

the forces between them are of the same order of magnitude as nucleon-nucleon forces. However the details of the mechanism responsible for hyperon-hyperon forces may be different from those involved in nucleon-nucleon interactions. Thus, for example, second order forces arising from the exchange of a single π or K meson between two Λ^0 particles are forbidden by isotopic invariance of strong interactions. In these cases the forces may arise in fourth order as a result of the exchange of two π or K mesons:



The main part of these forces has a non-exchange character. The absence of forces due to the exchange of a single particle eliminates the theoretical basis for the introduction of repulsion at short distances as is done in the nucleon-nucleon case.

This different character of hyperon-hyperon forces should affect the behavior of a system of many hyperons. In particular it is possible that conditions may exist favorable to the formation of a hyperon system with a large mass defect which would be stable against transformation into the proton-neutron state. For a system of Λ particles and nucleons the stability condition against transition to the nucleon state has the form

$$L(m_\Lambda - m_N) + B(A + L) + T_N + T_\Lambda + U < 0, \quad (1)$$

where A and L are the number of nucleons and Λ particles; m_N , T_N , m_Λ , T_Λ are the masses and kinetic energies of the nucleons and Λ particles respectively; B is the absolute value of the binding energy per nucleon in nuclear matter; and U is the potential energy due to the interaction between the particles.

To estimate the conditions necessary for the fulfillment of the inequality (1), a calculation of binding energy of a system of nucleons and Λ particles was carried out under certain assumptions about the forces. It was assumed that a Wigner-type short range force acts between two Λ hyperons and between a Λ hyperon and a nucleon which gives rise to zero binding energy for the $\Lambda\Lambda$ and ΛN systems. In the interaction of nucleons with each other only a repulsion at $r_0 = 0.4f$ was taken into account. The nucleons and hyperons were treated as a degenerate Fermi gas. Under these assumptions it was found that when the Λ particles are distributed with constant density inside a sphere of radius $R = r_0 L^{1/3}$, condition (1) is satisfied for $r_0 \approx 0.9f$. Here the mini-

imum energy is obtained if the nucleons are distributed inside a sphere of the same radius and the ratio of nucleons to Λ particles is $A/L \approx 1.6f$.

When condition (1) is satisfied the proton-neutron state will be metastable against a transition to the hyperon-nucleon state.

At present only incomplete information exists about the hyperon-nucleon interaction and none about the hyperon-hyperon forces. Neither can meson theory give an unambiguous answer to this question. Therefore it is not possible to draw any definite conclusions about the behavior of a system of many heavy particles or about the saturation properties of such a system; however neither should one discount the possibility that a stable baryon system other than the proton-neutron state might exist.

¹K. A. Brueckner, *Revs. Modern Phys.* **30**, 561 (1958).

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393

DYNAMICAL MODEL IN THE THEORY OF THE BROWNIAN MOTION

V. B. MAGALINSKIĬ

Moscow State University

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LET us consider a dynamical model that has been discussed repeatedly¹⁻³ in connection with the problem of the reciprocal relations of dynamical processes and statistical laws: an oscillator with mass m and frequency ω_0 linearly coupled with a set of a large number of independent harmonic oscillators with frequencies ω_k ($k = 1, 2, \dots, N$; $N \gg 1$). In the present note we give a simple derivation of some general relations in the theory of the Brownian motion on the basis of this model.

The Hamiltonian of the system in question is written in the form

$$H = \frac{p^2}{2m} + \frac{1}{2} m \omega_0^2 q^2 + \frac{1}{2} \sum_k (p_k^2 + \omega_k^2 q_k^2) + gq \sum_k \alpha_k q_k, \quad (1)$$

where q and p are the coordinate and momentum of the oscillator with frequency ω_0 , q_k and p_k

are those of the k -th oscillator of the medium, and $g\alpha_k$ is the coefficient of the coupling with the k -th oscillator.

Solving the system of Hamilton's equations corresponding to Eq. (1), we get the equation of motion for q :

$$m\ddot{q} + m\omega_0^2 q + \int_0^t K(t-\tau) q(\tau) d\tau - qK(0) + q_0 K(t) = f(t), \tag{2}$$

where

$$K(t) = g^2 \sum_k \alpha_k^2 \omega_k^{-2} \cos \omega_k t, \tag{3}$$

$$f(t) = -g \sum_k \alpha_k \left(q_{k0} \cos \omega_k t + \frac{p_{k0}}{\omega_k} \sin \omega_k t \right), \tag{4}$$

and q_{k0} and p_{k0} are the initial values of the canonical variables.

It can be seen from Eq. (2) that the force caused by the interaction of the particle with the medium falls in a natural way into two parts: the random force (4) (the "impulses"), which does not depend on the state of the particle, and the dissipative force, which is connected with the position of the particle by a functional relation of the type of a persistent action.

Let us consider the correlation of the "impulses," $\kappa(t-\tau) = \overline{f(t)f(\tau)}$, where the averaging is over all the microscopic states of the oscillators of the medium. If the medium is a thermostat, i.e., if these states are canonically distributed, we have

$$\overline{p_{i0} p_{k0}} = \delta_{ik} E(\omega_k, \Theta),$$

$$q_{i0} q_{k0} = \delta_{ik} E(\omega_k, \Theta) \omega_k^{-2}, \quad \overline{q_{i0} p_{k0}} = 0,$$

where

$$E(\omega_k, \Theta) = \frac{1}{2} \hbar \omega_k \coth(\hbar \omega_k / 2\Theta)$$

is the average energy of the k -th oscillator at temperature Θ . In this case it is obvious that

$$\kappa(t-\tau) = g^2 \sum_k \alpha_k^2 \omega_k^{-2} E(\omega_k, \Theta) \cos \omega_k(t-\tau). \tag{5}$$

Assuming the frequency spectrum of the oscillators of the medium sufficiently dense and replacing the sums by integrals by the rule

$$\sum_k F_k = A \int_0^\infty F(\omega) \omega^2 d\omega,$$

we get

$$K(t) = Ag^2 \int_0^\infty \alpha^2(\omega) \cos \omega t d\omega, \tag{3'}$$

$$\kappa(t) = Ag^2 \int_0^\infty \alpha^2(\omega) E(\omega, \Theta) \cos \omega t d\omega. \tag{5'}$$

Let us find the macroscopic physical meaning of the coefficients $\alpha(\omega)$. We introduce the impedance $Z(\omega)$ of the system by the relation

$$\Phi(\omega) = Z(\omega) \dot{q}(\omega), \tag{6}$$

where $\Phi(\omega)$ and $\dot{q}(\omega)$ are the Fourier components of the dissipative force

$$\Phi(t) = \int_0^t K(t-\tau) \dot{q}(\tau) d\tau$$

and the velocity $\dot{q}(t)$, respectively. From the definition (6) and Eq. (3') it at once follows that

$$K(t) = \frac{2}{\pi} \int_0^\infty R(\omega) \cos \omega t d\omega, \quad \alpha^2(\omega) = 2R(\omega) / \pi Ag^2, \tag{7}$$

where $R(\omega) = \text{Re } Z(\omega)$.

Substituting Eq. (7) in Eq. (3') we get, finally, the general formula that connects the correlation of the fluctuation force with the dissipative properties of the system:

$$\kappa(t) = \frac{2}{\pi} \int_0^\infty R(\omega) E(\omega, \Theta) \cos \omega t d\omega. \tag{8}$$

The relation (8) known as the quantum Nyquist formula, was first obtained by Callen and Welton⁴ by an application of quantum-mechanical perturbation theory to calculate the energy exchange between a linear system with prescribed dissipative properties ($Z(\omega)$) and a thermostatic oven.

In conclusion I regard it as my duty to express my gratitude to Professor Ya. P. Terletskiĭ for his interest in this work.

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⁴H. B. Callen and T. A. Welton, *Phys. Rev.* **83**, 34 (1951).