THERMOELECTRIC COEFFICIENTS OF METALS IN STRONG MAGNETIC FIELDS AND THE EFFECT OF DRAG OF ELECTRONS BY PHONONS

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The behavior of the thermoelectric tensor in strong magnetic fields, when the electron Larmor frequency is greater than the collision frequency, is considered by the methods proposed by Lifshitz, Azbel', and Kaganov.[1-3] Drag of the electrons by phonons is taken into account, and it is shown that this latter effect significantly changes the asymptotic values of the tensor (for large values of the magnetic field) and also its dependence on the direction of the magnetic field relative to the crystal axes (in the case of a complex topology of the Fermi surface).

THE asymptotic behavior of the thermoelectric tensor in strong magnetic fields was studied by I. M. Lifshitz and Peschanskiĭ.[4] However, they did not take into account the phenomenon of the drag on electrons by phonons. The aim of the present work is the consideration of this latter effect. We shall consider the region of low temperatures, where $T \ll \Theta$ (Θ is the Debye temperature and T the temperature of the specimen).

1. THE KINETIC EQUATIONS FOR ELECTRONS AND PHONONS

The linearized kinetic equations for the electron and phonon distribution functions, $n(\mathbf{p})$ and $N(\mathbf{q})$, in the presence of a temperature gradient, a magnetic field directed along the z axis, and a chemical potential gradient $\nabla\mu$, have the form

$$\frac{\partial(n-n_0)}{\partial t} = -\frac{\partial(n-n_0)}{\partial\tau}\omega^* - t_{ed}^{-1}\hat{W}_{ed}(n-n_0) + x\frac{\partial n_0}{\partial x}\frac{\mathbf{v}\nabla T}{T} + \frac{\partial n_0}{\partial x}\frac{\mathbf{v}\nabla\mu}{T} + \int\frac{d^3q}{(2\pi\hbar)^3}V_{\mathbf{p},\mathbf{p}+\mathbf{q}}\{[(N_\mathbf{q}+1)(1-n_\mathbf{p})n_{\mathbf{p}+\mathbf{q}} - n_\mathbf{p} \times (1-n_{\mathbf{p}+\mathbf{q}})N_\mathbf{q}]\,\delta(\varepsilon_{\mathbf{p}+\mathbf{q}}-\varepsilon_\mathbf{p}-\hbar\omega_\mathbf{q}) + [n_{\mathbf{p}+\mathbf{q}}(1-n_\mathbf{p})N_{-\mathbf{q}} - n_\mathbf{p}(1-n_{\mathbf{p}+\mathbf{q}})(N_{-\mathbf{q}}+1)]\,\delta(\varepsilon_{\mathbf{p}+\mathbf{q}}-\varepsilon_\mathbf{p}+\hbar\omega_\mathbf{q})\};$$

$$\frac{\partial(N-N_0)}{\partial t} = -(N-N_0)t_{fd}^{-1} + \xi\frac{\partial N_0}{\partial\xi}\frac{\mathbf{s}\nabla T}{T} + \int\frac{2d^3p}{(2\pi\hbar)^3}V_{\mathbf{p},\mathbf{p}+\mathbf{q}} \times [n_{\mathbf{p}+\mathbf{q}}(1-n_\mathbf{p})(N_\mathbf{q}+1) - n_\mathbf{p}(1-n_{\mathbf{p}+\mathbf{q}})N_\mathbf{q}] \times \delta(\varepsilon_{\mathbf{p}+\mathbf{q}}-\varepsilon_\mathbf{p}-\hbar\omega_\mathbf{q}).$$

By the substitutions

$$n(\mathbf{p}) = n_0(\varepsilon - \mathbf{p}\mathbf{v}_e(\mathbf{p}))\text{ and }N(\mathbf{q}) = N_0(\hbar\omega_\mathbf{q} - \mathbf{q}\mathbf{v}_f(\mathbf{q}))$$

and for small $\mathbf{v}_e$ and $\mathbf{v}_f$, these equations take the form

$$\frac{\partial}{\partial t}\left(\frac{\mathbf{p}\mathbf{v}_e}{T}\right) = -\frac{\partial}{\partial\tau}\left(\frac{\mathbf{p}\mathbf{v}_e}{T}\right)\omega^* - x\frac{\mathbf{v}\nabla T}{T} - \frac{\mathbf{v}\nabla\mu}{T} - t_{ed}^{-1}\hat{W}_{ed}\left(\frac{\mathbf{p}\mathbf{v}_e}{T}\right) + \int\frac{d^3q}{(2\pi\hbar)^3}V_{\mathbf{p},\mathbf{p}+\mathbf{q}}\left\{\frac{\mathbf{v}_e(\mathbf{p})-\mathbf{v}_f(\mathbf{q})}{T}\mathbf{q}\left[\frac{n(x+\xi)}{n(x)}(N(\xi)+1) + \frac{n(x-\xi)}{n(x)}N(\xi)\right]\delta(\varepsilon_{\mathbf{p}+\mathbf{q}}-\varepsilon_p) + \frac{\mathbf{v}_e(\mathbf{p}+\mathbf{q})-\mathbf{v}_e(\mathbf{p})}{T}(\mathbf{p}+\mathbf{q}) \times \left[\frac{n(x+\xi)}{n(x)}(N(\xi)+1)\,\delta(\varepsilon_{\mathbf{p}+\mathbf{q}}-\varepsilon_p-\hbar\omega_\mathbf{q}) + \frac{n(x-\xi)}{n(x)}N(\xi) \times \delta(\varepsilon_{\mathbf{p}+\mathbf{q}}-\varepsilon_p+\hbar\omega_\mathbf{q})\right]\right\};$$

$$\frac{\partial}{\partial t}\left(\frac{\mathbf{q}\mathbf{v}_f}{T}\right) = \int\frac{2d^3p}{(2\pi\hbar)^3}\left\{\frac{\mathbf{v}_e(\mathbf{p})\mathbf{p}}{T}\left[\frac{n(x)(1-n(x-\xi))}{N(\xi)} - \frac{n(x+\xi)(1-n(x))}{N(\xi)}\right] - \frac{\mathbf{v}_f(\mathbf{q})\mathbf{q}}{T}\frac{n(x+\xi)(1-n(x))}{N(\xi)}\right\} \times V_{\mathbf{p},\mathbf{p}+\mathbf{q}}\,\delta(\varepsilon_{p+\mathbf{q}}-\varepsilon_p) - t_{fd}^{-1}\frac{\mathbf{q}\mathbf{v}_f}{T} - \xi\frac{\mathbf{s}\nabla T}{T}. \quad (1)$$

Here τ is a quantity defining the phase of the electron in the Larmor orbit; $\omega^* = 2\pi/T_0$, where T_0 is the characteristic time for motion around the orbit, after which the momentum of the electron changes by a quantity of the order of the period of the reciprocal lattice or (in the case of a closed trajectory) returns to its initial value (see references 2 and 3), $\mathbf{p}$ is the quasi-momentum of the electron, $\mathbf{v} = \partial\epsilon/\partial\mathbf{p}$ is its velocity, $\mathbf{q}$ is the quasi-momentum of the phonon, $\mathbf{s} = \partial(\hbar\omega)/\partial\mathbf{q}$ is its group velocity, $\hat{W}_{ed} \sim 1$ and t_{ed} are the dimensionless collision operator and the characteristic relaxation time of electrons on lattice defects and on each other. The latter is of the order $\hbar E/T^2$, where E is an energy of atomic order[8] and can be shown to be important at low temperatures for sufficiently pure metals; t_{fd} is the relaxation time of phonons on lattice defects and on each other; $\xi = \hbar\omega_q/T$; $x = (\epsilon_p - \mu)/T$; T is the temperature in energy units. In the latter system, and in what follows, $n(x)$ and $N(\xi)$ denote the equilibrium distributions for electrons and phonons. When $\mathbf{v}_e$ does not appear under the integral sign, the inelasticity is taken into account in $n(x \pm \xi)$, and is neglected in the δ-function, which is valid. The vectors $\mathbf{v}_e(\mathbf{p})$ and $\mathbf{v}_f(\mathbf{p})$ have the meaning of drift velocities of electrons and phonons, respectively. Their interaction tends to equalize these velocities. In this case, the drag on the electrons by the phonons is expressed by an integral containing $(\mathbf{v}_e - \mathbf{v}_f)$.

If (for its estimate) we consider $\mathbf{v}_e - \mathbf{v}_f$ to be constant, then the remaining integral I, multiplied by T/p, determines the order of magnitude of the reciprocal of the relaxation time for scattering of electrons on phonons in terms of the momentum t_5^{-1}.

The law of conservation of energy for $\Theta \gg T \gg ms^2$ gives $\epsilon(\mathbf{p}+\mathbf{q}) - \epsilon(\mathbf{p}) = \mathbf{v}\cdot\mathbf{q} + \frac{1}{2}\sum m_{\alpha\beta}^{-1} \times q_\alpha q_\beta = 0$. Here $m_{\alpha\beta}^{-1} = \partial^2\epsilon/\partial p_\alpha \partial p_\beta$. We set $\mathbf{q} = \mathbf{q}_\perp + q_\parallel \mathbf{v}/v$, where $\mathbf{q}_\perp \cdot \mathbf{v} = 0$. Since $q \ll mv$, then, in first approximation, $q_\parallel = 0$, and in second, $q_\parallel = -\sum' m_{\alpha\beta}^{-1} q_\alpha q_\beta / 2v$, where the primes on the sums indicate that the directions of α and β are orthogonal to $\mathbf{v}$. After integration over $q_\parallel$ in I, we get an integral of the form

$$\int d^2 q_\perp (\mathbf{q}_\perp + q_\parallel \mathbf{v}/v)\,\Phi(\mathbf{q}). \tag{2}$$

In $\Phi(\mathbf{q})$, we can set $\mathbf{q} = \mathbf{q}_\perp$, and since $\Phi(\mathbf{q}) = \Phi(-\mathbf{q})$, there remains only the integral over the second component. Since $v_{p,p+q} \sim q$, while $q_\parallel \sim q_\perp^2$, then $t_5^{-1} \sim T^5$, i.e., actually, the time of equalization of the drift velocities of the electrons and phonons is connected with momentum relaxations (see references 5 — 7), which is physically understandable, since the drag on the electrons by the phonons takes place as a result of the change in their momentum due to interaction with the phonons. The integral containing the difference $\mathbf{v}_e(\mathbf{p}+\mathbf{q}) - \mathbf{v}_e(\mathbf{p})$ expresses the trend to equalization of the drift velocities of electrons with different momenta by the agency of their interaction with phonons. Since $\epsilon_{p+q} - \epsilon_p = \hbar\omega_q$, while the angular separation of the vectors $\mathbf{p}+\mathbf{q}$ and $\mathbf{p}$ is insignificant, then this is actually the energy relaxation of electrons due to phonons. In a manner similar to the above, it is not difficult to show that the corresponding reciprocal of the relaxation time is $t_3^{-1} \sim T^3$.

The drift velocity of the phonon can be eliminated from (1) in the stationary case. Inasmuch as we are interested here in the case in which the electron current $j = 0$, we can neglect the integral term containing $\mathbf{v}_e$ in the phonon equation. It can be shown that consideration of this term in the calculation of the electric conductivity leads to the "renormalization" of the relaxation time, so that the effective relaxation time of the electrons is

$$\tau_e^{-1} = t_e^{-1} - \alpha t_5^{-1} t_f t_{fe}^{-1},$$

where $t_f^{-1} = t_{fd}^{-1} + t_{fe}^{-1}, \quad t_e^{-1} = t_{ed}^{-1} + t_{ef}^{-1}$;

t_{ef} is the relaxation time of electrons on phonons, t_{fe} is the relaxation time of phonons on electrons, determined by the integral for $\mathbf{v}_f \cdot \mathbf{q}/T$ in the phonon equation (1), and α is a numerical coefficient. If the metal is "dirty," i.e., if $t_f \ll t_{fe}$, or $t_e \ll t_{ef}$, then $\tau_e = t_e$. If the metal is "clean," then τ_e has the same temperature dependence as $t_e(t_{ef})$, and differs only numerically from it.

Physically, the latter effect represents the change in the scattering of electrons by phonons as a result of the fact that the drift velocity of the latter increases because of the presence of drift in the electrons ("mutual" drag).

Turning to our case, we get for $\mathbf{v}_e$ the equations

$$\frac{\partial}{\partial\tau}\left(\frac{\mathbf{p}\mathbf{v}_e}{T}\right)\omega^* + t_0^{-1}\hat{W}\left\{\frac{\mathbf{v}_e\mathbf{p}}{T}\right\} = -\frac{\mathbf{v}\nabla\mu}{T} - x\frac{\mathbf{v}\nabla T}{T} - \frac{\nabla T}{T}\mathbf{F}; \tag{3}$$

$$\mathbf{F} = \int \frac{d^3q}{(2\pi\hbar)^3} t_f(\mathbf{q})\,\xi s V_{\mathbf{p},\mathbf{p}+\mathbf{q}} \left[\frac{n(x+\xi)}{n(x)}(N(\xi)+1) + \frac{n(x-\xi)}{n(x)}N(\xi)\right]\delta(\varepsilon_{p+q} - \varepsilon_p). \tag{4}$$

Here

$$t_0^{-1}\hat{W} = t_{ed}^{-1}\hat{W}_{ed} + t_{ef}^{-1}\hat{W}_{ef}.$$

We can get the order of the quantity F by equating the integral for it to the integral determining the relaxation time t_5:

$$F \sim \frac{p}{q} t_f s t_5^{-1} \sim s\frac{\Theta}{T} t_f t_5^{-1}. \tag{5}$$

2. SOLUTION OF THE KINETIC EQUATION IN THE CASE OF SCATTERING OF THE ELECTRONS BY LATTICE DEFECTS OR BY EACH OTHER

We must solve the equation

$$\frac{\partial}{\partial\tau}\left(\frac{\mathbf{p}\mathbf{v}_e}{T}\right)\omega^* + t_0^{-1}\hat{W}\left(\frac{\mathbf{p}\mathbf{v}_e}{T}\right) = -x\frac{\mathbf{v}\nabla T}{T} - \frac{\nabla T}{T}\mathbf{F}. \tag{6}$$

Solving it, we can find the current $j_i^{(T)} = \sum_k \beta_{ik}\nabla_k T$; here, the desired thermoelectric tensor α_{ik}, which is determined from the relation $\nabla_i\mu = -e\sum_k \alpha_{ik}\nabla_k T$ for the condition $j = 0$, is

$$\alpha_{ik} = -\sum_l \sigma_{il}\beta_{lk}.$$

The tensor σ_{ik} is known (see references 2 and 3); therefore it is necessary to find the tensor β_{ik}.

We shall seek a solution of Eq. (6) in the form

$$\mathbf{p}v_c/T = -t_0 \sum_i \psi_i \nabla_i T/T.$$

For ψ_i, we obtain the equation

$$\frac{\partial \psi_i}{\partial \tau} + \gamma_0 \hat{W}(\psi_i) = \gamma_0 x v_i + \gamma_0 F_i. \qquad (6')$$

Here $\gamma_0 = 1/\omega^* t_0 \ll 1$. To each of the two terms of inhomogeneity in Eq. (6′), we can juxtapose ψ_{i1}, and ψ_{i2}, respectively.

In accord with this, we also have

$$\beta_{ik} = \beta_{ik}^{(1)} + \beta_{ik}^{(2)}, \qquad \alpha_{ik} = \alpha_{ik}^{(1)} + \alpha_{ik}^{(2)}.$$

In this case, quantities with index (1) are defined directly by the effect of the temperature gradient on the electron, and those with index (2) by the drag of electrons by phonons. The asymptotic value of the tensor $\alpha^{(1)}$ was found in reference 4. If $\alpha_{ik} = \gamma_0^{p_{ik}} a_{ik}$, then the order of the $a_{ik}^{(1,2)}$ can easily be estimated.* Let

$$\beta_{ik}^{(1)} = \gamma_0^{r_{ik}} b_{ik}^{(1)}, \qquad \sigma_{ik} = \gamma_0^{s_{ik}} c_{ik};$$

Then $a^{(1)} \sim b^{(1)}/ec$. But $b^{(1)}/c$ is of the order of the ratio of the right sides of the kinetic equations determining $\beta^{(1)}$ and σ, multiplied by T/μ (in view of the presence of the factor x in the inhomogeneity of the equation for $\beta^{(1)}$) so that

$$a_{ik}^{(1)} \sim T/e\mu. \qquad (7)$$

Equating the same two terms in the inhomogeneity of the equation and taking (5) into account, and also the fact that F_i is an (almost) even function of x, we find

$$a^{(2)} \sim a^{(1)} \frac{s}{v} \frac{\Theta}{T} \frac{t_f}{t_5} \frac{\mu}{T} \sim a^{(1)} \left(\frac{\Theta}{T}\right)^2 \frac{t_f}{t_5}, \qquad (8)$$

since $s/v = (\hbar s/a)\, a/\hbar v \approx \Theta/\mu$ (a is the lattice constant). If the phonons are scattered by the electrons, then $t_f \sim (\hbar/\Theta)\mu/T$, $t_5 \sim (\hbar/\Theta)(\Theta/T)^5$ (see references 5 — 7), so that,

$$a^{(2)} \sim a^{(1)} (T/\Theta)^2 \mu/\Theta. \qquad (8')$$

Therefore, for not very low temperatures, i.e.,

$$\Theta > T > T_\alpha \sim \Theta (\Theta/\mu)^{1/2} \qquad (9)$$

the drag effect predominates even in those elements of the tensor α_{ik} in which the asymptotic values of $\alpha_{ik}^{(1)}$ and $\alpha_{ik}^{(2)}$ are identical in γ_0. But, as will be seen below, the asymptotic value (in γ_0) of $\alpha_{ik}^{(2)}$ is always smaller than the asymptotic value of $\alpha_{ik}^{(1)}$. In what follows, only $\beta_{ik}^{(2)}$ and $\alpha_{ik}^{(2)}$ are computed. Therefore the index 2 is omitted for brevity.

For calculation of β_{ik}, it is necessary to solve

$$\frac{\partial \psi_i}{\partial \tau} + \gamma_0 \hat{W}(\psi_i) = \gamma_0 F_i, \qquad (6'')$$

$$\beta_{ik} = \lim_{G\to\infty} \frac{2et_0}{(2\pi\hbar)^3} \frac{1}{G} \iint d\varepsilon\, dp_z \frac{\partial n}{\partial \varepsilon} \frac{T_0 eH}{c} \int v_i \psi_k d\tau. \qquad (10)$$

Integration is carried out over all momentum space; G is the number of cells included in the limits of integration. As in references 2 and 3, it is necessary to treat the following cases separately ($|\epsilon - \mu| \sim T$):

1. The trajectories $\epsilon = \text{const}$, $p_z = \text{const}$ are closed and lie within the limits of a single cell of the reciprocal lattice.
2. There is a layer of open trajectories.
3. The approximation to "critical" directions (see reference 3).

1. Closed Trajectories Lying Within the Limits of a Single Cell

As in reference 2, we seek ψ_i in the form

$$\psi_i = \sum_{n=0}^{\infty} \gamma_0^n \psi_i^{(n)}; \qquad (11)$$

We then obtain the set of recurrent equations

$$\partial\psi_i^{(0)}/\partial\tau = 0, \quad \partial\psi_i^{(1)}/\partial\tau + \hat{W}(\psi_i^{(0)}) = F_i,$$
$$\partial\psi_i^{(n)}/\partial\tau + \hat{W}(\psi_i^{(n-1)}) = 0 \quad \text{при } n \geqslant 2. \qquad (12)$$

The periodicity in τ serves as an additional condition on these equations, i.e.,

$$\oint \hat{W}(\psi_i^{(0)})\, d\tau = \oint F_i d\tau, \qquad \oint \hat{W}(\psi_i^{(n)})\, d\tau = 0, \; n \geqslant 2. \qquad (13)$$

Integration is carried out over τ along the closed trajectory. Since all $\oint F_i d\tau \neq 0$, then all

$$\psi_i^{(0)} = C_i^{(0)}(\varepsilon, p_z) \neq 0.$$

We also take it into account that $\oint v_{x,y} d\tau \sim \oint dp_{y,x} = 0$; then the tensor in this case generally has the form

$$\beta_{ik} = \begin{vmatrix} \gamma_0 b_{xx} & \gamma_0 b_{xy} & \gamma_0 b_{xz} \\ \gamma_0 b_{yx} & \gamma_0 b_{yy} & \gamma_0 b_{yz} \\ b_{zx} & b_{zy} & b_{zz} \end{vmatrix}. \qquad (14)$$

The symmetry properties of the crystal can change the asymptotic value of the tensor β. Thus, for example, if there is mirror symmetry relative to the plane zx, then, inasmuch as both v_i and F_i are transformed in the symmetry transformation as p_i, and the operator $\partial/\partial\tau \sim v_y \partial/\partial p_x$ is odd, while $\hat{W}$

*As in references 1–3, we denote by γ_0 the small ratio $\gamma_0 = H_0/H$, where H_0 is a characteristic value of the magnetic field (for which the Larmor period is of the order of the relaxation time t_0).

is even relative to the transformation p_y, $p'_y \to -p_y$, $-p'_y$, the equation for ψ_i can be written in the form

$$\hat{L}(\psi_i) = \gamma_0 \hat{F}_i, \text{ where } L_{p_y, p'_y}(\mathbf{H}) = -L_{-p_y, -p'_y}(-\mathbf{H}),$$

i.e.,

$$\psi_{x,z}(p_y, \mathbf{H}) = \psi_{x,z}(-p_y, -\mathbf{H}),$$

$$\psi_y(p_y, \mathbf{H}) = -\psi_y(-p_y, -\mathbf{H}).$$

Therefore β_{xz}, β_{zx}, β_{xx}, β_{yy}, β_{zz} are even, and β_{xy}, β_{yx}, β_{zy}, β_{yz} are odd functions of $\mathbf{H}$, so that $\beta_{xz} \sim \gamma_0^2$; $\beta_{zx} \sim \gamma_0^2$; $\beta_{xx} \sim \gamma_0^2$; $\beta_{yy} \sim \gamma_0^2$; $\beta_{zy} \sim \gamma_0^2$.

This result is found to be in agreement with the requirements imposed in the macroscopic theory on the tensor β_{ik}. Actually, if we write

$$\beta_{ik} = c_{ik} + c_{kllm} H_{lm} + c_{iklml'm'} H_{lm} H_{l'm'},$$

where H_{lm} is an antisymmetric tensor, the dual to the pseudovector $\mathbf{H}$, and c is a tensor of the corresponding rank, and if we consider that, when $\mathbf{H} = (0, 0, H)$, only $H_{xy} = -H_{yx} \neq 0$, then it is easy to see, from the presence of mirror symmetry relative to the plane zx, that the components of the tensor β_{ik} which contain an even number of signs ik equal to y are even functions of $\mathbf{H}$, and vice versa.

Returning to the general case, we have (making use of the expression for σ^{-1} from reference 3):

$$\alpha_{ik} = \begin{vmatrix} a_{xx} & a_{xy} & a_{xz} \\ a_{yx} & a_{yy} & a_{yz} \\ a_{zx} & a_{zy} & a_{zz} \end{vmatrix}. \quad (15)$$

The case in which the Fermi surface is closed and the number of electrons is equal to the number of holes should be specially noted. In such a case (see reference 2), the asymptotic value of the tensor σ is such that

$$\alpha_{ik} = \begin{vmatrix} \gamma_0^{-1} a_{xx} & \gamma_0^{-1} a_{xy} & \gamma_0^{-1} a_{xz} \\ \gamma_0^{-1} a_{yx} & \gamma_0^{-1} a_{yy} & \gamma_0^{-1} a_{yz} \\ a_{zx} & a_{zy} & a_{zz} \end{vmatrix}. \quad (16)$$

2. Open Trajectories

Equations (12) must be solved in this case under the additional condition of finiteness, i.e.,

$$\overline{\hat{W}(\psi_i)} = \overline{F}_i, \text{ where } \bar{f} = \lim_{\tau\to\infty} \frac{1}{2\tau} \int_{-\tau}^{+\tau} f d\tau$$

and integration is carried out over the entire trajectory. This means that

$$\overline{\hat{W}(\psi_i^{(0)})} = \overline{F}_i, \qquad \overline{\hat{W}(\psi_i^{(n)})} = 0 \quad \text{for } n \geqslant 1.$$

Generally, all $\overline{F}_i \neq 0$, so that all $C_i^{(0)}(\epsilon, p_z) \neq 0$. However, if all trajectories have infinite extension in one direction, chosen to be the x axis, then

$$\bar{v}_x \sim \lim_{\tau\to\infty} \frac{1}{2\tau} \int dp_y = 0,$$

since p_y changes within finite limits. Consequently, in this case,

$$\beta_{ik} = \begin{vmatrix} \gamma_0 b_{xx} & \gamma_0 b_{xy} & \gamma_0 b_{xz} \\ b_{yx} & b_{yy} & b_{yz} \\ b_{zx} & b_{zy} & b_{zz} \end{vmatrix}. \quad (17)$$

The expressions for the α_{ik} have the form

$$\alpha_{ik} = \begin{vmatrix} \gamma_0^{-1} a_{xx} & \gamma_0^{-1} a_{xy} & \gamma_0^{-1} a_{xz} \\ a_{yx} & a_{yy} & a_{yz} \\ a_{zx} & a_{zy} & a_{zz} \end{vmatrix}. \quad (18)$$

3. Approximation to "Critical" Directions

Let us consider the case here in accord with the classification given by I. M. Lifshitz and Peschanskiĭ.[3] For simplicity of calculation, we set $\hat{W} = 1$.

a) Approximation to an isolated direction, in which a layer of open trajectories appears. In this approximation, there are greatly extended trajectories whose period of motion is $\tau_0 \sim T_0/\vartheta$, where ϑ is the angle between the direction of the magnetic field and the direction in which the open trajectories occur.

The equation for ψ_i has the form

$$\partial\psi_i/\partial\tau + \gamma\psi_i = \gamma F_i, \qquad \gamma = \tau_0/t_0 \sim \gamma_0/\vartheta. \quad (19)$$

Applying the Fourier method (see reference 2), we obtain

$$\beta_{ik} = -\frac{2et_0}{(2\pi\hbar)^3} \lim_{G\to\infty} \frac{1}{G} \iint d\varepsilon dp_z \frac{\partial n}{\partial x} \frac{eH\tau_0}{c} \Big\{ v_i^{(0)} F_k^{(0)} + \gamma \sum_{n=1}^{\infty} \frac{\gamma\left(v_i^{(n)} F_k^{(-n)} + v_i^{(-n)} F_k^{(n)}\right) + in\left(v_i^{(n)} F_k^{(-n)} - v_i^{(-n)} F_k^{(n)}\right)}{n^2 + \gamma^2} \Big\}. \quad (20)$$

Here, $v_i^{(n)}$, $F_k^{(n)}$ are the Fourier components of the corresponding quantities as functions of τ. Taking it into account that $v_0^{(n)} \sim \vartheta$, while the factor $1/\vartheta$ which comes from τ_0, is compensated by the factor $1/G$, and that $v_x^{(0)} = v_y^{(0)} = 0$, we obtain

$$\beta_{ik} = \begin{vmatrix} \gamma_0 b_{xx} & \gamma_0 b_{xy} & \gamma_0 b_{xz} \\ b_{yx} & b_{yy} & b_{yz} \\ b_{zx} & b_{zy} & b_{zz} \end{vmatrix}. \quad (21)$$

Here, all the b_{ik} are functions of the ratio $\eta = \gamma_0/\vartheta$, while $\beta_{ik}(\infty) = \text{const}$, and as $\eta \to 0$:

$$b_{ik} = \begin{vmatrix} b_{xx}(0) & b_{xy}(0) & b_{xz}(0) \\ \eta b'_{yx} & \eta b'_{yy} & \eta b'_{yz} \\ b_{zx}(0) & b_{zy}(0) & b_{zz} \end{vmatrix}. \quad (22)$$

Hence we obtain for the tensor α_{ik}:

$$\alpha_{ik} = \begin{vmatrix} \gamma_0^{-1} a_{xx} & \gamma_0^{-1} a_{xy} & \gamma_0^{-1} a_{xz} \\ a_{yx} & a_{yy} & a_{yz} \\ a_{xz} & a_{zy} & a_{zz} \end{vmatrix}, \quad (23)$$

where all a_{ik} are functions of η.

As $\eta \to \infty$, all $a_{ik} \to a_{ik}(\infty) = \text{const}$, and as $\eta \to 0$,

$$a_{ik} = \begin{vmatrix} \eta a'_{xx} & \eta a'_{xy} & \eta a'_{xz} \\ a_{yx}(0) & a_{yy}(0) & a_{yz}(0) \\ a_{xz}(0) & a_{zy}(0) & a_{zz}(0) \end{vmatrix} \tag{24}$$

in agreement with Eqs. (15) and (18).

b) Approximation to the direction in which the layer of open trajectories disappears. If we denote by ϑ the angle between the direction of the magnetic field and the critical direction, then, since the contribution to the tensor β_{ik} from the open trajectories $\sim \vartheta$, in this case $\beta_{ik} = g_{ik} + \vartheta d_{ik}$, where the tensor g_{ik} has a structure of the type (14), and the tensor d_{ik} has the type (17). By means of Eq. (29) of reference 3, we get the following expression for α_{ik}:

$$\alpha_{ik} = \begin{vmatrix} a_{xx} + \vartheta\gamma_0^{-1}c_{xx} & a_{xy} + \vartheta\gamma_0^{-1}c_{xy} & a_{xz} + \vartheta\gamma_0^{-1}c_{xz} \\ a_{yx} & a_{yy} & a_{yz} \\ a_{zx} & a_{zy} & a_{zz} \end{vmatrix}. \tag{25}$$

3. CASE OF SCATTERING OF ELECTRONS BY PHONONS

In view of the presence of two relaxation times for electrons relative to phonons, t_{ef} (see Sec. 1), it is necessary to investigate the role of the operators of relaxation for the direction $\hat{W}_5/t_5$ and the energy $\hat{W}_3/t_3$. If we introduce the function $n_1 = -\mathbf{v}_e \cdot \mathbf{p}\partial n_0/\partial\epsilon$, and multiply the electron equation in the system (1) by $\partial n_0/\partial x$, then the term containing $\mathbf{v}_e \cdot (\mathbf{p}+\mathbf{q}) - \mathbf{v}_e(\mathbf{p})$, has the form

$$\int M(x\Omega;\ x'\Omega')\, n_1(x'\Omega')\, dx'd\Omega' - n_1(x\Omega)\int K(x\Omega;\ x''\Omega'')\, dx''d\Omega''.$$

Here Ω is the set of angular coordinates of the momentum $\mathbf{p}$. We write

$$\int M(x\Omega,\ x'\Omega')\, n_1(x'\Omega')\, dx'd\Omega' = \int M(x\Omega,\ x'\Omega')\,[n_1(x'\Omega') - n_1(x'\Omega)] \times dx'd\Omega' + \int dx' \int M(x\Omega,\ x'\Omega')\, n_1(x'\Omega)\, d\Omega'.$$

The first of the integrals can be combined with $\hat{W}_5$, which represents a relaxation in direction, such that

$$t_3^{-1}\hat{W}_3(x\Omega,\ x'\Omega') = \delta(\Omega - \Omega')\left\{\int M(x\Omega,\ x'\Omega'')\, d\Omega'' - \delta(x - x') \times \int K(x\Omega,\ x''\Omega'')\, dx''d\Omega''\right\}.$$

Here,

$$M(x\Omega,\ x'\Omega') = V_{p,p'}\Delta\left\{N_0(x'-x)\frac{n_0(x)}{n_0(x')}\delta(x'-x-\xi_{p'-p}) + N_0(x-x')e^{x-x'}\frac{n_0(x)}{n_0(x')}\delta(x'-x+\xi_{p'-p})\right\},$$

$$K(x\Omega,\ x'\Omega') = V_{pp'}\Delta\left\{N_0(x-x')\frac{n_0(x')}{n_0(x)}\delta(x'-x+\xi_{p'-p}) + N_0(x'-x)e^{x'-x}\frac{n_0(x')}{n_0(x)}\delta(x'-x-\xi_{p'-p})\right\}$$

(Δ = number of states in the interval $dxd\Omega$).

It is easy to establish the fact that, with accuracy up to terms $\sim T/\mu$, we have $K(x\Omega,\ x'\Omega') = M(x'\Omega,\ x\Omega')$, so that

$$t_3^{-1}\hat{W}_3(x\Omega,\ x'\Omega') = \delta(\Omega-\Omega')\left\{L(x,\ x') - \delta(x-x') \times \int L(x'',\ x)\, dx''\right\},$$

$$\iint \hat{W}_3(x\Omega,\ x'\Omega')\, t_3^{-1} f(x')\, dx'dx \equiv 0. \tag{26}$$

This property of the operator $\hat{W}_3$ expresses the fact that a change in the total number of electrons in a solid angle can be brought about only as a result of collisions with a change in momentum.

We set $n_1 = \varphi(p_z,\ \tau)\,\chi(\epsilon)\,\partial n_0/\partial\epsilon$, which is possible in view of the δ-like character of the function $\partial n_0/\partial\epsilon$. The current is

$$j = \frac{2e}{(2\pi\hbar)^3}\iint dp_z d\tau \int \mathbf{v}\varphi(p_z,\ \tau)\,\chi(\varepsilon)\frac{eHT_0}{c}\frac{\partial n_0}{\partial\varepsilon}\, d\varepsilon = \frac{2e}{(2\pi\hbar)^3}\iint dp_z d\tau \frac{eHT_0}{c}\,\mathbf{v}(\mu)\,\varphi(p_z\tau),$$

since $\chi(\epsilon)$ can always be chosen in such a fashion that

$$\int \frac{\partial n_0}{\partial\varepsilon}\chi(\varepsilon)\, d\varepsilon = 1.$$

The kinetic equation for n_1 has the form

$$\frac{\partial n_1}{\partial\tau}\omega^* + t_5^{-1}\hat{W}_5(n_1) + t_3^{-1}\hat{W}_3(n_1) = -R\frac{\partial n_0}{\partial\varepsilon}. \tag{27}$$

If $R(x) = R(-x)$, then the corresponding function $n_1(x) = n_1(-x)$, since $W(x,\ x') = W(-x,\ -x')$ (see reference 6).

Integrating Eq. (27) with respect to ϵ with account of Eq. (26), we obtain an equation for $\varphi(p_z\tau)$:

$$\omega^*\partial\varphi/\partial\tau + \hat{W}'(\varphi)/t_5 = R(0), \qquad W'(p_z\tau;\ p'_z\tau') = \iint dx'dx W_5(xp_z\tau;\ x'p'_z\tau')\,\chi(\varepsilon')\,\partial n_0/\partial\varepsilon'. \tag{28}$$

Consequently, the function φ is determined by the directional relaxation time t_5. If $R(x) = -R(-x)$, then, correspondingly, $n_1(-x) = -n_1(x)$ and is determined from Eq. (27), where it is possible to discard the term $\hat{W}_5/t_5$ since $1/t_5 \ll 1/t_3$.

Inasmuch as the inhomogeneity in the kinetic equation, which determines the electrical conductivity and the drag effect, is an even function of x, then the tensors σ_{ik} and $\beta_{ik}^{(2)}$ contain $\gamma_5 \sim 1/\omega t_5$, and the coefficients in the expansion in powers of γ_5 are proportional to t_5. In the calculation of the tensor $\beta_{ik}^{(1)}$, it is necessary to take it into account that the even part (in x) in the equation determining the tensor is smaller than the odd by the factor T/μ, but in the calculation of the current from the odd part, a factor of the same order of magnitude appears.

We note that in view of the sharp difference between t_3 and t_5, a case can occur in which $t_3 < t_{ed}$ and $t_5 > t_{ed}$. In such a case we must replace t_5 by t_{ed}. Moreover, in certain magnetic fields it can happen that $1/\omega t_3 = \gamma_3 \gg 1$, while $\gamma_5 = (\omega t_5)^{-1} \gg 1$. In such a case, the asymptotic value will evidently be an expansion in γ_5 and γ_3^{-1}. We shall not linger over these cases. Then,

$$\beta_{ik}^{(1)} = t_3 f_{ik}(\gamma_3) + t_5 g_{ik}(\gamma_5).$$

Since $t_5 \gg t_3$, i.e., $\gamma_5 \ll \gamma_3$, then the zero order terms (in γ) are proportional to t_5, and terms of second order $\sim \gamma_3^2 t_3$. The two first-order terms are equal, since $\gamma_3 t_3 = \gamma_5 t_5 = \gamma t$. Therefore, the tensor $\alpha_{ik}^{(2)}$ remains the same as in Sec. 2, if we set $\gamma_0 = \gamma_5$ in all formulas for $\alpha_{ik}^{(2)}$. The tensors $\beta_{ik}^{(1)}$, and correspondingly, the $\alpha_{ik}^{(1)}$, have the following form (see reference 4):

1) The case of close trajectories lying within the limits of a single cell:

$$\beta_{ik}^{(1)} = \begin{vmatrix} \gamma_3^2 t_3 c_{xx} & \gamma t c_{xy} & \gamma t c_{xz} \\ \gamma t c_{yx} & \gamma_3^2 t_3 c_{yy} & \gamma t c_{yz} \\ \gamma t c_{zx} & \gamma t c_{zy} & t_5 c_{zz} \end{vmatrix}. \tag{29}$$

Using σ_{ik} from references 2 and 3 we get

$$\alpha_{ik}^{(1)} = \begin{vmatrix} a_{xx} & \gamma_3 a_{xy} & a_{xz} \\ \gamma_3 a_{yx} & a_{yy} & a_{yz} \\ \gamma_5 a_{zx} & \gamma_5 a_{zy} & a_{zz} \end{vmatrix}. \tag{30}$$

The tensor for $\alpha_{ik}^{(2)}$ has the form (15). In the case of a closed Fermi surface, for an equal number of electrons and holes, the $\alpha_{ik}^{(1,2)}$ have the form (16) if we set $\gamma_0 = \gamma_5$.

2) Open trajectories. In correspondence with the above, we have

$$\beta_{ik}^{(1)} = \begin{vmatrix} \gamma_3^2 t_3 c_{xx} & \gamma t c_{xy} & \gamma t c_{xz} \\ \gamma t c_{yx} & t_5 c_{yy} & t_5 c_{yz} \\ \gamma t c_{zx} & t_5 c_{zy} & t_5 c_{zz} \end{vmatrix}, \tag{31}$$

$$\alpha_{ik}^{(1)} = \begin{vmatrix} \gamma_3 \gamma_5^{-1} a_{xx} & \gamma_5^{-1} a_{xy} & \gamma_5^{-1} a_{xz} \\ \gamma_3 a_{yx} & a_{yy} & a_{yz} \\ \gamma_3 a_{zx} & a_{zy} & a_{zz} \end{vmatrix}, \tag{32}$$

$\alpha_{ik}^{(2)}$ has the form (18) with $\gamma_0 = \gamma_5$.

3) Approximation to the critical directions.

a) Approximation to the direction in which the layer of open trajectories disappears. As above, in Sec. 2, $\beta_{ik}^{(1)} = g_{ik}^{(1)} + \vartheta d_{ik}^{(1)}$, while $g_{ik}^{(1)}$ has the form of (29), and $d_{ik}^{(1)}$ has the form of (31). Taking this into account, we get the following expression for $\alpha_{ik}^{(1)}$:

$$\alpha_{ik}^{(1)} = \begin{vmatrix} a_{xx} + \gamma_3 \gamma_5^{-1} \vartheta c_{xx} & \gamma_3 a_{xy} + \vartheta \gamma_5^{-1} c_{xy} & a_{xz} + \vartheta \gamma_5^{-1} c_{xz} \\ \gamma_3 a_{yx} & a_{yy} & a_{yz} \\ \gamma_5 a_{zx} + \vartheta \gamma_3 c_{zx} & \gamma_5 a_{zy} + \vartheta c_{zy} & a_{zz} \end{vmatrix}; \tag{33}$$

$\alpha_{ik}^{(2)}$ has the form of (25) with $\gamma_0 = \gamma_5$.

b) Approximation to the isolated direction in which a layer of open trajectories arises. If ϑ is the angle between the z axis and the critical direction, while $\eta_3 = \gamma_3/\vartheta$, $\eta_5 = \gamma_5/\vartheta$, then the tensor $\alpha_{ik}^{(2)}$ has a structure of the type (23), (24), if we set $\gamma_0 = \gamma_5$, $\eta = \eta_5$ in the latter equations.

In the approximation of the tensor $\alpha_{ik}^{(1)}$ we consider three regions:

1) $\eta_3 \ll 1$; $\eta_5 \ll 1$. With consideration of the relation between γ_3 and γ_5, we get the expression

$$\beta_{ik}^{(1)} = \begin{vmatrix} \gamma_3^2 t_3 c_{xx} & \gamma t c_{xy} & \gamma t c_{xz} \\ \gamma t c_{yx} & \eta_3^2 t_3 c_{yy} & \eta t c_{yz} \\ \gamma t c_{zx} & \eta t c_{zy} & t_5 c_{zz} \end{vmatrix} \tag{34}$$

for $\beta_{ik}^{(1)}$, and by means of $\sigma_{ik}(\gamma_5, \eta_5)$ from reference 3, we determine $\alpha_{ik}^{(1)}$:

$$\alpha_{ik}^{(1)} = \begin{vmatrix} a_{xx} & \gamma_5^{-1} \eta_3 \eta_5 a_{xy} & \gamma_5^{-1} \eta_5 a_{xz} \\ \gamma_3 a_{yx} & a_{yy} & a_{yz} \\ \gamma_5 a_{zx} & \eta_5 a_{zy} & a_{zz} \end{vmatrix}. \tag{35}$$

2) $\eta_3 \gg 1$; $\eta_5 \ll 1$. In this case,

$$\beta_{ik}^{(1)} = \begin{vmatrix} \gamma_3^2 t_3 c_{xx} & \gamma t c_{xy} & \gamma t c_{xz} \\ \gamma t c_{yx} & t_3 \left(c_{yy} + \eta_3 \eta_5 c'_{yy}\right) & \eta t c_{yz} \\ \gamma t c_{zx} & \gamma t c_{zy} & t_5 c_{zz} \end{vmatrix} \tag{36}$$

and correspondingly,

$$\alpha_{ik}^{(1)} = \begin{vmatrix} a_{xx} + \eta_3 \eta_5 b_{xx} & \gamma_3^{-1} \left(a_{xy} + \eta_3 \eta_5 b_{xy}\right) & \gamma_5^{-1} \eta_5 a_{xz} \\ \gamma_3 a_{yx} & a_{yy} & a_{yz} \\ \gamma_3 \eta_5 a_{zx} & \eta_5 a_{zy} & a_{zz} \end{vmatrix}. \tag{37}$$

3) $\eta_3 \gg 1$; $\eta_5 \gg 1$. Under these conditions,

$$\beta_{ik}^{(1)} = \begin{vmatrix} \gamma_3^2 t_3 c_{xx} & \gamma t c_{xy} & \gamma t c_{xz} \\ \gamma t c_{yx} & t_5 c_{yy} & t_5 c_{yz} \\ \gamma t c_{zx} & t_5 c_{zy} & t_5 c_{zz} \end{vmatrix}, \tag{38}$$

$$\alpha_{ik}^{(1)} = \begin{vmatrix} \gamma_3 \gamma_5^{-1} a_{xx} & \gamma_5^{-1} a_{xy} & \gamma_5^{-1} a_{xz} \\ \gamma_3 a_{yx} & a_{yy} & a_{yz} \\ \gamma_3 a_{zx} & a_{zy} & a_{zz} \end{vmatrix}. \tag{39}$$

As is seen, the elements of the tensor $\alpha_{ik}^{(2)}$ always have an asymptotic value (asymptotic in γ and η) no higher than the asymptotic value of the corresponding elements of the tensor $\alpha_{ik}^{(1)}$. Thus, as has already been pointed out, the coefficients in the expansion in γ and η in $\alpha^{(2)}$, (if the phonons are scattered by the electrons) are of order $(T/\Theta)^3/e$, while in $\alpha^{(1)}$, they are of order $\sim T/e\mu$ [(7), (8′)]; above the temperature T_γ [Eq. (9)] the drag effect predominates, except for the case (37), when $\alpha^{(1)}$ can exceed $\alpha^{(2)}$ [Eqs. (23) and (24)].

In the isotropic model (for this the asymptotic values of $\alpha^{(2)}$ and $\alpha^{(1)}$ coincide for scattering of electrons by other than phonons, expanding $\beta^{(1,2)} = \sum_r \beta_r^{(1,2)}\gamma_0$, we get $\beta_r^{(2)}/\beta_r^{(1)} = (T/T_\gamma^{(r)})^2$, where

$$T_\gamma^{(r)} \approx 5.4 \left| \frac{(r+2)-(1-r)\delta}{(n/n_0)^{1/3}} + \frac{1-r}{(m^*/m_0)(v/v_0)} \right|^{1/2} \times \left(\frac{(s/s_0)^3 n/n_0}{v/v_0} \right)^{1/2} \text{deg};$$

$$m^* = (d^2\varepsilon/dp^2)^{-1}, \qquad \delta = -d(\ln t_0)/d(\ln p).$$

Here n is the concentration of electrons, $m_0 = 10^{-27}$ gm;

$$s_0 = 3\cdot 10^5 \text{ cm-sec}^{-1}, \quad v_0 = 10^8 \text{cm-sec}^{-1}, \quad n_0 = 3\cdot 10^{22} \text{cm}^{-3}.$$

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