

$$f(x, p) = \exp(p_i q_i) \left\{ \frac{n\sigma}{4\pi (mc)^3 K_3(\sigma)} + \frac{\sigma^2 \tau_{ik} \xi_i \xi_k}{8\pi m^4 c^5 K_3(\sigma)} \right. \\ \left. + \frac{\sigma^{1/2}}{24\pi m^5 c^6 K_4(\sigma)} \left[ R_{ikl} + \frac{(mc)^2 [K_4^2(\sigma) - K_5(\sigma) K_3(\sigma)]}{[K_3^2(\sigma) - K_4(\sigma) K_2(\sigma)]} \right] \right. \\ \left. \times (\nu_i U_k U_l + \nu_k U_l U_i + \nu_l U_i U_k) \right] \\ \times \left[ \xi_i \xi_k \xi_l - \frac{K_4(\sigma)}{K_3(\sigma)} (\xi_i \delta_{kl} + \xi_k \delta_{il} + \xi_l \delta_{ik}) \right],$$

where the  $q_i$  have the same meaning as in reference 2, and  $\nu_i$  is an additional term in the vector of material flow density, due to the dissipative process.

Inasmuch as  $T_{ijl} = -m^2 c^2 (cnU_l + \nu_l)$ , the parameters that determine the state of the gas are  $n$ ,  $T$ ,  $U_i$ ,  $\tau_{ik}$ , and  $R_{ikl}$ . Using Eq. (3) and the requirement that the mean energy be expressed only in terms of the Maxwellian portion of the distribution function, it is easy to show that  $\tau_{ij} = 0$ .

In conclusion, I consider it my pleasant duty to thank Prof. V. L. German for interest in the work and for valuable advice, and to G. I. Budker and S. I. Braginskiĭ for very valuable discussions.

\*The Greek indices run through three values, and the Latin ones through four; repeated indices imply summation.

<sup>1</sup>H. Grad, Commun. on Pure and Appl. Math. 2, 331 (1949).

<sup>2</sup>S. T. Belyaev and G. I. Budker, Dokl. Akad. Nauk SSSR 107, 807 (1956), Soviet Phys.-Doklady 1, 218 (1956).

Translated by J. G. Adashko

99

## ELECTRON RESONANCE IN CROSSED ELECTRIC AND MAGNETIC FIELDS

I. M. LIFSHITZ and M. I. KAGANOV

Technical Institute, Academy of Sciences,  
Ukrainian S.S.R.

Submitted to JETP editor May 21, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) 37, 555-556  
(August, 1959)

IT is known that a free electron placed in crossed electric and magnetic fields drifts in the direction perpendicular to the electric and magnetic fields. The drift velocity, i.e., the mean velocity of the

electron's motion\* (apart from a part dependent on its initial velocity), is equal to  $\bar{\mathbf{v}} = cH^{-2} [\mathbf{E} \times \mathbf{H}]$ , where  $\mathbf{E}$  and  $\mathbf{H}$  are the intensities of the electric and magnetic fields. In addition, the electron executes an oscillatory motion along the electric field, at a frequency  $eH/mc$ ; that is, the frequency in crossed fields does not depend on the electric field.<sup>1</sup>

The situation is different for an electron in a metal or semiconductor. The complicated dispersion law has a pronounced effect on the character of the motion of a conduction electron. We shall start from the classical equation of motion, the Lorentz generalized equation,

$$d\mathbf{p}/dt = e \{ \mathbf{E} + c^{-1} [\mathbf{v} \times \mathbf{H}] \}, \quad \mathbf{v} = \partial \epsilon / \partial \mathbf{p}. \quad (1)$$

It is easy to show that the integrals of the motion in this case are

$$\epsilon^*(\mathbf{p}) \equiv \epsilon(\mathbf{p}) - \mathbf{v}_0 \mathbf{p} = \text{const},$$

$$\mathbf{v}_0 = cH^{-2} [\mathbf{E} \times \mathbf{H}], \quad p_z = \text{const}. \quad (2)$$

The  $z$  axis is taken along the magnetic field;  $\mathbf{v}_0$  coincides with the mean velocity of the electron's motion, i.e., with the drift velocity, only in case the trajectory of the electron in momentum space, as determined by Eq. (2), is closed. In fact, on introducing the velocity  $\mathbf{v}_0$  in Eq. (1) we get

$$d\mathbf{p}/dt = (e/c) [\mathbf{v} - \mathbf{v}_0] \times \mathbf{H}. \quad (3)$$

From this it is clear that  $\bar{\mathbf{v}} = \mathbf{v}_0$  if  $\overline{d\mathbf{p}/dt} = 0$ . This happens in the case of closed trajectories.<sup>2</sup>

Equation (3) shows that the motion in crossed fields of a particle with the dispersion law  $\epsilon = \epsilon(\mathbf{p})$  can be treated as motion in a magnetic field alone of a particle with the dispersion law†

$$\epsilon^*(\mathbf{p}) = \epsilon(\mathbf{p}) - \mathbf{v}_0 \mathbf{p}. \quad (4)$$

Therefore the results obtained before are easily transferred to this case. In particular, the period  $T^*$  of revolution of an electron around a closed orbit is<sup>2,3</sup>

$$T^* = - (c/eH) \partial S^* / \partial \epsilon^*. \quad (5)$$

Here  $S^*$  is the area bounded by the curve determined by Eq. (2); it depends, naturally, on the electric field. It is interesting to note that this dependence disappears in the case of a quadratic dispersion law: the presence of the term  $-\mathbf{v}_0 \mathbf{p}$  in the Eq. (2) merely perturbs the trajectory without changing its area. Thus the dependence of the period of revolution on the electric field is an effect specific to an electron with a complicated (non-quadratic) dispersion law. It should be noticed that cases are possible in which a conduction

electron in a magnetic field executes a finite motion (its trajectory in momentum space being a closed curve), whereas in crossed fields it executes an infinite one, since the trajectory (2) is an open curve.

The explicit dependence of the period on the electric field can be obtained only for a definite law of dispersion. However, if  $E/H \ll 1$ , it is possible to obtain the result

$$T^* \approx T \left\{ 1 - (c v_0 / e H T) \oint v_{\perp}^{-2} dl (\mathbf{n} + \mathbf{p}/R) \right\}. \quad (6)$$

Here  $T$  is the period of revolution in a magnetic field,  $\mathbf{n} = \mathbf{v}_{\perp} / |\mathbf{v}_{\perp}|$  is the normal to the trajectory of the electron in a magnetic field, and  $R$  is the radius of curvature of the trajectory. The integration extends over the trajectory in a magnetic field. Thus  $\Delta T/T \sim (c/v)(E/H)$ .

Once we know the frequency of revolution of the electron ( $\omega^* = 2\pi/T^*$ ), it is easy to write down the distance between quantum energy levels in the classical approximation:<sup>4,5</sup>

$$\Delta \varepsilon^* = \hbar \omega^* = 2\pi |e| \hbar H / c (\partial S^* / \partial \varepsilon^*).$$

In connection with the dependence of the frequency of revolution of an electron in crossed fields on the size of the electric field, an interesting peculiarity should apparently occur in diamagnetic resonance in those semiconductors in which the dependence of the energy of the current carriers on the quasimomentum is appreciably nonquadratic: the resonance frequency should depend on the electric current passed through the specimen.

A nonquadratic dependence of the energy on the components of the quasi-momentum occurs not infrequently near the edge of the conduction band. Often it is a consequence of the crystal symmetry. Here the quadratic dependence on the magnitude of the momentum is retained near the edge of the band, but the angular dependence becomes complicated. Thus the energy spectrum of "holes" in Ge and Si crystals has the form<sup>6</sup>

$$\varepsilon = A p^2 \pm [B^2 p^4 + C^2 (p_x^2 p_y^2 + p_x^2 p_z^2 + p_y^2 p_z^2)]^{1/2},$$

where  $A$ ,  $B$ , and  $C$  are constants.

To observe such effects in metals is in all probability impossible, since in a metal (in consequence of the large electrical conductivity) it is impossible to produce any appreciable electric field. To estimate the order of magnitude of the effect, we must start from formula (6), remembering however that the resonance frequencies are determined not by all the electrons but by those that have extremal effective masses.<sup>7</sup> It can be shown that for these electrons no effect

linear in the electric field is present because of the symmetry of the trajectory. Therefore, apparently,  $\Delta\omega/\omega \sim (c/v)^2 (E/H)^2$ .

\*We have in mind the mean velocity in a plane perpendicular to the magnetic field.

<sup>†</sup>Except for an unimportant constant,  $\varepsilon^*$  coincides with the total energy of the particle.

<sup>1</sup>L. D. Landau and E. M. Lifshitz, *Теория поля (Field Theory)*, Gostekhizdat, 1948 [Transl: Addison-Wesley, 1951].

<sup>2</sup>Lifshitz, Azbel', and Kaganov, *J. Exptl. Theoret. Phys. (U.S.S.R.)* **31**, 63 (1956), *Soviet Phys. JETP* **4**, 41 (1957).

<sup>3</sup>W. Shockley, *Phys. Rev.* **79**, 191 (1950).

<sup>4</sup>I. M. Lifshitz, Report at a session of the physics-mathematics section, Academy of Sciences, Ukr. S.S.R., 1951; I. M. Lifshitz and A. M. Kosevich, *J. Exptl. Theoret. Phys. (U.S.S.R.)* **29**, 730 (1955), *Soviet Phys. JETP* **2**, 636 (1956).

<sup>5</sup>L. Onsager, *Phil. Mag.* **43**, 1006 (1952).

<sup>6</sup>Dresselhaus, Kip, and Kittel, *Phys. Rev.* **98**, 368 (1955).

<sup>7</sup>M. Ya. Azbel' and É. A. Kaner, *J. Exptl. Theoret. Phys. (U.S.S.R.)* **32**, 896 (1957), *Soviet Phys. JETP* **5**, 730 (1957).

Translated by W. F. Brown, Jr.

100

## IDENTIFICATION OF PARTICLES IN HIGH ENERGY STARS

G. I. KOPYLOV

Joint Institute for Nuclear Research

Submitted to JETP editor April 17, 1959

*J. Exptl. Theoret. Phys. (U.S.S.R.)* **37**, 557-558 (August, 1959)

THE identification of particles in high energy stars\* is often made by comparing the measurements of the momentum  $p_1$  of one of the particles with its possible limiting values under predetermined assumptions about the mass and the number of remaining particles  $2, 3, \dots, n$ . These last are united into one composite particle having some effective mass  $m_{\text{eff}}$ . The formula for the momentum of particle 1 at an angle of observation  $\theta_1$  under the assumption that the other particle has a mass  $m_{\text{eff}}$  gives the limiting pos-