NONRESONANCE ABSORPTION OF ELECTROMAGNETIC WAVES IN A MAGNETOACTIVE PLASMA

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The absorption (attenuation) outside the gyromagnetic resonance regions is analyzed from the general equation for three types of high-frequency waves. Both collisions and the specific plasma-absorption mechanisms are taken into account.

In an earlier paper by the present author[1] the propagation of electromagnetic waves in a plasma in a fixed magnetic field was subjected to a kinetic analysis, in which the thermal motion of the electrons was taken into account. A dispersion equation was obtained for the propagation of the three kinds of characteristic waves: ordinary, extraordinary, and plasma. However, absorption was not considered in reference 1. This gap has been filled to some extent in papers by Sitenko, Stepanov, and Tkalich.[2-4] These authors, however, considered only particular cases. Hence it seems desirable to make a more general analysis, taking account of both collisions and the specific plasma attenuation mechanisms which were first treated by Landau.[5] This approach leads to a number of new results (attenuation of the ordinary wave at low frequencies, a more detailed description of collisions, etc.). Furthermore, a number of points in the calculations given in references 1 — 4 are supplemented or refined.

In the present work chief attention is given to absorption outside the gyromagnetic resonance regions. Absorption in these regions will be considered in another paper (together with the evaluation of the absorption coefficients given in the formulas that follow).

1. We start from a linearized system consisting of the kinetic equation and the electrodynamic equations*

$$\frac{\partial f}{\partial t} + \mathbf{v}\nabla_r f - \frac{e}{m}\mathbf{E}\nabla_v f_0 - \frac{e}{mc}[\mathbf{v}\times\mathbf{H}_0]\nabla_v f + \nu f = 0,$$

$$\operatorname{curl}\mathbf{H} = -\frac{4\pi e}{c}\int \mathbf{v} f d\tau_v + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}, \quad \operatorname{curl}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t},$$

$$\operatorname{div}\mathbf{E} = -4\pi e\int f d\tau_v, \quad \operatorname{div}\mathbf{H} = 0, \qquad (1.1)$$

where f_0 is the equilibrium distribution function, f is a small deviation from f_0, e and m are the mass and charge of the electron, and **E** and **H** are the self-consistent electric and magnetic fields. Collisions of electrons with other particles are taken into account directly by introducing the term νf in the first equation in (1.1), where ν is the effective number of collisions of electrons with other particles. We assume that f_0 is Maxwellian and characterized by a temperature T

$$f_0 = N(m/2\pi\kappa T)^{3/2}\exp(-mv^2/2\kappa T), \qquad (1.2)$$

where N is the equilibrium electron density (N is independent of the coordinates because the plasma is uniform) and κ is the Boltzmann constant.

The system above can be solved by the method given by Landau.[5] In considering propagation in a uniform infinite medium we expand the variables **E**, **H**, and f in Fourier integrals in the coordinates (for example, $\mathbf{E} = \int \mathbf{E}_\mathbf{k} e^{i\mathbf{k}\cdot\mathbf{r}} d\tau_k$) and consider the equations for the Fourier components characterized by given values of the wave vector **k**. Then we apply the operational technique, assuming that the values of the nonequilibrium distribution at the initial time $f_\mathbf{k}(t=0)$ are given. The solution shows that the asymptotic nature of the behavior of the field is determined by the quantity e^{pt}, where $p = -i\omega - \gamma$ (ω is the frequency and γ the damping factor). Carrying out the appropriate calculations it is possible to obtain the dispersion equation which relates p and the wave vector **k**.

It is also of interest to consider another formulation of the problem: in this case, at an arbitrary time t the values of the nonequilibrium function $f_\omega(z=0)$ are given in the plane $z = 0$ (here and below it is assumed that propagation is along the z axis; f_ω is a spectral component of f).

In this formulation the problem can also be

*It is assumed that ion motion can be neglected. In the presence of a fixed magnetic field H_0 this assumption is justified only when $\omega \gg \Omega_H$, where ω is the frequency of the propagating wave and Ω_H is the gyromagnetic frequency of the ions.

solved by the Landau technique.* In this case the asymptotic behavior of the fields as a function of the coordinate z (for $z > 0$) is determined by $\exp(-qz + ikz)$, where k is the wave number and q is the amplitude of the absorption factor. In the analysis below we use the solution of the problem in the second formulation, although the corresponding formulas for the first formulation are also given.

We obtain the dispersion equation in the same way as in reference 1. The original system of equations differs from that considered earlier in that collisions are taken into account. Furthermore the boundary conditions are somewhat different from those in reference 1, in which the analysis has been carried out for the first formulation of the problem. This new approach provides results which are much more general. For this reason we do not repeat the entire derivation here. We summarize the important steps and indicate the assumptions which are made.

The dispersion equation can be written in the form

$$\begin{vmatrix} E_1[A_1]-1 & E_1[A_2] & E_1[A_3] \\ E_2[A_1] & E_2[A_2]-1 & E_2[A_3] \\ E_3[A_1] & E_3[A_2] & E_3[A_3]-1 \end{vmatrix} = 0, \qquad (1.3)$$

where the elements of the determinant are given by the relations ($j = 1, 2, 3$)

$$E_1[A_j] = \frac{4\pi e i}{\omega(c^2\tilde{k}^2 - \omega^2)}\{(c^2\tilde{k}^2\sin^2\alpha - \omega^2)I_1[A_j] + c^2\tilde{k}^2\sin\alpha\cos\alpha I_3[A_j]\},$$

$$E_2[A_j] = -\frac{4\pi e i\omega}{c^2\tilde{k}^2 - \omega^2}I_2[A_j],$$

$$E_3[A_j] = \frac{4\pi e i}{\omega(c^2\tilde{k}^2 - \omega^2)}\{c^2\tilde{k}^2\sin\alpha\cos\alpha I_1[A_j] + (c^2\tilde{k}^2\cos^2\alpha - \omega^2)I_3[A_j]\}. \qquad (1.4)$$

Here α is the angle between the wave vector **k** and the magnetic field **H**; $\tilde{k} = k + iq$. Furthermore we assume that the condition

$$|\delta| = |(\varkappa T/m)(\tilde{k}/\omega_H)^2\sin^2\alpha| \ll 1, \qquad (1.5)$$

is satisfied. For weakly damped waves this condition is violated only when there is a weak anisotropy and $\omega_H \ll \omega$ ($\omega_H = eH_0/mc$ is the electron gyromagnetic frequency). If (1.5) is satisfied the quantities $I_i[A_j]$ can be written in the form

$$I_1[A_1] = -(eN/2m)[J_0^+ + J_0^- + \delta(J_0^{++} + J_0^{--} - J_0^- - J_0^+)],$$

$$I_1[A_2] = -I_2[A_1] = -(ieN/2m)[J_0^- - J_0^+ + \delta\langle 2(J_0^+ - J_0^-) + J_0^{--} - J_0^{++}\rangle],$$

$$I_2[A_2] = -(eN/2m)[J_0^+ + J_0^- + \delta\langle 4J_0 - 3(J_0^+ + J_0^-) + J_0^{++} + J_0^{--}\rangle],$$

$$I_3[A_1] = I_1[A_3] = (\tilde{k}\sin\alpha\, eN/2m\omega_H)(J_1^- - J_1^+),$$

$$I_3[A_2] = -I_2[A_3] = -(i\tilde{k}\sin\alpha\, eN/m\omega_H)[J_1 - \tfrac{1}{2}(J_1^+ + J_1^-)],$$

$$I_3[A_3] = -(2\pi eN/m)[J_2 + \delta\langle -J_2 + \tfrac{1}{2}(J_2^+ + J_2^-)\rangle]. \qquad (1.6)$$

Here

$$J_0 = \sqrt{\frac{m}{2\pi\varkappa T}}\int_C \frac{1}{-i\omega + \nu + i\tilde{k}v_z\cos\alpha}\exp\left(-\frac{mv_z^2}{2\varkappa T}\right)dv_z. \qquad (1.7)$$

The integrals J_1 and J_2 are obtained from J_0 by adding the factors v_z and $mv_z^2/2\pi\kappa T$ respectively; the integrals denoted by ± are obtained by replacing the frequency ω by $\omega \mp \omega_H$ respectively in the denominator of the integrand while in the integrals denoted by ++ and −− the frequency ω is replaced by $\omega \mp 2\omega_H$.

The integration is carried out along a contour C which bypasses the singularities of the integrands from below (in the plane of v_z). We see that collisions can be taken formally into account by replacing $-i\omega$ with $-i\omega + \nu$. However this substitution cannot be made everywhere, but only in integrals such as that given in (1.7). The relations in (1.6) are obtained by expanding in terms of the small parameter δ. In this case account is taken of the thermal corrections in the first approximation (only terms proportional to the temperature T are retained).

Substituting (1.4) in (1.3) and making use of (1.6) we arrive at the dispersion equation

$$i\omega\omega_0^6\{2\pi J_2J_0^+J_0^- - (\tilde{k}^2\sin^2\alpha/2\omega_H^2)J_0^-J_1^{+2}$$
$$+ \delta[2\pi J_2\langle J_0(J_0^+ + J_0^-) - 4J_0^+J_0^- + J_0^+J_0^{--} + J_0^-J_0^{++}\rangle$$
$$+ 2\pi J_0^+J_0^-\langle -J_2 + \tfrac{1}{2}(J_2^+ + J_2^-)\rangle]\}$$
$$+ \omega_0^4\{-J_0^+J_0^-(c^2\tilde{k}^2\sin^2\alpha - \omega^2)$$
$$- \pi J_2(J_0^+ + J_0^-)\langle c^2\tilde{k}^2(1 + \cos^2\alpha) - 2\omega^2\rangle$$

*In solving the problem it is necessary to expand all variables in Fourier time integrals and to use the operational method with respect to the variable z. In the remaining calculations the method is the same as that used for solving the problem in the first formulation.

$$+ (c^2\tilde{k}^3/\omega_H)\sin^2\alpha\cos\alpha\langle J_1(J_0^- - J_0^+) + J_0^+J_1^- - J_1^+J_0^-\rangle$$

$$+ \tilde{k}^2\sin^2\alpha/\omega_N^2 + \langle c^2\tilde{k}^2(1+\cos^2\alpha) - 2\omega^2\rangle J_1^{+2}/4$$

$$+ \delta[(c^2\tilde{k}^2\sin^2\alpha - \omega^2)\langle 4J_0^+J_0^-$$

$$- J_0(J_0^+ + J_0^-) - J_0^+J_0^{--} - J_0^-J_0^{++}\rangle - \pi(c^2\tilde{k}^2\cos^2\alpha - \omega^2)$$

$$\times\langle 4J_2(J_0 - J_0^+ - J_0^-) + \tfrac{1}{2}(J_2^+ + J_2^-)(J_0^+ + J_0^-)$$

$$+ J_2(J_0^{++} + J_0^{--})\rangle - \pi(c^2\tilde{k}^2 - \omega^2)\langle J_2(J_0^{++} + J_0^{--})$$

$$+ (J_0^+ + J_0^-)(-2J_2 + \tfrac{1}{2}(J_2^+ + J_2^-))\rangle]\}$$

$$+ (i\omega_0^2/2\omega)(c^2\tilde{k}^2 - \omega^2)\{-(c^2\tilde{k}^2\sin^2\alpha - 2\omega^2)(J_0^+ + J_0^-)$$

$$+ (2c^2\tilde{k}^3\sin^2\alpha\cos\alpha/\omega_H)(J_1^- - J_1^+)$$

$$- 4\pi(c^2\tilde{k}^2\cos^2\alpha - \omega^2)J_2 - \delta[\omega^2\langle -4J_0 + 4(J_0^+ + J_0^-)$$

$$- 2(J_0^{++} + J_0^{--})\rangle$$

$$+ c^2\tilde{k}^2\sin^2\alpha(-J_0^+ - J_0^- + J_0^{++} + J_0^{--})$$

$$+ 4\pi(c^2\tilde{k}^2\cos^2\alpha - \omega^2)\langle -J_2 + \tfrac{1}{2}(J_2^+ + J_2^-)\rangle]\}$$

$$- (c^2\tilde{k}^2 - \omega^2)^2 = 0. \qquad (1.8)$$

Here $\omega_0 = \sqrt{4\pi e^2N/m}$ is the characteristic frequency for the plasma oscillations in the isotropic case. In (1.8) we have retained thermal terms which are at least of order T; this corresponds to taking account of the thermal motion in the first approximation.

2. Using (1.8), we now consider the absorption of various kinds of waves, excluding frequency regions close to the electron gyromagnetic frequency and its multiples. We shall be interested in weakly attenuated waves (the regions of strong attenuation will also be indicated in most cases). In accordance with these remarks we assume that the following relations are satisfied:

$$\omega \gg k\sqrt{\varkappa T/m}\cos\alpha, \qquad \omega \gg \nu, \qquad (2.1)$$

$$|\omega - \omega_H| \gg k\sqrt{\varkappa T/m}\cos\alpha, \qquad |\omega - \omega_H| \gg \nu,$$

$$|\omega - 2\omega_H| \gg k\sqrt{\varkappa T/m}\cos\alpha, \qquad |\omega - 2\omega_H| \gg \nu. \qquad (2.2)$$

When (2.1) and (2.2) are taken into account, the integrals in (1.7) can be expanded

$$J_0 = \frac{1}{\nu - i\omega} - \frac{\tilde{k}^2\varkappa T}{(\nu - i\omega)^3 m}\cos^2\alpha$$

$$+ \frac{1}{k\cos\alpha}\sqrt{\frac{m\pi}{2\varkappa T}}\exp\left(-\frac{m\omega^2}{2\varkappa Tk^2\cos^2\alpha}\right),$$

$$J_1 = -\frac{\varkappa T\tilde{k}\cos\alpha}{m(\nu - i\omega)^2} + \frac{\omega}{k^2\cos^2\alpha}\exp\left(-\frac{m\omega^2}{2\varkappa Tk^2\cos^2\alpha}\right),$$

$$J_2 = \frac{1}{2\pi(\nu - i\omega)} - \frac{3\varkappa T\tilde{k}^2}{2\pi m(\nu - i\omega)^3}\cos^2\alpha$$

$$+ \frac{\omega^2}{\sqrt{\pi}k^3\cos^3\alpha}\left(\frac{m}{2\varkappa T}\right)^{3/2}\exp\left(-\frac{m\omega^2}{2\varkappa Tk^2\cos^2\alpha}\right). \qquad (2.3)$$

The corresponding formulas for the J^+ integrals can be obtained by replacing ω by $\omega - \omega_H$, for the J^- by replacing ω by $\omega + \omega_H$, for J^{++} by replacing ω by $\omega - 2\omega_H$, etc. In the terms which contain exponentials we replace $\nu - i\omega$ by $-i\omega$ and $\tilde{k}$ by k (for weakly attenuated waves $q \ll k$). Substituting into (1.8) a relation of the type given in (2.3), we can obtain the dispersion equation. However, this equation is too unwieldy to be given here in general form.

In view of the fact that $q \ll k$, we can in general neglect absorption at the outset [we assume that $\nu = 0$ and omit the exponential terms in (2.3)]. Thus we arrive at the equation

$$\beta^2 vRn^6 - (1 - u - v + uv\cos^2\alpha)n^4 + [2(1-v)^2$$

$$+ uv(1+\cos^2\alpha) - 2u]n^2$$

$$+ (1-v)[u - (1-v)^2] = 0. \qquad (2.4)$$

Equation (2.4), which is cubic in n^2 ($n = ck/\omega$ is the index of refraction), describes the propagation of the extraordinary ($n^2 = n_1^2$), ordinary ($n^2 = n_2^2$), and plasma waves ($n^2 = n_3^2$). In writing this equation we have used the customary notation: $v = \omega_0^2/\omega^2$, $u = \omega_H^2/\omega^2$ and $\beta^2 = \kappa T/mc^2$. The parameter β represents the ratio of the mean thermal velocity of the electrons to the velocity of light c. In the nonrelativistic case being considered here the relation $\beta^2 \ll 1$ always holds. For example, in the solar corona $\beta^2 \sim 10^{-4}$ and in the ionosphere $\beta^2 < 10^{-6}$. Because β^2 is so small we need retain in (2.4) only those terms independent of β^2 and those of order β^2. We retain only the term that contains the highest power of the quantity ck, i.e., c^6k^6. The approximation used here is adequate for studying the important features of the characteristic waves. For the quantity R in (2.4) we have

$$R = \frac{3\sin^4\alpha}{1 - 4u} + \sin^2\alpha\cos^2\alpha\left\langle 1 + \frac{5 - u}{(1-u)^2}\right\rangle$$

$$+ 3(1-u)\cos^4\alpha. \qquad (2.5)$$

We shall not dwell here on a number of questions related to the interpretation of Eq. (2.4) and associated problems; the appropriate discussion can be found in the review in reference 7 and in various papers of the author.[1,8,9] In the work cited curves have been given to show the features of wave propagation in a hot magnetoacoustic plasma. We shall limit ourselves to several pertinent remarks.

If we exclude from consideration the transition region between the extraordinary (ordinary) and plasma waves,[7,9] we have for waves 1 and 2 the well-known relation:[6]

$$(1-u-v+uv\cos^2\alpha)\,n^4-[2(1-v)^2+uv(1+\cos^2\alpha)-2u]\,n^2+(1-v)[(1-v)^2-u]=0. \qquad (2.6)$$

It is possible to speak of a pure plasma wave in regions of u and v for which the following condition is satisfied:

$$1-u-v+uv\cos^2\alpha\approx 0. \qquad (2.7)$$

For n_3^2 we obtain an expression which follows from (2.5) (only terms with n^4 and n^6 are taken into account):

$$n_3^2\approx(1-u-v+uv\cos^2\alpha)/\beta^2vR. \qquad (2.8)$$

Equation (2.8) does not hold at very small values of the numerator, in which case it is necessary to take account of n^2 terms, and at large values of the numerator, in which case $\beta^2n^2\sim 1$. In the latter case strong absorption may occur. An analysis of (2.4) shows that there are values of the parameters u and v and the angle α for which certain roots may be negative ($n^2<0$); in the region defined by the condition in (2.7), certain roots can even be complex. Inasmuch as dissipation processes have been neglected in making the transition to (2.4) the suppression of the field indicates that propagation is impossible. The situation here is similar to that which obtains above points of reflection for low-frequency radio waves incident on the ionosphere from below. In such cases there are frequency regions for which propagation is impossible. If the thermal motion in the plasma is taken into account the width of the forbidden zone can be smaller than the frequency ω itself. The existence of these narrow gaps in the frequency spectrum in gyromagnetic resonance regions for quasi-transverse propagation ($\alpha\approx\pi/2$) was first noted by Gross.[10]

We now consider the absorption. In this case, in the general dispersion equation, obtained by substituting (2.3) in (1.8), we retain the terms that depend on the collision frequency ν and exponential terms [cf. (2.3)]. Because the conditions in (2.1) and (2.2) are satisfied, the resulting relation for the absorption factor can be written as a sum of two terms: one depends on collisions and the other takes account of the absorption effect due to remote interactions. From (2.1), (2.2), and (2.4) we have (we recall that $\tilde{k}=k+iq$)

$$\frac{q}{k}\{-6\beta^2vRn^6+4(1-u-v+uv\cos^2\alpha)n^2-2[2(1-v)^2+(1+\cos^2\alpha)uv-2u]\,n^2\}$$
$$=s\Big\{-2\beta^2vn^6\Big[3\cos^4\alpha-\frac{\sin^2\alpha\cos^2\alpha\,(u-9)}{(1-u)^3}+3\cos^4\alpha+\frac{12u\sin^4\alpha}{1-4u}\Big]+n^4(u-3+2v)+n^2(2v^2-8v-2u+6)$$
$$-3v^2+6v+u-3\Big\}+v\Big\{(u-1)\cos^2\alpha\,n^4+(1-u-v)(1+\cos^2\alpha)\,n^2$$
$$+u-(1-v)^2\Big\}\sqrt{\frac{\pi}{2}}\,\frac{1}{(\beta n\cos\alpha)^3}\exp\Big(-\frac{1}{2\beta^2n^2\cos^2\alpha}\Big)+v(u-1)\Big\{\frac{\sin^2\alpha\,n^4}{2u}+\Big[v\Big(\frac{1+\cos^2\alpha}{2}+\frac{\sin^2\alpha}{1+\sqrt{u}}\Big)$$
$$-\Big\langle\frac{\sin^2\alpha}{2}+1+\frac{\sin^2\alpha}{\sqrt{u}}(1-\sqrt{u})+\frac{\sin^2\alpha}{2\cos^2\alpha}(1-\sqrt{u})^2(1+\cos^2\alpha)\Big\rangle\Big]n^2$$
$$+\frac{v^2}{1+\sqrt{u}}-v\Big(\frac{2+\sqrt{u}}{1+\sqrt{u}}\Big)+1+\frac{\sin^2\alpha}{2u\cos^2\alpha}(1-\sqrt{u})^2\Big\}\sqrt{\frac{\pi}{2}}\,\frac{1}{\beta n\cos\alpha}\exp\Big(-\frac{(1-\sqrt{u})^2}{2\beta^2n^2\cos^2\alpha}\Big)+v(u-1)\Big\{\frac{\sin^2\alpha\,n^4}{2u}$$
$$+\Big[v\Big(\frac{1+\cos^2\alpha}{2}+\frac{\sin^2\alpha}{1-\sqrt{u}}\Big)-\Big\langle\frac{\sin^2\alpha}{2}+1-\frac{\sin^2\alpha}{\sqrt{u}}(1+\sqrt{1u})+\frac{\sin^2\alpha}{2\cos^2\alpha}(1+\sqrt{u})^2(1+\cos^2\alpha)\Big\rangle\Big]n^2+\frac{v^2}{1-\sqrt{u}}$$
$$-v\Big(\frac{2-\sqrt{u}}{1-\sqrt{u}}\Big)+1+\frac{\sin^2\alpha}{2u\cos^2\alpha}(1+\sqrt{u})^2\Big\}\sqrt{\frac{\pi}{2}}\,\frac{1}{\beta n\cos\alpha}\exp\Big(-\frac{(1+\sqrt{u})^2}{2\beta^2n^2\cos^2\alpha}\Big), \qquad (2.9)$$

where $s=\nu/\omega$.

We now consider some particular cases. Although the specific absorption [which is characterized by the exponential terms in (2.9)] is small (in accordance with our original assumptions), it is of interest when we are concerned with small values of q, i.e., when the numbers in the exponential are not very large in absolute value. For example, in the factor $\exp(-\frac{1}{2}\beta^2n^2\cos^2\alpha)$ the quantity ($\beta n\cos\alpha$) cannot be too small. Since $\beta n\cos\alpha\lesssim\sqrt{\kappa T/m}/v_{ph}$ ($v_{ph}=c/n$ is the phase velocity), when $\beta^2\ll 1$ attenuation due to this term can have a noticeable effect if the wave is a slow wave, i.e., when $v_{ph}\ll c$ or

$$n^2 \gg 1. \tag{2.10}$$

The factor $\exp[-(1-\sqrt{u})^2/2\beta^2 n^2 \cos^2\alpha]$ in Eq. (2.9) may become large at large values of n^2 and near the gyromagnetic frequency ω_H when $u \to 1$ (a similar picture obtains close to $2\omega_H$ and so on). Thus the effects of specific absorption can be discussed in two cases which sometimes overlap: slow waves or frequencies near the gyromagnetic resonance frequencies. We here consider the case in which the condition in (2.10) is satisfied. However, we have left terms with the indicated exponential factor in (2.9) and below, since they may make a considerable contribution when the frequency of the wave is substantially (say, several percent) different from the gyromagnetic frequency ω_H. In addition, we consider the particular case in which the condition in (2.10) is satisfied precisely in the region of the first gyromagnetic resonance.

Even if we go to $n^2 \gg 1$ [(2.10)], it does not mean that we must always neglect terms with small powers of n^2 in (2.9).

Such a procedure would deprive us of the possibility of calculating the absorption of the ordinary wave at low frequencies when (as for plasma waves) the case $n^2 \gg 1$ is possible. A simple analysis indicates that in (2.9) the quantity proportional to $\beta^2 n^6$ in the first term, which reflects the contribution of collisions, can be omitted. Then, making several omissions which are permissible when (2.10) is satisfied, we find

$$(q/k)[-6\beta^2 vRn^6 + 4(1-u-v+uv\cos^2\alpha)n^4]$$
$$= s[(u+2v-3)n^4 + 2v^2n^2] + v[(u-1)\cos^2\alpha$$
$$+ (1-u-v)(1+\cos^2\alpha)n^2 - v^2]\sqrt{\frac{\pi}{2}}\frac{1}{(\beta n\cos\alpha)^3}$$
$$\times \exp\left(-\frac{1}{2\beta^2 n^2\cos^2\alpha}\right) + \sqrt{\frac{\pi}{8}}\frac{v(u-1)\sin^2\alpha\, n^4}{u\beta n\cos\alpha}$$
$$\times\left\{\exp\left(-\frac{(1-\sqrt{u})^2}{2\beta^2 n^2\cos^2\alpha}\right) + \exp\left(-\frac{(1+\sqrt{u})^2}{2\beta^2 n^2\cos^2\alpha}\right)\right\}. \tag{2.11}$$

For plasma waves we must take account of the condition in (2.7); in other words, as follows from (2.8) and the last formula in (2.12), the absorption need not necessarily be weak. Furthermore, were we to have $v \gg 1$, the requirement for the propagation of plasma waves (2.7) would lead precisely to $u\cos^2\alpha = 1$. When $u < 1$, the last equality is not satisfied at all. When $u \gg 1$ the angle α must have a definite value, close to $\alpha = \pi/2$. Hence the case $u\cos^2\alpha = 1$ (for $v \gg 1$) is a special one and will not be considered here. Assuming that v is not large, we can neglect in (2.11) all n^2 terms and terms that do not contain n^2. Finally, taking account of (2.4) and (2.7), we have

$$q_3 = \frac{k(1-u)}{1-u-v+uv\cos^2\alpha}\left\{\frac{s}{2}\left(1+\frac{2uv\sin^2\alpha}{(1-u)^2}\right)\right.$$
$$+\sqrt{\frac{\pi}{8}}v\left[\frac{1}{\beta^3 n^3\cos\alpha}\exp\left(-\frac{1}{2\beta^2 n^2\cos^2\alpha}\right)\right.$$
$$+\frac{\sin^2\alpha}{2\beta n\cos\alpha\, u}\left[\exp\left(-\frac{(1-\sqrt{u})^2}{2\beta^2 n^2\cos^2\alpha}\right)\right.$$
$$\left.\left.\left.+\exp\left(-\frac{(1+\sqrt{u})^2}{2\beta^2 n^2\cos^2\alpha}\right)\right]\right]\right\}. \tag{2.12}$$

Strictly speaking this relation holds only for weak attenuation ($q \ll k$); however, it can also be used to obtain qualitative results when the absorption is strong. It is apparent from Eq. (2.12) that when $\beta n\cos\alpha \sim 1$ the absorption is not weak ($q \sim k$). For propagation α close to $\pi/2$, the quantity $\beta n\cos\alpha$ may be small even for very slow waves; however, in these cases it does not follow that we can neglect (1.5), since violation of this condition means strong absorption when $u \gtrsim 1$. It should be noted that in (2.12) the term containing the factor $\exp[-(1+\sqrt{u})^2/2\beta^2 n^2\cos^2\alpha]$ is not important and is given here to show the contribution due to such terms. From Eq. (2.12) it can be shown that the absorption of plasma waves is especially strong when $u \approx 1$. It should be noted however, from Eq. (2.8), that when $u \approx 1$, for values of α which are not close to 0 or $\pi/2$, we have $n_3^2 = -(1-u)^2/4\beta^2\cos^2\alpha$; obviously, in this region the propagation of Wave 3 generally is impossible, because $n_3^2 < 0$ for both $u > 1$ and $u < 1$.

We now derive the corresponding results, using the first formulation of the problem of propagation of electromagnetic waves in a plasma (cf. Sec. 1); the field falls off asymptotically as $e^{-\gamma t}$, where γ is the damping factor. Calculations, as well as general considerations, yield the following formula for the case of weak absorption

$$\gamma = q\, d\omega/dk. \tag{2.13}$$

In the isotropic case this relation can be written $\gamma = gU_{gp}$, where U_{gp} is the group velocity.* In the anisotropic case $d\omega/dk$ is equal to the projection of the group velocity in the direction of the wave vector **k**. Using (2.7) and (2.8) (neglecting certain terms), we find for Wave 3

*An elementary derivation of (2.13) can be found in reference 11. Although there is no question that (2.13) holds outside the gyromagnetic-resonance regions, it may be violated inside these regions.

$d\omega/dk = c(1 - u - v + uv\cos^2\alpha)/n(2 - u - v)$

and, from (2.7) and (2.13) then obtain a formula for the damping factor of the plasma wave

$$\gamma_3 = \left(1 + \frac{uv\sin^2\alpha}{(1-u)^2}\right)^{-1}\left\{\frac{\nu}{2}\left(1 + \frac{2uv\sin^2\alpha}{(1-u)^2}\right) + \sqrt{\frac{\pi}{8}}\omega v\left(\frac{1}{\beta^3 n^3\cos\alpha}\exp\left(-\frac{1}{2\beta^2 n^2\cos^2\alpha}\right) + \frac{\sin^2\alpha}{2\beta n\cos\alpha\, u}\left[\exp\left(-\frac{(1-\sqrt{u})^2}{2\beta^2 n^2\cos^2\alpha}\right) + \exp\left(-\frac{(1+\sqrt{u})^2}{2\beta^2 n^2\cos^2\alpha}\right)\right]\right)\right\}. \quad (2.14)$$

The part of this expression which reflects the contribution of specific attenuation effects coincides with the results obtained by Sitenko and Stepanov.[2]

However, the collision term differs from that derived in reference 4. In particular, from Eq. (36) of reference 4 it follows that $\gamma \to 0$ when $u \to 1$ and $\alpha \neq 0$, whereas in our case $\gamma \to \nu$ under the same conditions. In an earlier paper by the author[1] a formula analogous to (2.14) was given without proof, but the coefficient similar to that in front of the curly brackets of (2.14) was given incorrectly.

In analogy with the spatial absorption, the attenuation is not necessarily weak ($\gamma \sim \omega$) when $\beta n\cos\alpha \sim 1$. The other remarks made in connection with (2.12) also apply to (2.14).

We now consider the attenuation of ordinary waves at relatively low frequency; in particular we assume that

$$\omega_0 \gg \omega, \quad \omega_H\cos\alpha \gg \omega, \quad \omega_0 \gg \omega_H.$$

These inequalities can be written in the form

$$v \gg 1, \; u\cos^2\alpha \gg 1, \; v \gg u. \quad (2.15)$$

The third of these conditions is not necessarily a strong one. Investigations of the ordinary waves under the conditions given in (2.15) are of interest basically in connection with the problem of propagation of whistlers.[12] The propagation is found to be quasi-longitudinal[6] and, from Eq. (2.6), leads to the equation*

$$u\cos^2\alpha\, n^4 - 2vn^2 - v^2 = 0. \quad (2.16)$$

The second term is smaller than the others in absolute magnitude, so that as a first approximation we have

$$n_2^2 = v/\sqrt{u}\cos\alpha = \omega_0^2/\omega\omega_H\cos\alpha, \quad (2.17)$$

which is used as a basis for the generally accepted analysis of the propagation of whistlers.[12] From Eqs. (2.15) and (2.17) it follows that $n_2^2 \gg 1$. Under these same conditions we have $n_1^2 = -v/\sqrt{u}\cos\alpha$, and propagation is impossible for the extraordinary wave.*

In calculating the absorption coefficient we can, by virtue of the condition $\omega_H \gg \omega$, omit terms that contain the parameter u in the exponent. We are left with the basic terms in Eq. (2.11); after some simple transformations, which take account of Eqs. (2.15) — (2.17), we have

$$q_2 = \frac{k}{2\sqrt{u}\cos\alpha}\left\{s + \sqrt{\frac{\pi}{8}}\frac{\sin^2\alpha}{(\beta n\cos\alpha)^3}\exp\left(-\frac{1}{2\beta^2 n^2\cos^2\alpha}\right)\right\}. \quad (2.18)$$

In this case, because $\omega_H\cos\alpha \gg \omega$ the absorption is weak even when $\beta n\cos\alpha \sim 1$ and $s \sim 1$. The specific absorption described by (2.18) may apparently be realized for propagation of whistlers in the upper layers of the atmosphere in periods of intense solar corpuscular disturbance.[13] If we neglect collisions, (2.18) coincides with the results obtained by Shafranov[14] by a completely different method [cf. the appendix to reference 14, Eq. (21)]. In the latter paper the absorption was calculated by considering the intensity of the Cerenkov radiation of electrons in a plasma and applying the Kirchoff formula. Thus, whereas the ordinary absorption due to collisions of electrons with ions is due to bremsstrahlung of moving electrons in the ion field,[6] the specific absorption is due to Cerenkov radiation,[14] and also to "magnetic bremsstrahlung" losses.

We now obtain a formula for the damping of the ordinary wave. Using (2.13) and a result which follows from (2.17), $d\omega/dk = 2v_{ph}$, we find the attenuation γ to be

$$\gamma_2 = \frac{\nu}{\sqrt{u}\cos\alpha} + \omega\sqrt{\frac{\pi}{8u}}\frac{\sin^2\alpha}{\beta^3 n^3\cos^4\alpha}\exp\left(-\frac{1}{2\beta^2 n^2\cos^2\alpha}\right). \quad (2.19)$$

In conclusion we consider one more example, which arises in the propagation of slow waves. It should actually be related to gyromagnetic resonance although here, as in the preceding examples, $n^2 \gg 1$. At the outset we consider purely longitudinal propagation, with $\alpha = 0$. The attenuation of the plasma waves can then be found from (2.12) and (2.14), with the latter formula becoming the Landau expression.[5] However, when $\alpha = 0$, the extraordinary wave can also be slow when ω is close to

*The effect of the thermal corrections on propagation is not very important here; in any case, it need not be taken into account in calculating absorption.

*It should be noted that when $\alpha = 0$ the wave described by (2.7) should be called the extraordinary wave. The case $\alpha = 0$ is exceptional and for any small values $\alpha \neq 0$ this wave becomes the ordinary wave.

ω_H. Starting from our general relations, we find the corresponding formulas for absorption (attenuation) in this case. From (2.6) with $\alpha = 0$, neglecting the effect of thermal corrections on propagation, we have for Wave 1

$$n_1^2 = 1 - v/(1 - \sqrt{u}) \approx v/(\sqrt{u} - 1). \qquad (2.20)$$

The approximate equality refers to the case $u \approx 1$. We assume further that v is not very large, taking $v \sim 1$. Then, we are justified in assuming (neglecting difference terms of the type $u - 1$) that $u = 1$. The attenuation of Wave 1 for $\alpha = 0$ is determined by collisions and the term with the factor $\exp[-(1-\sqrt{u})^2/2\beta^2 n^2]$. From Eq. (2.9) with $\alpha = 0$, we have after certain simplifications based on (2.20)

$$q_1 = k\left\{\frac{s}{u-1} + (\sqrt{u} - 1)\sqrt{\frac{\pi}{8}}\frac{1}{\beta n}\exp\left(-\frac{(1-\sqrt{u})^2}{2\beta^2 n^2}\right)\right\}. \qquad (2.21)$$

When $s = 0$ this formula becomes the corresponding formula given by Shafranov[14] (cf. also reference 15). Using Eq. (2.13) it is easy to calculate the value of the damping factor γ. In the present case $d\omega/dk = 2cn(\sqrt{u} - 1)^2/v$ and

$$\gamma = \nu + (\sqrt{u} - 1)^2\sqrt{\frac{\pi}{2}}\frac{\omega}{\beta n}\exp\left(-\frac{(1-\sqrt{u})^2}{2\beta^2 n^2}\right). \qquad (2.22)$$

As Shafranov has emphasized,[14] damping of the kind being considered here may be especially important at large values of v. In this case the conditions which determine the degree to which the frequency ω approaches ω_H may not be especially stringent, so that it may be necessary to introduce corrections in the formulas which have been derived. It is obvious that absorption of this kind also obtains for small values of the angle α; these are determined by the condition $|1 - v| \gg v \sin^2\alpha / |1 - \sqrt{u}|$ with the supplementary requirement $v/(\sqrt{u} - 1) \gg 1$. However, when $\alpha \neq 0$ it is necessary to take account of attenuation of the ordinary wave even for very small values of α. The situation is to a considerable degree analogous to that which obtains for the low-frequency ordinary waves considered earlier, although in the latter the specific absorption is due to the nonresonance factor $\exp(-\frac{1}{2}\beta^2 n^2 \cos^2\alpha)$, whereas in the present case it is due to the resonance factor $\exp[-(1-\sqrt{u})^2/2\beta^2 n^2 \cos^2\alpha]$.

The author is indebted to Prof. V. L. Ginzburg for his continued interest in this work.

[1] B. N. Gershman, JETP **24**, 659 (1953).

[2] A. G. Sitenko and K. N. Stepanov, JETP **31**, 642 (1956), Soviet Phys. JETP **4**, 512 (1957).

[3] K. N. Stepanov, JETP **35**, 283 (1958), Soviet Phys. JETP **8**, 195 (1959).

[4] K. N. Stepanov, JETP **35**, 283 (1958), Soviet Phys. (U.S.S.R.) **28**, 1789 (1958), Soviet Phys.–Tech. Phys. **3**, 1649 (1959).

[5] L. D. Landau, JETP **16**, 574 (1946).

[6] Al'pert, Ginzburg, and Feĭnberg, Распространение радиоволн (Propagation of Radiowaves), Gostekhizdat, 1953.

[7] Gershman, Ginzburg, and Denisov, Usp. Fiz. Nauk **61**, 561 (1957).

[8] B. N. Gershman, Сб. памяти А. А. Андронова (Collection in memory of A. A. Andronov), p. 599, U.S.S.R. Acad. Sci. Press, 1955; JETP **31**, 707 (1956), Soviet Phys. JETP **4**, 582 (1957).

[9] B. N. Gershman and M. S. Kovner, Изв. высш. уч. заведений. Радиофизика (Bull. of the Insts. of Higher Learning, Radiophysics) **2**, No. 1, 28 (1959).

[10] E. P. Gross, Phys. Rev. **82**, 232 (1951).

[11] V. V. Zheleznyakov, Изв. высш. уч. заведений. Радиофизика (Bull. of the Insts. of Higher Learning, Radiophysics) **2**, No. 1, 14 (1959).

[12] L. R. O. Storey, Phil. Trans. **246**, 113 (1953).

[13] B. N. Gershman, Изв. высш. уч. заведений. Радиофизика (Bull. of the Insts. of Higher Learning, Radiophysics) **1**, Nos. 5 – 6, 49 (1958).

[14] V. D. Shafranov, JETP **34**, 1475 (1958), Soviet Phys. JETP **7**, 1019 (1958).

[15] V. P. Silin, Труды ФИАН (Trans. Phys. Inst. Acad. Sci.) **6**, 201 (1955).

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136

ERRATA TO VOLUME 9

On page 868, column 1, item (e) should read:

(e). Ferromagnetic weak solid solutions. By way of an example, we consider the system Fe-Me with A2 lattice, where Me = Ti, V, Cr, Mn, Co, and Ni. For these the variation of the moment m with concentration c is

$$dm/dc = (N_d)_{Me} \mp 0.642 \ \{8\ (2.478 - R_{Me}) + 6\,|2.861 - R_{Me}| \mp [\,8\,(2.478 - R_{\mathrm{Fe}}) + 6\,(2.861 - R_{\mathrm{Fe}})]\},$$

where the signs − and + pertain respectively to ferromagnetic and paramagnetic Me when in front of the curly brackets, and to metals of class 1 and 2 when in front of the square brackets. The first term and the square brackets are considered only for ferromagnetic Me. We then have dm/dc = −3 (−3.3) for Ti, −2.6 (−2.2) for V, −2.2 (−2.2) for Cr, −2 (−2) for Mn, 0.7 (0.6) for Ni, and 1.2 (1.2) for Co; the parentheses contain the experimental values.

ERRATA TO VOLUME 10

Page	Reads	Should Read
224, Ordinate of figure	10^{23}	10^{29}
228, Column 1, line 9 from top	3.6×10^{-2} mm/min	0.36 mm/min
228, Column 1, line 16 from top	0.5 mm/sec	0.05 mm/min
329, Third line of Eq. (23a)	+(1/4 cosh r + ...	+1/4(cosh r + ...
413, Table II, line 2 from bottom	−0.0924 ±	−1.0924 ±
413, Table II, line 3 from bottom	+1.8730 ±	+0.8370 ±
479, Fig. 7, right, 1st line	92 hr	9.2 hr
499, Second line of Eq. (1.8)	$+\tilde{k}\,\sin^2\alpha/\omega_N^2 + \langle c^2\tilde{k}^2 \ldots$	$+\left(\tilde{k}/\omega_H\right)^2 \sin^2\alpha\ \langle c^2\tilde{k}^2 \ldots$
648, Column 1, line 18 from top	18 × 80 mm	180 × 80 mm
804, First line of Eq. (17)	$-{}^1/_3\left(\alpha_x^2\alpha_y^2 + \ldots\right.$	$\ldots - 3\left(\alpha_x\alpha_y^2 + \ldots\right.$
967, Column 1, line 11 from top	$\sigma(N', \pi) \approx 46\,(N', N')$	$\sigma(N', \pi) > \sigma(N', N')$
976, First line of Eq. (10)	$= \frac{e^2}{3r^2c^4}$	$= \frac{e^2}{3\hbar^2c^2}$
978, First line of Eq. (23)	$\left[\frac{(2\gamma^2 - 1)^2}{(\gamma^2 - 1)\sin^4(\theta/2)}\right.$	$\left[\frac{(2\gamma^2 - 1)^2}{(\gamma^2 - 1)^2\sin^4(\theta/2)}\right.$