

where

$$\begin{aligned} \Phi_1 &= k_{0\pi}^2 q^{-2} k_-^2 k_+^2 \sin^2 \theta + 2 [K_- (k_{0\pi} - K_-) q^2 \\ &\quad - (k_-^2 + k_- k_+) (k_-^2 + k_- k_+ + (2K_- - k_{0\pi}) q)], \\ \Phi_2 &= q^{-2} k_- k_+ \sin^2 \theta [k_{0\pi}^2 (K_- K_+ - k_0^2) + 2k_0^2 q^2] \\ &\quad + (2/k_+) \{q^2 k_- (K_+ (k_{0\pi} - K_-) - k_0^2) - K_+ \\ &\quad \times (k_- + k_+ \cos \theta) [K_- (k_-^2 + k_- k_+) + (2k_-^2 - k_{0\pi} K_-) q]\}, \\ \Phi_3 &= 2q^{-2} k_- k_+^2 \sin^2 \theta k_{0\pi} K_- q + 2k_- (k_{0\pi} - K_-) q^2 - 2(k_- \\ &\quad + k_+ \cos \theta) [K_- (k_-^2 + k_- k_+ + (2K_- - k_{0\pi}) q) - k_0^2 q], \\ \Phi_4 &= 2q^{-2} k_-^2 k_+ \sin^2 \theta k_{0\pi} K_+ q \\ &\quad + 2(K_- / k_+) q^2 [K_+ (k_{0\pi} - K_-) - k_0^2] - (2/k_+) (k_-^2 \\ &\quad + k_- k_+) [K_+ (k_-^2 + k_- k_+ + (2K_- - k_{0\pi}) q) + k_0^2 q], \\ \mathbf{q} &\equiv -\mathbf{x} = \mathbf{k}_- + \mathbf{k}_+, \quad \cos \theta = k_- k_+ / k_- k_+, \quad k_{0\pi} = m_{\pi^0} c / \hbar, \\ k_+ &= \frac{-2bk_- \cos \theta \pm a \sqrt{b^2 - k_0^2 (a^2 - 4k_-^2 \cos^2 \theta)}}{a^2 - 4k_-^2 \cos^2 \theta}, \\ a &= 2(k_{0\pi} - K_-), \quad b = k_{0\pi} (k_{0\pi} - 2K_-) + 2k_0^2, \\ K_{\pm} &= \sqrt{k_0^2 + k_{\pm}^2}. \end{aligned} \quad (5)$$

Here  $E_{\pm} = c\hbar K_{\pm}$ ,  $\mathbf{p}_{\pm} = \hbar \mathbf{k}_{\pm}$  stand for the total energy and momentum of the electron and positron;  $k_0 = m_0 c / \hbar$  is the rest mass;  $d\Omega_+$  is the solid angle of positron emission;  $s_+$ ,  $s_- = \pm 1$  are the eigenvalues of the projection operator  $\boldsymbol{\sigma} \cdot \mathbf{k}_{\pm} / k_{\pm}$ . For  $s_- = 1$  ( $s_+ = 1$ ) the electron (positron) has right polarization and for  $s_- = -1$  ( $s_+ = -1$ ) left polarization. The corresponding expressions for  $\Phi_i$  ( $i = 1, 2, 3, 4$ ) are also easy to obtain for the case of a scalar  $\pi^0$  meson. Formula (4) gives the angle and energy dependence of the degree of longitudinal polarization of the created pairs and the correlation between polarizations (the terms proportional to  $s_- s_+$ ,  $l s_-$ ,  $l s_+$ ) in the decay (1); this could be of value in the determination of the properties of the  $\pi^0$  meson. It follows from (4) and (5) that for extremely relativistic electrons and positrons (when  $k_-, k_+ \gg k_0$  and  $\Phi_1 = \Phi_2$ ,  $\Phi_3 = \Phi_4$ ) the probability for the decay (1) will differ from zero only if both the electron and positron of a pair are right polarized ( $s_- = s_+ = 1$ ) or left polarized ( $s_- = s_+ = -1$ ). In that case the  $\pi^0$ -decay with the emission of a left polarized electron ( $s_- = -1$ ) and right polarized positron ( $s_+ = +1$ ) or vice versa ( $s_- = +1$ ,  $s_+ = -1$ ) is forbidden since the probability (4) vanishes.

In conclusion we express gratitude to Prof. A. A. Sokolov for his interest in this work.

<sup>1</sup>Budagov, Viktor, Dzhelapov, Ermolov, and Moskalev, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 1575 (1958), Soviet Phys. JETP **8**, 1101 (1959).

<sup>2</sup>Sargent, Cornelius, Rinehart, Lederman, and Rogers, Phys. Rev. **98**, 1349 (1955).

<sup>3</sup>R. H. Dalitz, Proc. Phys. Soc. **A64**, 667 (1951).

<sup>4</sup>N. M. Kroll and W. Wada, Phys. Rev. **98**, 1355 (1955).

<sup>5</sup>A. A. Sokolov, Введение в квантовую электродинамику (Introduction to Quantum Electrodynamics), M., Fizmatgiz, 1958.

Translated by A. M. Bincer  
111

### ATTRACTION OF SMALL PARTICLES SUSPENDED IN A LIQUID AT LARGE DISTANCES

L. P. PITAEVSKIĬ

Institute of Terrestrial Magnetism, Ionosphere,  
and Radio Wave Propagation, Academy of  
Sciences, U.S.S.R.

Submitted to JETP editor May 15, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **37**, 577-578  
(August, 1959)

IN the present note we derive formulas for the interaction energy connected with the Van-der-Waals forces of interaction between uncharged particles suspended in a liquid. The distance between the particles will be assumed large compared with their dimensions.

In principle this problem can be solved on the basis of the general theory of Van-der-Waals forces in dielectrics.<sup>1</sup> However, as shown earlier,<sup>2</sup> the expression for the interaction forces of arbitrary bodies in a medium can be derived by simple transformation from the corresponding expression for the interaction forces in vacuum. Indeed, the expression for the additional pressure in a medium of dielectric constant  $\epsilon$  can be obtained from the expression for the pressure in vacuum by multiplying the integrand in the integral with respect to frequency\* (this integral determines the pressure) by  $\epsilon^{3/2}$ , by replacing the dielectric constant of the interacting bodies  $\epsilon_1$  by  $\epsilon_1/\epsilon$ , and by increasing all the linear dimensions by a factor of  $\sqrt{\epsilon}$ . In accordance with this, the energy  $U$  of the interaction of the particles in the medium can be obtained from the energy of interaction in vacuum  $U_0$  by replacing the dielectric constant of the particles  $\epsilon_1$  by  $\epsilon_1/\epsilon$ , their volume  $V$  by

$\epsilon^{3/2} V$ , and the distance  $R$  between them by  $\sqrt{\epsilon} R$ .

On the other hand, the energy of interaction of small particles in vacuum is given directly by the formulas of London or Casimir-Polder (at distances which are respectively smaller or larger than the characteristic wavelengths  $\lambda_0$  in the spectrum), because only the smallness of the dimensions of the interacting systems is used in the derivation of these equations. We shall find it convenient to rewrite these equations in the following form:<sup>3</sup>

$$U_0 = (3\hbar / \pi R^6) \int_0^\infty \alpha^2(i\xi) d\xi, \quad \text{for } R \ll \lambda_0,$$

$$U_0 = (23 \hbar c / 4\pi R^7) \alpha^2(0), \quad \text{for } R \gg \lambda_0, \quad (1)$$

where  $\alpha(\omega)$  is the complex polarizability of the particles. Considering that the polarizability of spherical particles of volume  $V$  with dielectric constant  $\epsilon_1$  is given by

$$\alpha(\omega) = \frac{3}{4\pi} \frac{\epsilon_1(\omega) - 1}{\epsilon_1(\omega) + 2} V, \quad (2)$$

and performing the transformation indicated in the beginning of the article, we obtain a final equation for the interaction energy in the liquid

$$U = \frac{27 \hbar V^2}{16 \pi^3 R^6} \int_0^\infty \left[ \frac{\epsilon_1(i\xi) - \epsilon(i\xi)}{\epsilon_1(i\xi) + 2\epsilon(i\xi)} \right]^2 d\xi, \quad \text{for } R \ll \lambda_0,$$

$$U = \frac{207}{64 \pi^3} \frac{V^2}{R^7} \frac{\hbar c}{V \epsilon(0)} \left[ \frac{\epsilon_1(0) - \epsilon(0)}{\epsilon_1(0) + 2\epsilon(0)} \right]^2, \quad \text{for } R \gg \lambda_0. \quad (3)$$

We note that for the second equation in (3) to be applicable it is enough that the dimensions of the particles be small only compared with the distance between them (and not compared with  $\lambda_0$ ).

Equations (3) can be also rewritten in a different form, taking into account the fact that the change in the dielectric constant of a liquid, due to the presence of  $N$  particles per unit volume, is equal to

$$\delta\epsilon = 3NV(\epsilon_1 - \epsilon) \epsilon / (\epsilon_1 + 2\epsilon). \quad (4)$$

(where  $NV \ll 1$ ; see, for example, reference 3, problems of Sec. 9).

Using (4), we rewrite (3) in the form

$$U = \frac{3\hbar}{16\pi^3 R^6} \int_0^\infty \left( \frac{\partial \epsilon(i\xi)}{\partial N} \right)^2 \frac{d\xi}{\epsilon^2(i\xi)}, \quad \text{for } R \ll \lambda_0,$$

$$U = \frac{23 \hbar c}{64\pi^3 R^7 \epsilon^{3/2}(0)} \left( \frac{\partial \epsilon(0)}{\partial N} \right)^2, \quad \text{for } R \gg \lambda_0 \quad (5)$$

[ $\epsilon(\omega)$  is the dielectric constant of the mixture and  $N$  is the number of particles per unit volume].

We note that in this form Eqs. (5) describe not only the interaction of macroscopic particles sus-

ended in a liquid, but also the interaction of particles with dimensions on the order of interatomic distances, as well as the interaction between molecules of a dissolved substance in a solution. Here, however, the value of the quantity  $\partial \epsilon(\omega) / \partial N$  — the derivative of the dielectric constant of the solution with respect to the concentration — can naturally no longer be calculated from Eq. (4), but must be obtained directly from experiment.

\*We use the equations for the absolute temperature zero, and the influence of the temperature on the interaction forces under ordinary conditions is very small.

<sup>1</sup>I. E. Dzyaloshinskiĭ and L. P. Pitaevskiĭ, J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 1797 (1959), Soviet Phys. JETP **9**, 1282 (1959).

<sup>2</sup>Dzyaloshinskiĭ, Lifshitz, and Pitaevskiĭ, J. Exptl. Theoret. Phys. (U.S.S.R.) **37**, 229 (1959), Soviet Phys. JETP **10**, 161 (1960).

<sup>3</sup>L. D. Landau and E. M. Lifshitz, *Электродинамика сплошных сред, (Electrodynamics of Continuous Media)*, Gostekhizdat, 1957.

Translated by J. G. Adashko

112

## A SIMPLE MODEL IN THE THEORY OF SUPERCONDUCTIVITY

Yu. B. RUMER

Institute of Radiophysics and Electronics,  
Siberian Branch, Academy of Sciences,  
U.S.S.R.

Submitted to JETP editor May 16, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **37**, 578-580  
(August, 1959)

THE key to the understanding of superconductivity lies in the Cooper phenomenon, i.e., in the fact that two electrons with opposite momenta and spins near the Fermi surface can form bound states. These states obviously represent bosons, which form a condensate at low temperatures.

In constructing a theory of superconductivity it is then natural to take the Bose condensation of these bosons explicitly into account. In analogy to the theory of superfluidity of Bogolyubov we therefore introduce the boson creation and annihilation operators  $c^+(q)$  and  $c(q)$ .

The Hamiltonian which takes account of the