} = \sqrt{\Theta_c/T} \mathbf{ak}$ .

In (10) we have omitted the term containing the time derivative of the nonequilibrium correction to the distribution function; this is valid for  $\omega\tau_{SS} \ll 1$ .

From (10) we have

$$n' = \tau_{ss}(\mu\dot{H}/T)\varphi(\mathbf{x}), \quad (12)$$

where  $\varphi(\mathbf{x})$  is a dimensionless function which cannot be determined without solving the integral equation (10).

When the temperature dependence of  $n'$  is known the temperature dependence of  $\mu''$  can be determined. This requires calculating the variation of the entropy  $S$  per unit time resulting from the departure of the spin-wave distribution from equilibrium. A familiar formula<sup>5</sup> gives the amount of absorbed magnetic field energy:

$$Q = (\omega\mu''/4\pi)\overline{H^2}. \quad (13)$$

This equation in conjunction with  $T\dot{S} = Q$  gives  $\mu''$  ( $\mu_2''$  in the present case). We shall therefore determine  $T\dot{S}$ , using the following expression for the entropy of a nonequilibrium Bose gas:<sup>6</sup>

$$S = \sum_{\mathbf{k}} \{(n_{\mathbf{k}} + 1) \ln(n_{\mathbf{k}} + 1) - n_{\mathbf{k}} \ln n_{\mathbf{k}}\}.$$

By differentiating this expression with respect to time and taking into account the energy conservation law

$$\sum_{\mathbf{k}} \epsilon_{\mathbf{k}} n_{\mathbf{k}}^0 = 0$$

[a corollary of the linearized kinetic equation (9)], we obtain

$$Q \equiv T\dot{S} = -T \sum_{\mathbf{k}} (\dot{n}_{\mathbf{k}})_{ss} n_{\mathbf{k}}'/n_{\mathbf{k}}^0 (n_{\mathbf{k}}^0 + 1). \quad (14)$$

From (14), (12), and (13) we then have

$$\mu_2'' = \kappa_2 (\mu^2/a^3\theta_c) (T/\theta_c)^{1/2} \omega \tau_{ss}, \quad (15)$$

where  $\kappa_2$  is a numerical constant that can be determined only by solving the kinetic equation exactly.

Equation (15) is valid for  $1/\tau_{sl} \ll \omega \ll 1/\tau_{ss}$ . A comparison with (7) shows that  $\mu_1''$  and  $\mu_2''$  coincide when  $\omega \approx (\tau_{sl} \tau_{ss})^{-1/2}$ . At lower frequencies  $\mu_1'' > \mu_2''$  and at higher frequencies  $\mu_1'' < \mu_2''$ . As the frequency approaches  $1/\tau_{ss}$  the value of  $\mu_2''$  must be reduced since the spin system "cannot" depart from equilibrium. This can be taken into account by retaining the term  $\partial n'/\partial t$  in (10). We then obtain

$$\mu_2'' = \kappa_2 \frac{\mu^2}{a^3\theta_c} \left(\frac{T}{\theta_c}\right)^{1/2} \frac{\omega \tau_{ss}}{1 + \omega^2 \tau_{ss}^2}. \quad (16)$$

A comparison of (15) and (11) shows that for frequencies in the interval  $(\tau_{sl} \tau_{ss})^{-1/2} \ll \omega \ll 1/\tau_{ss}$  the imaginary part  $\mu''$  of the magnetic susceptibility is independent of temperature and is linearly dependent on frequency:

$$\mu'' \approx \hbar\omega/\mu M_0. \quad (17)$$

We shall now consider the case of relatively high temperatures  $T \gg T_0$ . As previously we shall assume that the temperature is considerably below both the Curie temperature  $\Theta_c$  and the Debye temperature  $\Theta_0$ . In this temperature region exchange forces play the principal part in the interaction between spin waves. It is therefore convenient to write the kinetic equation as follows:

$$\frac{\partial n_{\mathbf{k}}}{\partial t} = \left(\frac{\partial n_{\mathbf{k}}}{\partial t}\right)_{ss}^{\text{exch}} + \left(\frac{\partial n_{\mathbf{k}}}{\partial t}\right)_{ss}^{\text{rel}} + \left(\frac{\partial n_{\mathbf{k}}}{\partial t}\right)_{sl}. \quad (18)$$

We cannot drop the small relativistic term, which is responsible for energy dissipation in one of the frequency regions. The large exchange interaction between spin waves does not change their number; this is reflected mathematically in the fact that the Bose distribution with an arbitrary value of the chemical potential  $\gamma$  causes vanishing of the first term in the right member of (18). We recall that the equilibrium chemical potential of the spin-wave gas is zero.

The solution of (18) must be found in the form

$$n = n_0 \left(\frac{\epsilon - \gamma}{T}\right) + n', \quad |n'| \ll n_0, \quad (19)$$

taking into account the time dependence of the energy  $\epsilon$ , chemical potential  $\gamma$  and spin-wave temperature  $T$ . We shall use (2) for the energy of a spin wave, regarding it as valid also for  $\mathbf{k} = 0$ . This is permissible if we assume that the ferromagnet has a large anisotropy constant or is in a relatively strong magnetic field. The spin-wave energy for  $\mathbf{k} = 0$  without an external alternating field will hereinafter be denoted by  $\epsilon_0$  ( $\epsilon_0 = \mu H_{\text{eff}}$ ,  $H_{\text{eff}} = H_0 + \beta M_0$ ).

From (18) and (19) we easily obtain the following relations between the variable part of the spin temperature  $T$ , the chemical potential  $\gamma$  and the alternating magnetic field:

$$(A_{T\gamma} - i\omega q C_s)T' + (TA_{\gamma\gamma} - i\omega B)\gamma = B\mu\dot{H},$$

$$(A_{TT} - i\omega C_s)T' + (A_{T\gamma} - i\omega C_s)\gamma = qC_s\mu\dot{H}. \quad (20)$$

Here

$$B = T^2/8\pi\theta_c^{3/2}\epsilon_0^{1/2}, \quad (21)$$

and the  $A$  coefficients were calculated in reference 7, where magnetic-moment relaxation in a ferromagnetic dielectric was studied. In the given temperature interval ( $T \gg T_0$ ,  $\Theta_0^2/\Theta_c$ )\*

$$A_{T\gamma} = (\hbar/6\pi^2\rho a^5) (\gamma_1\mu M_0/\theta_c)^2 (T/\theta_c)^2,$$

$$A_{TT} = (2\pi^2\hbar/45\rho a^5) [2\beta_1^2 + (2\beta_1 + \beta_2)^2] (T/\theta_c)^4,$$

$$A_{\gamma\gamma} = (\chi/\hbar) (\mu M_0/\theta_c)^2 (T^2/\epsilon_0\theta_c), \quad \chi \approx 0.1.$$

\*The notation is that of references 2 and 3.

$T'$  and  $\gamma$  can be calculated by means of (20). The formula

$$M = M_0 - \mu \sum_k n_k \left( \frac{\varepsilon - \gamma}{T} \right)$$

for the magnetic moment is then used to give the imaginary part of the magnetic susceptibility. Retaining only the principal terms, we obtain

$$\mu'' = \frac{5\pi}{4\chi} \frac{\hbar\omega}{a^3 M_0^2} \frac{1 + \omega^2 \tau_2^2}{(1 + \omega^2 \tau_1^2)^2 + \omega^2 \tau_3^2} \quad (T \gg T_0), \quad (23)$$

where

$$\begin{aligned} 1/\tau_1 &= (T A_{\gamma\gamma} A_{TT} / BC_s)^{1/2} \\ &\approx (\mu M_0 / \hbar) (\Theta_0^2 / m s^2 \Theta_c)^{1/2} (T / \Theta_c)^{1/2} (T / \varepsilon_0)^{1/2}, \\ 1/\tau_2 &= A_{TT} / C_s \sim (\Theta_0 / \hbar) (\Theta_0 / m s^2) (T / \Theta_c)^{1/2}, \\ 1/\tau_3 &= T A_{\gamma\gamma} / B \sim (T / \hbar) (\mu M_0 / \Theta_c)^2 (\Theta_c / \varepsilon_0)^{1/2}. \end{aligned} \quad (24)$$

The frequency dependence of the imaginary part of the susceptibility is shown in (23), where all relaxation times satisfy the following sequence of inequalities:

$$\tau_{ss} \ll \tau_1 \ll \tau_2 \ll \tau_3,$$

and the reciprocal spin-spin relaxation time  $1/\tau_{ss}$  is now given by<sup>3</sup>

$$1/\tau_{ss} \sim (\Theta_c / \hbar) (T / \Theta_c)^4. \quad (24')$$

At low frequencies

$$\mu'' \sim \hbar\omega / \mu M_0 \quad (\omega\tau_3 \ll 1; T \gg T_0).$$

We assume that  $M_0 \sim \mu/a^3$ .

It can be shown that an additional loss mechanism exists at frequencies  $1/\tau_1 \ll \omega \ll 1/\tau_{ss}$ , due to the departure of the spin-wave distribution from equilibrium. (This is analogous to the second mechanism at low temperatures.) However, it is easily seen that this mechanism always (at all frequencies) results in values of  $\mu''$  which are considerably smaller than those calculated from (23). This results from the fact that the exchange forces, which play the principal part in this temperature region, cannot produce dissipation without the cooperation of relativistic interactions, since the former do not change the magnetic moment.

This section will be concluded by specifying the imaginary part of the longitudinal magnetic susceptibility in different frequency and temperature ranges. From (7), (16), and (23) we have

$$\mu'' \sim \begin{cases} \frac{M_0 \mu}{\Theta_c} \left( \frac{T}{\Theta_c} \right)^{1/2} \frac{\omega \tau_{sl}}{1 + \omega^2 \tau_{sl}^2}, & \omega \sqrt{\tau_{sl} \tau_{ss}} \ll 1, & T \ll T_0; \\ \frac{\mu M_0}{\Theta_c} \left( \frac{T}{\Theta_c} \right)^{1/2} \frac{\omega \tau_{ss}}{1 + \omega^2 \tau_{ss}^2}, & \omega \sqrt{\tau_{sl} \tau_{ss}} \gg 1, & T \ll T_0; \\ \frac{\hbar\omega}{\mu M_0} \frac{1 + \omega^2 \tau_2^2}{(1 + \omega^2 \tau_1^2)^2 + \omega^2 \tau_3^2}, & \omega \tau_{ss} \ll 1, & T \gg T_0. \end{cases}$$

The relaxation times are given by (8), (11), (24), and (24'). We recall that  $T_0 \sim \mu M_0 (\Theta_c / \mu M_0)^{3/7}$ .

## 2. HIGH FREQUENCIES

Since at frequencies  $\omega \gg 1/\tau_{ss}$  spin waves cannot depart from equilibrium, the attenuation of the magnetic field can be regarded as the absorption of photons by an equilibrium spin-wave gas.<sup>8,9</sup>

The interaction Hamiltonian of the field and spin waves is obviously

$$\hat{\mathcal{H}}_{int} = \mu \int \hat{H}_z \hat{a}^*(\mathbf{r}) \hat{a}(\mathbf{r}) dv.$$

The expansion of the operators  $\hat{a}(\mathbf{r})$  and  $\hat{a}^*(\mathbf{r})$  in terms of the spin waves is given by (12) – (20) of reference 2, and the magnetic field operator is<sup>10</sup>

$$\hat{H}_z = i \sqrt{2\pi \hbar \omega / V} \{ \hat{q}_\omega e^{i\omega x/c} - \hat{q}_\omega^* e^{-i\omega x/c} \}.$$

The operators  $\hat{q}_\omega$  and  $\hat{q}_\omega^*$  have the nonvanishing matrix elements

$$q_{n_\omega, n_\omega - 1}^* = \sqrt{n_\omega} e^{i\omega t}, \quad q_{n_\omega, n_\omega + 1} = \sqrt{n_\omega + 1} e^{-i\omega t}; \quad (25)$$

$n_\omega$  is the number of protons with frequency  $\omega$ .

The magnetic field wave vector  $\mathbf{q}$  ( $|\mathbf{q}| = \omega/c$ ) is parallel to the  $x$  axis.

The interaction Hamiltonian in terms of spin-wave creation and annihilation operators is

$$\begin{aligned} \hat{\mathcal{H}}_{int} &= i\mu \sqrt{2\pi \hbar \omega V} \sum_{\mathbf{k}, \mathbf{k}'} \{ v_k^* u_{\mathbf{k}'} \hat{a}_{\mathbf{k}}^* \hat{a}_{\mathbf{k}'} q_\omega \\ &+ (u_{\mathbf{k}}^* u_{\mathbf{k}'} + v_k^* v_{\mathbf{k}'}) \hat{a}_{\mathbf{k}} \hat{a}_{\mathbf{k}'} q_\omega \} + \text{compl. conj.} \end{aligned} \quad (26)$$

This equation reveals two possible processes for photon absorption—through splitting into two spin waves (the first term in the brace) or through absorption by a spin wave (the second term). The first process corresponds to the momentum conservation law

$$\mathbf{k} + \mathbf{k}' = \mathbf{q}, \quad (27)$$

while the second process corresponds to  $\mathbf{k} = \mathbf{k}' + \mathbf{q}$ . However, a photon cannot be absorbed by a spin wave since for this process energy and momentum conservation would require the velocity of the spin wave to be greater than that of light. To be sure, if the dielectric constant of the ferroelectric is greater than unity this process is possible in principle (analogously to Cerenkov emission), but it is easily seen that its mean probability will then be exponentially small. Photon absorption by spin waves will therefore not be considered hereinafter.

The splitting of a photon into two spin waves can, of course, occur only if the energy of the photon is greater than twice the minimum spin-wave energy, i.e.,

$$\hbar\omega > 2\mu H_{eff}, \quad H_{eff} = H_0 + \beta M_0$$

when it is assumed that the magnetic field  $H_0$  is in the direction of easiest magnetization.

For  $\hbar\omega < 2\mu H_{\text{eff}}$  (when the external field frequency can satisfy the condition  $\omega \gg 1/\tau_{\text{SS}}$ ) the absorption of the magnetic field energy is mainly associated with inelastic scattering of photons by spin waves. This process occurs in the second perturbation approximation and its probability is very small. When  $\omega\tau_{\text{SS}} \gg 1$  energy is, of course, also absorbed through the mechanisms considered in the first section.

However, for  $\hbar\omega \geq 2\mu H_{\text{eff}}$  increased absorption of magnetic field energy should be observed because of the new mechanism. We shall hereinafter consider frequencies greater than  $2\mu H_{\text{eff}}/\hbar$ .

Thus

$$\mathcal{H}_{\text{int}} = i\mu \sqrt{2\pi\hbar\omega V} \sum_{\mathbf{k}, \mathbf{k}'} v_{\mathbf{k}} u_{\mathbf{k}'}^* a_{\mathbf{k}}^* a_{\mathbf{k}'} q_{\omega} + \text{compl. conj.} \quad (28)$$

Non-zero matrix elements of the Hamiltonian (28) correspond to transitions  $n_{\mathbf{k}}, n_{\mathbf{k}'}, n_{\omega} \rightarrow n_{\mathbf{k}+1}, n_{\mathbf{k}'+1}, n_{\omega}-1$  and the inverse transitions, and give the transition probability

$$4\pi^2 V \omega \mu^2 |v_{\mathbf{k}} u_{\mathbf{k}'} + v_{\mathbf{k}'} u_{\mathbf{k}}|^2 (n_{\mathbf{k}}+1)(n_{\mathbf{k}'}+1) n_{\omega} \delta(\epsilon_{\mathbf{k}} + \epsilon_{\mathbf{k}'} - \hbar\omega);$$

the  $\delta$  function insures conservation of energy.

The photon absorption coefficient is defined as the difference between the probabilities of all photon absorption and emission processes:

$$\Gamma = \sum_{\mathbf{k}, \mathbf{k}'} 4\pi^2 V \omega \mu^2 \{ (n_{\mathbf{k}}+1)(n_{\mathbf{k}'}+1) - n_{\mathbf{k}} n_{\mathbf{k}'} \} \times |v_{\mathbf{k}} u_{\mathbf{k}'} + u_{\mathbf{k}} v_{\mathbf{k}'}|^2 \delta(\epsilon_{\mathbf{k}} + \epsilon_{\mathbf{k}'} - \hbar\omega).$$

Here  $n_{\mathbf{k}}, n_{\mathbf{k}'}$  are understood to be equilibrium Bose functions. The summation over  $\mathbf{k}$  and  $\mathbf{k}'$  in the last equation is subject to the momentum conservation expressed by (27).

Since the wave vector number of a spin wave is considerably greater than that of a photon,\* we have

$$J(\nu, \eta) \approx \begin{cases} 16/15 \sqrt{\nu}; & \nu \gg 1; \\ \frac{2}{3} \sqrt{\eta} \left\{ 1 + \frac{3}{2} \frac{\nu - \sqrt{2\eta}}{\sqrt{2\eta}} \ln \frac{\sqrt{2\eta}}{\nu - \sqrt{2\eta}} \right\}, & 0 < \frac{\nu - \sqrt{2\eta}}{\sqrt{2\eta}} \ll 1; \\ \frac{2}{3} \sqrt{\eta} \left\{ 1 - \frac{3}{2} \frac{\sqrt{2\eta} - \nu}{\sqrt{2\eta}} \ln \frac{\sqrt{2\eta}}{\sqrt{2\eta} - \nu} \right\}, & 0 < \frac{\sqrt{2\eta} - \nu}{\sqrt{2\eta}} \ll 1; \\ (\pi/8 \sqrt{2}) \sqrt{\eta} (\nu - \eta)^2, & \nu \geq \eta. \end{cases}$$

Substituting our results into (29), we can determine the frequency dependence of the photon absorption coefficient in all limiting cases. At high frequencies we have

\* $k \sim (\hbar\omega/\Theta_c)^{1/2}/a$  is comparable with the wave vector  $q$  only at a frequency that satisfies the condition  $\omega \sim c^2 \hbar/a^2 \Theta_c \sim 10^{22} \text{ sec}^{-1}$ . We of course always have  $\omega \ll \hbar c^2/\Theta_c a^2$ .

$$\Gamma = 4\pi^2 V \omega \mu^2 \sum_{\mathbf{k}} (n_{\mathbf{k}} + n_{-\mathbf{k}} + 1)$$

$$\times |v_{\mathbf{k}} u_{-\mathbf{k}} + u_{\mathbf{k}} v_{-\mathbf{k}}|^2 \delta(\epsilon_{\mathbf{k}} + \epsilon_{-\mathbf{k}} - \hbar\omega).$$

Since  $\epsilon_{\mathbf{k}}, u_{\mathbf{k}}$  and  $v_{\mathbf{k}}$  are even functions of the wave vector the last equation leads to

$$\Gamma = (2\pi)^{-3} 32\pi^2 V^2 \omega \mu^2 \int |v_{\mathbf{k}} u_{\mathbf{k}}|^2 \left( n_{\mathbf{k}} + \frac{1}{2} \right) \delta(2\epsilon_{\mathbf{k}} - \hbar\omega) d\tau_{\mathbf{k}}.$$

By using (12) - (20) of reference 2 we therefore obtain

$$\Gamma = \frac{\mu^2 (2\pi\mu M_0)^{1/2}}{2\Theta_c^{1/2} a^3 \hbar} \coth \frac{\hbar\omega}{2T} J(\nu, \eta),$$

$$\nu = \hbar\omega/4\pi\mu M_0, \quad \eta = H_{\text{eff}}/2\pi M_0. \quad (29)$$

Here

$$J(\nu, \eta) = \int_0^1 \frac{x^2 [Vx^2 + \nu^2 - (x + \eta)]^{1/2}}{V(1-x)(x^2 + \nu^2)} dx, \quad (30)$$

when  $\nu > \sqrt{\eta^2 + 2\eta}$  and

$$J(\nu, \eta) = \int_0^{(\nu^2 - \eta^2)/2\eta} \frac{x^2 [Vx^2 + \nu^2 - (x + \eta)]^{1/2}}{V(1-x)(x^2 + \nu^2)} dx, \quad (31)$$

when  $\nu < \sqrt{\eta^2 + 2\eta}$ .

We note that the condition  $\hbar\omega/2 > \mu H_{\text{eff}}$  gives  $\nu > \eta$ . When  $\nu < \eta$  ( $\hbar\omega < 2\mu H_{\text{eff}}$ ) we have  $\Gamma = 0$  in this approximation. We shall calculate the integrals (30) and (31) in different limiting cases.

1.  $\eta \gg 1$  (strong magnetic field or large anisotropy energy):\*

$$J(\nu, \eta) \approx \begin{cases} 16/15 \sqrt{\nu}, & \nu \gg \eta; \\ \frac{1}{3\eta} \left\{ 1 - \frac{5}{4\eta} + \frac{3}{2} (\eta + 1 - \nu) \ln(\nu - \eta - 1) \right\}, & \nu \geq \eta + 1; \\ \frac{1}{3\eta} \left\{ 1 - \frac{5}{4\eta} + \frac{3}{2} (\eta + 1 - \nu) \ln(\eta + 1 - \nu) \right\}, & \nu \leq \eta + 1; \\ 16(\nu - \eta)^3/105\eta, & \nu \geq \eta. \end{cases}$$

2.  $\eta \ll 1$  (weak magnetic field and small anisotropy energy):†

$$\Gamma = \frac{16}{15} \frac{\mu^2}{a^3 \hbar} \left( \frac{2\pi\mu M_0}{\Theta_c} \right)^{1/2} \left( \frac{2\pi\mu M_0}{\hbar\omega} \right)^{1/2} \coth \frac{\hbar\omega}{2T} (\hbar\omega \geq 2\pi\mu M_0, \mu H). \quad (32)$$

\*We note that in this case  $\sqrt{\eta^2 + 2\eta} \approx \eta + 1$  and  $\nu$  is also considerably greater than unity since  $\nu > \eta$ .

†In this case  $\sqrt{\eta^2 + 2\eta} \approx \sqrt{2\eta}$ , and  $\nu$  can be either greater or smaller than unity.

We note that for  $\hbar\omega/2 = \sqrt{(\mu H)^2 + 4\pi\mu^2 M_0 H}$  the derivative of the absorption coefficient with respect to frequency becomes infinite.

The attenuation coefficient of the magnetic field is obviously equal to half the photon absorption coefficient ( $\delta = \Gamma/2$ ). The imaginary part  $\mu''$  of the magnetic susceptibility is thus easily obtained by means of the dispersion equation for an electromagnetic wave with  $\omega = qc/\sqrt{\epsilon\mu_{||}}$ .

Since the magnetic field is parallel to the magnetic moment there is no need for taking the gyrotropy of the magnetic susceptibility tensor into account. Assuming  $\omega = \omega_0 - i\delta$ ,  $\mu_{||} = \mu'_{||} + i\mu''_{||}$ ,  $\epsilon \sim \mu' \sim 1$ , we obtain  $\mu'' = 2\delta/\omega$ , i.e.,  $\mu'' = \Gamma/\omega$ . Thus (7), (15) – (17), (29), and (32) solve our problem of determining the imaginary part of the longitudinal magnetic susceptibility for a ferromagnetic dielectric over a broad range of temperatures and frequencies.

In the limit  $\omega \rightarrow \infty$  the magnetic susceptibility approaches unity, while its imaginary part approaches zero. The frequency dependence of  $\mu''$  at very high frequencies is\*

$$\mu'' \approx \frac{8V\sqrt{2}\mu^2}{15a^3} \frac{(2\pi\mu M_0)^2}{\Theta_c^{7/2}(\hbar\omega)^{3/2}} \quad (\hbar\omega \gg \frac{\hbar}{\tau_{ss}}, T, \mu H_{eff}).$$

Thus the decrease of  $\mu''$  at higher frequencies is inversely proportional to  $\omega^{3/2}$  and is independent of temperature.

We are not aware of any sufficiently thorough experiments (with a fully magnetized sample† at

\*Our treatment is, of course, subject to the high frequency limit  $\omega \ll \Theta_c/\hbar$ , since we have not taken into account the excitation of vibrations of sublattices with respect to each other (see the Introduction).

†In a polydomain sample most absorption of the magnetic field will obviously be accounted for by the motion of domain boundaries.

low temperatures) with which our results might be compared. Our entire discussion shows that such experimental investigations could greatly extend our knowledge of the character of interactions between different quasi-particles (spin waves and phonons) in a ferromagnetic dielectric.

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