allel to the direction of motion of the mesons. Setting $s_\mu s'_\mu = -1$ in Eq. (3) we find

$$dw_1^s = dw_{-1}^s = (e^4 d\Omega / 4c\hbar^2 L^3 K_\mu^2)(1 + \cos^2\theta). \qquad (4)$$

In order to find the transition probability $\mu^+\mu^- \rightarrow e^+e^-$ in the third ortho-state where the projection of the total spin of the mesons onto their direction of motion is zero, we introduce the appropriate symmetric combination of the spinor amplitudes directly into Eq. (1) and write it in the form

$$dw_0^s = (e^4 d\Omega / 8c\hbar^2 L^3 K_\mu^2)(b_e^+ \alpha_{1\nu} b'_e b'^+_e \alpha_{2\nu} b_e) \cdot 2^{-1/2} \{b_\mu^+(1) \alpha_{1\nu} b'_\mu(1) + b_\mu^+(-1) \alpha_{1\nu} b'_\mu(-1)\} \cdot 2^{-1/2} \{b'^+_\mu(1) \alpha_{2\nu} b_\mu(1) + b'^+_\mu(-1) \alpha_{2\nu} b_\mu(-1)\}. \qquad (5)$$

For simplicity let us choose the z axis along the direction of motion of the mesons. One can then show that $b(s) = \rho_3\sigma_1 b(-s)$. With this fact in mind we obtain from Eq. (5), after summing over the electron and positron spins, the following expression for the transition probability in the third ortho-state:

$$dw_0^s = (e^4 d\Omega / 4c\hbar^2 L^3 K_\mu^2) \times (1 - k_\mu^2 / K_\mu + k_{0\mu}^2 / K_\mu^2)(1 - \cos^2\theta). \qquad (6)$$

A comparison of Eqs. (4) and (6) shows that the transition probability in the orthostate depends strongly on the value of the projection of the total spin of the mesons onto their direction of motion. When this projection equals ±1 the electrons are emitted mainly along the direction of motion of the mesons, whereas when this projection equals 0 the electrons are emitted mainly in a direction perpendicular to the line of motion of the mesons.

In the nonrelativistic approximation ($k_\mu \rightarrow 0$, $\cos^2\theta \rightarrow 1/3$) the $\mu^+\mu^- \rightarrow e^+e^-$ transition probability is the same in all three ortho-states and is equal to

$$w^s = 4\pi e^4 / 3c\hbar^2 L^3 k_{0\mu}, \qquad (7)$$

which agrees with the result of Zel'dovich. However as the meson energy increases w_0^S decreases much faster then w_1^S, w_{-1}^S and in the extreme relativistic limit (when $K_\mu \gg k_{0\mu}$) the probability w_0^S vanishes.

As can be seen from Eqs. (3), (4), and (6) the $\mu^+\mu^- \rightarrow e^+e^-$ transition probability in the parastate is zero not only in the nonrelativistic approximation but in general.

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AN INVESTIGATION OF THE QUANTIZED OSCILLATIONS IN THE MAGNETIC SUSCEPTIBILITY OF BISMUTH AT EXTREMELY LOW TEMPERATURES

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It can be deduced from galvanomagnetic measurements that bismuth belongs to the group of metals having equal numbers of electrons and holes. We can now regard it as established that a portion of the Fermi surface for groups of electrons is well described by Shoenberg's three-ellipsoid model,[1] proposed on the basis of the measurement of quantized oscillations of the magnetic susceptibility of bismuth at helium temperatures.

Experiments by other authors on oscillations of magnetic susceptibility,[2,3] electrical resistance and Hall emf[4,5,6] in high magnetic fields, related to another part of the Fermi surface, have not until now yielded positive results. We thought that these oscillations were not observed at helium temperatures because of their small amplitude, which would be sufficiently enhanced for observation at much lower temperatures. For this purpose we developed the apparatus[7] and measured the anisotropy of magnetic susceptibility of bismuth at extremely low temperatures. These experiments are of interest in themselves since, as far as we know, the magnetic susceptibility of metals and semiconductors has not before been studied at these low temperatures.

The apparatus, which is a torsion balance, is shown in Fig. 1. The salt pill, 1, with the heat link,[8] 2, and the sample holder, 3, are fixed to the balance suspension system. The glass sleeve for the balance, 4, consists of two tubes separated

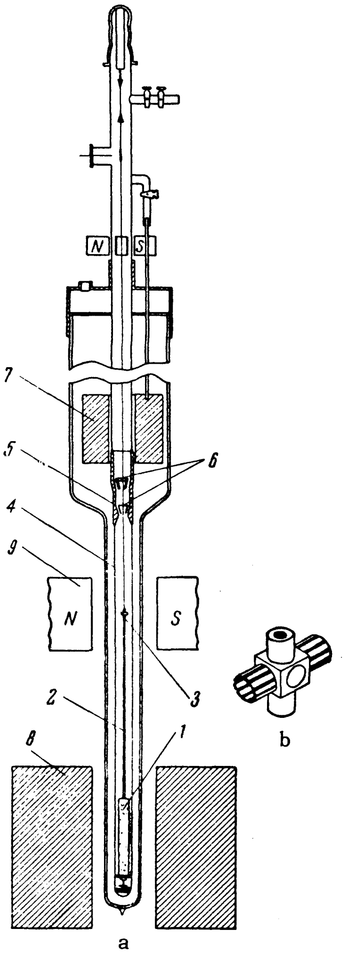

FIG. 1. a – Schematic drawing of the apparatus. b – Specimen holder.

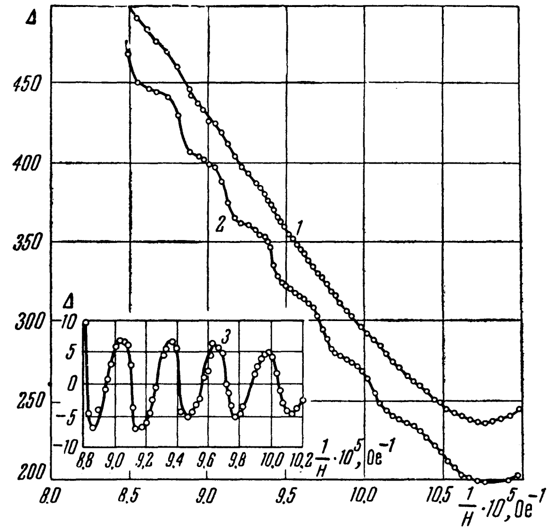

FIG. 2. Dependence of the moment of the forces, Δ, (in arbitrary units) acting on the specimen, on the magnetic field strength for $\psi = 81.5°$. Curve 1 is for T = 1.6°K, curve 2 for T = 0.07°K. Curve 3 represents the high frequency component deduced, at T = 0.07°K.

by the copper connector, 5, inside which two copper pieces are freely fixed, with holes in their centers, to shield the specimen and salt from radiation. The sleeve, 4, is in the liquid helium bath and can be evacuated by the charcoal pump, 7. After adiabatic demagnetization of the salt the balance was lowered until the pill was in the center of the magnetic shield, 8, while the specimen was in the center of the pole pieces, 9, of the electromagnet. In a single-run adiabatic demagnetization of the salt and cooling of the specimen could be repeated 6 or 7 times. The temperature was deduced from the susceptibility of the salt. The apparatus heated up from 0.06° to 0.1° K in 60 to 70 minutes, so that curves could be plotted of the dependence of the moment of the forces, Δ, due to the anisotropy of the specimen, acting in the plane perpendicular to the balance axis, on the direction of the uniform magnetic field H, within a temperature interval of about 0.02° K.

We used a cylindrical single crystal of bismuth of diameter 3.6 mm and length 7 – 8 mm, grown from Hilger bismuth, purified by thirty vacuum recrystallizations. The trigonal axis was perpendicular to the axis of the balance and the binary axis parallel to it. The error in orientation of the axes was not more than 0.3 – 0.4°.

At the lowest temperatures high frequency oscillations can be seen, superimposed on the curves of the low frequency oscillations. Figure 2 shows the dependence of Δ on H for a particular value of ψ, the angle between the direction of **H** and the trigonal axis of the specimen.

The frequency of oscillation of the susceptibility (or of Δ) with changing H is proportional to the area of the corresponding extreme cross-section, S_m, of the Fermi surface perpendicular to **H**.[9] The angular dependence of S_m for the

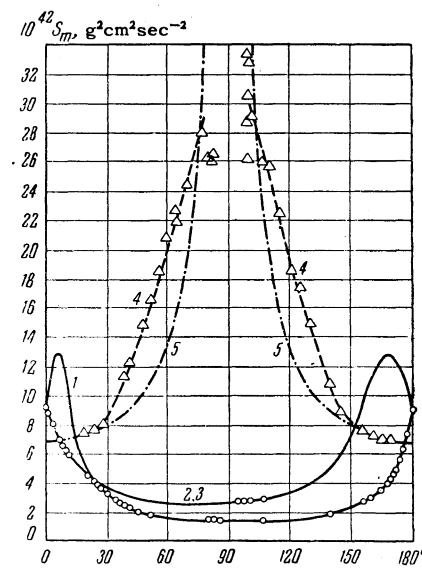

FIG. 3. Angular dependence of S_m for the new oscillations (curve 4) and for the oscillations connected with Shoenberg's three-ellipsoid model (curves 1, 2, 3).

new oscillations is shown in Fig. 3 by the dashed curve. It appears that these oscillations correspond to a group of holes which have a Fermi surface in the form of a surface of revolution, stretched out along the trigonal axis.* On the assumption that this surface is closed, a rough calculation leads to a value of the "hole" concentration $n \approx 0.5 \times 10^{18}\ \mathrm{cm}^{-3}$ and an effective mass along the trigonal axis $m_3^* = (\frac{1}{2}\pi)(\partial S_m/\partial E) \approx 0.06\ m_0$ (m_0 is the free electron mass). It is most probable that the high frequency oscillations in the angular range $105° > \psi > 75°$ belong to another group of charge carriers. (Measurements in much greater magnetic fields are necessary for studying this question.) This is in agreement with the suggestion that there must be at least three charge carrier groups in bismuth.[10]

We are most grateful to A. M. Kosevich for discussion of the results, to A. I. Shal'nikov for his interest in the work and to M. V. Volkova for assistance with the measurements.

*More exact measurements show that the part of this surface, corresponding to the angular range $180° \geqslant \psi \geqslant 105°$ and $75° \geqslant \psi \geqslant 0°$, approximates an ellipsoid.

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POLARIZATION EFFECTS IN THE $\pi^0 \to e^- + e^+ + \gamma$ *DECAY*

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A number of events were recently observed[1,2] which corresponded to a charge exchange scattering of π^- mesons on hydrogen ($\pi^- + p \to \pi^0 + n$) followed by the decay of the π^0 meson into a Dalitz pair and a photon:

$$\pi^0 \to e^- + e^+ + \gamma. \tag{1}$$

The probability for the decay (1), summed over the polarizations of the particles in the final state, was calculated by Dalitz[3] and Kroll and Wada.[4] In this paper we give the results of a calculation of the probability for the π^0 meson to decay according to the mode (1), taking into account the spin states (longitudinal polarizations) of the electron-positron pair and of the photon.

The direct interaction Hamiltonian for process (1) is given by

$$H_{\mathrm{int}} = eg\,\psi_{\pi^0}\,\{\psi_{e^-}^{+} O_i D^{-1} (\boldsymbol{\alpha}\mathbf{A}^{+})\,\psi_{e^+} + (\psi_{e^-}^{+} \boldsymbol{\alpha}\mathbf{A}^{+} D^{-1})\,O_i \psi_{e^+}\}. \tag{2}$$

Here ψ_{π^0}, $\psi_{e^-}^{+}$, ψ_{e^+} and A^+ are the wave functions of the π^0 meson, electron, positron and photon respectively; D is the Dirac operator; $\boldsymbol{\alpha} = \rho_1 \boldsymbol{\sigma}$ is a Dirac matrix; if the π^0 meson is pseudoscalar $O_i = \rho_2$ and if it is scalar $O_i = \rho_3$.

The field amplitude of a circularly polarized photon is given by the formula[5]

$$\mathbf{a}_l^{+} = (\beta - il\,[\mathbf{n}\times\beta])/\sqrt{2}, \qquad \mathbf{n} = \varkappa/\varkappa, \tag{3}$$

where $\boldsymbol{\kappa}$ is the photon wave vector and $\boldsymbol{\beta} \perp \mathbf{n}$ is an arbitrary unit vector. For a right-circularly polarized photon $l = 1$ (spin parallel to $\boldsymbol{\kappa}$) and for a left-circularly polarized photon $l = -1$ (spin antiparallel to $\boldsymbol{\kappa}$). We made use of formula (21.15) in Sokolov's[5] book to calculate the matrix elements of the decay (1) with longitudinal polarization of the pair taken into account. We obtain the following expression, in the rest system of the π^0 meson (pseudoscalar), for the probability for the decay (1) with prescribed polarizations of the particles:

$$dW(s_-, s_+, l, \theta) = \frac{e^2 g^2}{\hbar^2 c 4 (2\pi)^3} \frac{k_+^2 d\Omega_+ (dk_-)}{k_{0\pi} k_+ K_- (k_{0\pi} - K_-) + k_{0\pi} K_- k_- K_+ \cos\theta}$$

$$\times \{\Phi_1 + s_- s_+ \Phi_2 + l s_- \Phi_3 + l s_+ \Phi_4\}, \tag{4}$$