COMMENTS ON THE COVARIANTS IN THE BETA-DECAY INTERACTION

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IN the description of weak interactions, in particular the β-decay interaction, for which the interaction Hamiltonian density is

$$H' = \sum_i H_i' = \sum_i g_i (\bar{\psi}_p O_i \psi_n)(\bar{\psi}_e O_i (1 + \lambda_i \gamma_5)\psi_\nu) + \text{Herm. conj.}$$
$$i = S, V, T, A, P, \qquad (1)$$

only first order perturbation theory is normally used. This is justified by the weakness of the interaction, i.e., by the smallness of the dimensionless expansion parameter $\beta = gK^2/(2\pi)^2$ if a momentum cut-off K is introduced ($\hbar = c = 1$). This condition is satisfied if $r_0 = 1/K \sim 1/m_\pi$ or $\sim 1/m_N$ (m_π and m_N are the π meson and nucleon masses respectively) for $g \sim 10^{-32}\ cm^2$. However it is conceivable that shorter distances are relevant[1] for weak interactions, i.e., higher order corrections may have to be taken into account.* (This question was first discussed for the β-decay interaction by Heisenberg.[2])

Polubarinov[3] investigated n-p scattering due to an interaction of the form $(\bar{p}p)(\bar{n}n)$ and found that the inclusion of higher order approximations substantially modified the results of the first order approximation.

In this note we consider the effect of higher order approximations to interaction (1) on the β decay of the neutron and show that for certain interaction covariants the form of the matrix element does not change from that given by first order perturbation theory. This statement is also valid for all other processes which are allowed by the first order approximation to H', e.g., $\tilde{\nu} + p \rightarrow n + e^+$, provided that one also takes into account the interaction $(\bar{p}n)(\bar{\mu}\nu)$ for the indicated processes, taken in the same form as (1).

Let us assume that $K \gg m_N$, since for $K \sim m_N$ the contribution of higher order terms is small for a conventional value of g. This condition, together with the assumption that the energy of the particles in the center of mass system is $\ll K$, permits one to ignore logarithmically divergent terms in the integrals compared to quadratically divergent ones. For example, in a typical integral

$$\int \frac{\mathrm{Sp}\,\{[i\gamma(q + p/2) - m_1]\, O_i\, [i\gamma(-q + p/2) - m_2] O_j\}}{[(q + p/2)^2 + m_1^2][(q - p/2)^2 + m_2^2]}\, d^4 q$$

we can ignore the term proportional to $m_1 m_2$. Hence the S-matrix will not contain any terms with a linear dependence on the mass of any particle. This fact makes it possible to show that the S matrix for the β decay of the neutron, in the case of interest to us when only a single covariant j is involved ($H' = H_j$), will have the form

$$S^{(j)} = g_j \sum_i \varphi_i^{(j)} (\bar{\psi}_p O_i \psi_n)(\bar{\psi}_e O_i (1 + \Lambda_i^{(j)} \gamma_5)\psi_\nu), \qquad (2)$$

where $\varphi_i^{(j)}$, $\Lambda_i^{(j)}$ are scalar functions of $|g_j|^2$, λ_j, K and the invariants formed from the 4-momenta of the particles (they depend on $|g_j|^2$ because for the processes under study only odd approximations to H' are relevant); ψ_n, ψ_ν, etc., are the field operators in the interaction representation.

Let us introduce the transformation

$$\psi_p \rightarrow \gamma_5 \psi_p,\ \psi_n \rightarrow \gamma_5 \psi_n,\ m_p \rightarrow -m_p,\ m_n \rightarrow -m_n,$$
$$g_j \rightarrow \varepsilon_j g_j,\ \lambda_j \rightarrow \lambda_j, \qquad (3)$$
$$\bar{\psi}_p \rightarrow -\psi_n^T C^{-1},\ \psi_n \rightarrow C\psi_p^T,\ m_p \rightleftarrows m_n,$$
$$g_j \rightarrow -k_j g_j,\ \lambda_j \rightarrow \lambda_j. \qquad (4)$$

$m_n = m_p$; ϵ_j and k_j are determined from the relations $\gamma_5 O_j \gamma_5 = -\epsilon_j O_j$ and $C^{-1} O_j C = -k_j O_j^T$. The superscript T denotes transposition of spinor indices. The transformations (3) and (4) leave the interaction Hamiltonian and the commutation relations invariant and should not change the form of $S^{(j)}$. (The free field Lagrangian may be written in a form invariant under (3) and (4).) It therefore follows that for j = V, T, A in the sum (2) only the one term with $\varphi_j^{(j)}$ survives. For j = S both $\varphi_S^{(S)}$ and $\varphi_P^{(S)}$ may be different from zero. An estimate of the "admixture" $\varphi_P^{(S)}$ using perturbation theory gives zero in third order and $\varphi_P^{(S)} < 0.01\,\varphi_S^{(S)}$ in fifth order. The situation is analogous for j = P.

If $\lambda_j = \pm 1$ then it follows from the invariance of the S matrix under the transformation $\psi_\nu \rightarrow \pm\gamma_5 \psi_\nu$ that $\Lambda_i = \pm 1$ respectively.

In the case of the V-A interaction[4] it is easy to show, using relation (2) and the symmetry of the Hamiltonian under permutations, that again no difference from first order perturbation theory results. The same is true for the combinations S + P − T, 3 (S+P) + T.

Consequently, even if higher order corrections to the β decay interaction are important, under our assumptions it is still possible to decide from experiment what covariants are involved in the Hamiltonian (1).

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*This possibility, although it is not without difficulties, is of interest because it may reduce the number of Hamiltonians of type (1).

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ALPHA DECAY CONSTANTS OF NON-SPHERICAL NUCLEI

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In references 1 — 4, the author developed an analytical theory of alpha decay of non-spherical nuclei,* and simple formulas were given for the relative intensities of the fine structure lines. References 1 and 2 give expressions for wave functions which also make it possible to calculate the absolute decay probability. For even-even nuclei we obtain:

$$W = \frac{3\hbar\varkappa}{mR_0} w_\alpha \Gamma \left| \int_0^1 e^{\beta P_2(\mu)} d\mu \right|^2 \sum_J w_J, \qquad (1)$$

Where W is the decay probability per unit time; $\sum_J w_J$ is the sum of the relative decay probabilities to all rotation levels of the daughter nucleus, including decay to the ground state $w_0 = 1$;

$$\Gamma = \exp\left\{-2\int_{R_0}^{a_0} \sqrt{\varkappa_b^2 R_0/r - k^2}\, dr\right\}$$

$$= \exp\left\{-2\left(\frac{\varkappa_b^2 R_0}{k}\tan^{-1}\frac{\varkappa}{k} - \varkappa R_0\right)\right\};$$

$k = \sqrt{2mE}/\hbar$ is the wave number of an α particle at infinity; $\varkappa_b = \sqrt{4mZe^2/\hbar^2 R_0}$ is the wave number corresponding to the height of the Coulomb barrier for an α particle $2Ze^2/R_0$; m is the reduced mass; $\varkappa = \sqrt{\varkappa_b^2 - k^2}$; a_0 is the return point corresponding to the decay energy E to the ground state; the quantity β is given by the relation

$$\beta = [{}^4/_5\, \varkappa R_0 (1 - k^2/2\varkappa_b^2) - i2k^3R_0/5\varkappa_b^2]\, \alpha_2$$

and is connected with the quadrupole deformation α_2 contained in the equation $R(\mu) = R_0\{1 + \Sigma \alpha_n P_n(\mu)\}$ for the form of the daughter-nucleus surface.

Let us explain the meaning of the "internal probability of formation of an α-particle" w_α. The wave function of the mother nucleus can be expressed in the form:

$$\Psi = \sum_{ik} \psi_{ik}(\mathbf{r})\, \varphi_i^\alpha \varphi_k, \qquad (2)$$

where φ_i^α and φ_k are the wave functions of the stationary state of internal motion of an α particle and a daughter nucleus respectively; **r** is the radius vector of an alpha particle relative to the center of mass of the daughter nucleus. Only the term $\varphi_{00}(\mathbf{r})\,\varphi_0^\alpha \varphi_0$ appears in alpha decay to the ground state. The function $\varphi_{00}(\mathbf{r})$ goes over continuously to the external wave function found in reference 1. Considering the nuclear substance to be homogenous and the mean free path of an alpha particle in it to be small compared to the size of the nucleus, it is natural to assume $\varphi_{00}(\mathbf{r}) = \text{const}$. Then

$$w_\alpha = \int |\psi_{00}|^2 dV = \frac{4}{3}\pi R_0^3 |\psi_{00}|^2 < 1.$$

Since there must be many excited states with short alpha-particle mean free paths in sum (2), we have

$$w_\alpha \ll 1. \qquad (3)$$

With the help of formula (1), we can, from experimental data, determine w_α, for which the calculated results are shown in Table I.† Condition (3) is fulfilled only with $R_0 = 1.4\,A^{1/3} \times 10^{-13}$ cm but when $R_0 = 1.0\,A^{1/3} \times 10^{-13}$ the value of w_α increases by four or five orders of magnitude. With this, the good constancy of w_α in the entire region of alpha active nuclei confirms the reasonableness of the basic premises of the theory. Therefore, for alpha decay $R_0 \geq 1.4\,A^{1/3} \times 10^{13}$ cm.

For non-even nuclei the wave functions derived in reference 2 give