Letters to the Editor

ON AN ESTIMATE OF ENERGY CHARACTERISTICS OF SHOWER PRODUCING PARTICLES

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It has been shown that the values of the transverse momenta of particles $p_\perp$ (in units of μc, where μ is the particle mass) produced in cosmic ray showers are distributed in such a way that $p_\perp \sim 1$,[1-3] while separate values do not differ greatly from the average. This experimental fact, together with the assumption about the symmetrical emission of shower particles in the c.m.s., makes it possible to determine the parameter $\gamma_c = 1/\sqrt{1-\beta_c^2}$, where β_c is the velocity of the center of mass with respect to the laboratory system of coordinates. Correspondingly, we shall consider two variants of the symmetrical emission in c.m.s.: (a) exact angular symmetry (to each particle with emission angle θ_i' with respect to the axis there corresponds another with angle $\theta_j' = \pi - \theta_i'$); (b) an equal number of particles is contained on the two sides of the plane perpendicular to the motion of the center of mass.

Making use of the relation between the angles of emission of particles in c.m.s. and in the laboratory system

$$\cot\theta_i' = \gamma_c \cot\theta_i - \sqrt{\gamma_i^2 - 1}\,\sqrt{z_i^2 + \cot^2\theta_i},\qquad z_i^2 = (p_{\perp i}^2 + 1)/p_{\perp i}^2, \tag{1}$$

we can obtain for case (a)

$$\gamma_c = \varphi_1/\sqrt{\varphi_1^2 - \varphi_2^2},\qquad \varphi_1 = \sum_{i=1}^{n_s} \sqrt{z_i^2 + \cot^2\theta_i},\qquad \varphi_2 = \sum_{i=1}^{n_s} \cot\theta_i; \tag{2}$$

In case (b), for the medium value (Y_m) of the value $Y_i = \sqrt{z_i^2 + \cot^2\theta_i}/\cot\theta_i$, the following relation is true:

$$\gamma_c = Y_m/\sqrt{Y_m^2 - 1}. \tag{3}$$

The fraction of the energy transferred to mesons can be estimated from the relation

$$K \approx 1.5 \sum_{i=1}^{n_s} \varepsilon_i'/E';\ \varepsilon_i' = \mu_i c^2 p_{\perp i} \times \{\gamma_c \sqrt{z_i^2 + \cot^2\theta_i} - \sqrt{\gamma_c^2 - 1}\cot\theta_i\}, \tag{4}$$

where, for a nucleon-nucleon collision, $E' = 2Mc^2(\gamma_c - 1)$.

Making use of Eqs. (1) – (4) for "narrow" showers ($\cot^2\theta_i \gg z_i^2 \approx 1$; $\gamma_c^2 \gg 1$), we can obtain the simplified relations

$$\text{a)}\quad \gamma_c = \Big[\sum_{i=1}^{n_s} \cot\theta_i \Big/ \sum_{i=1}^{n_s} z_i^2 \tan\theta_i\Big]^{1/2};$$

$$\text{b)}\quad \gamma_c = \left(\frac{\cot\theta_i}{z_i}\right)_m,\qquad \varepsilon_i' \approx \frac{\mu_i c^2}{2} p_{\perp i}\left\{\frac{\gamma_c z_i^2}{\cot\theta_i} + \frac{\cot\theta_i}{\gamma_c}\right\}. \tag{5}$$

For the sake of comparison, the table lists showers in which it was found to be possible to measure the energy of secondary particles.[4-8] This made it possible to use formulae (4) and (5) directly for an estimate of γ_c and K (columns 3 – 7) under the assumption that all shower particles are π mesons. In formulae (2) to (5), all information which can be obtained from the measurements of the angles and momenta of secondary particles in the laboratory system is used.

Allowance for the distribution of transverse momenta of the particles leads to lower values of

Type of Shower	γ_c	a) γ_c	b) γ_c	a) K	b) K	$p_{\perp i}$	$p_\perp = 1$; $z = 1.5$; a) γ_c	b) γ_c	a) K	b) K
1	2	3	4	5	6	7	8	9	10	11
2+15 (p) [4]	204	>60	220	≤0.4	0.1	≥1	160	200	0.1	0.1
3+39 (p) [5]*	44	33	27	0.8	1.2	2.3	38	25	0.5	0.8
2+16 (p) [6]†	52	35	34	0.2	0.2	1.2	32	42	0.2	0.2
2+14 (p) [7]	78	64	53	0.1	0.2	1.2	59	44	0.1	0.1
0+ 7 (p) [8]	29	10	20	0.2	0.1	1.3	13	15	0.1	0.1

*A more detailed analysis shows that, evidently, a collision of the primary nucleon with a group of nucleons in the nucleus has occurred in this case.

†In a subsequent study it was found that one of the particles (number 15) was not a π meson. In the estimate of γ_c (column 3 and 4), this particle was not taken into account.

γ_c as compared with the values obtained by the method of the Rome group[1] (column 2).

With the exception of one shower,[5] the inelasticity factor is substantially smaller than 1. The error in the determination of γ_c and K, arising out of the fluctuation of the angular distribution of particles can be taken into account analogously to the procedure in reference 9.

The average value of the transverse momentum (column 7) lies within the limits of 1 to 2. To explain how the assumption of constant transverse momentum influences the estimated values of γ_c and K, the latter were calculated from Eqs. (4) and (5) for the value $p_{\perp i} \approx 1$. The assumption of constant transverse momentum ($p_\perp \approx 1$) does not lead to substantial changes of the estimated values (columns 8 to 11). This makes it possible to generalize the described method for an estimate of the energy characteristics (γ_c and K) in showers in which only the angular distribution of secondary shower particles is known. It should be noted that the estimated values of γ_c found by such a method are in good agreement with the values obtained by Takibaev under the assumption of a power-law energy spectrum of produced mesons.

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ON THE METHODS OF BORN AND PAIS FOR FINDING PHASE SHIFTS

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As is well known, in cases in which the Born approximation for finding the phase shifts is not applicable, we must use some other more accurate method, for example the method of Pais. The purpose of this note is to give a brief derivation of the approximations of Born[1] and Pais[2] and also a numerical comparison of the phase shifts obtained by these two methods.

Let us write the Schrödinger equation in the following form:

$$y'' + [k^2 - U - l(l+1)/r^2]\, y = 0, \qquad (1a)$$

$$u'' + [k^2 - l(l+1)/r^2]\, u = 0, \qquad (1b)$$

$$v'' + [k^2 - (l(l+1) - a^2)/r^2]\, v = 0, \qquad (1c)$$

$k = mv/\hbar$, and a^2 is a certain constant. The interaction potential is connected with U by the relation $U = (2m/\hbar^2)V$. The exact solutions of Eqs. (1b) and (1c) are well-known:

$$u = \sqrt{\pi kr/2}\, J_{l+1/2}(kr), \qquad v = \sqrt{\pi kr/2}\, J_{\sqrt{(l+1/2)^2 - a^2}}(kr). \qquad (2)$$

They satisfy the following boundary conditions:

$$u(0) = 0, \quad u(\infty) \to \sin(kr - l\pi/2),$$

$$v(0) = 0, \quad v(\infty) \to \sin\left(kr - \frac{\pi}{2}\sqrt{(l+1/2)^2 - a^2} + \frac{\pi}{4}\right). \qquad (3)$$

The exact solution of Eq. (1a) satisfies the boundary conditions

$$y(0) = 0, \quad y(\infty) \to \sin(kr - l\pi/2 + \eta_l). \qquad (4)$$

If we now require that the solution v of Eq. (1c) satisfy the same boundary conditions (4) as the exact solution of Eq. (1a), then the constant a^2 must be

$$a^2 = 4\pi^{-2}[-\eta_l^2 + \pi(l + \tfrac{1}{2})\eta_l]. \qquad (5)$$

Then

$$v = \sqrt{\pi kr/2}\, J_{(l+1/2) - 2\eta_l/\pi}(kr). \qquad (6)$$

Let us multiply Eq. (1a) by u and Eq. (1b) by y, subtract one equation from the other, and integrate from zero to ∞, taking account of the boundary conditions (3) and (4) for u and y. Furthermore, in the integral that contains the interaction poten-