

$$\begin{aligned}
D_a^i &= (f m_a m_b / c^2) \{ [(2 / m_a) I^{(a)} \omega_{s_i}^{(a)} (3 \dot{b}_s - 2 \dot{a}_s) \\
&+ (2 / m_b) I^{(b)} \omega_{s_i}^{(b)} (4 \dot{a}_s - 3 \dot{b}_s)] | \mathbf{a} - \mathbf{b} |^{-5} \\
&+ (2 / m_a) I^{(a)} [3 \omega_{i_j}^{(a)} (a_j - b_j) (a_s - b_s) (\dot{a}_s - \dot{b}_s) \\
&+ 3 \omega_{j_s}^{(a)} (a_j - b_j) (a_i - b_i) (2 \dot{b}_s - \dot{a}_s)] | \mathbf{a} - \mathbf{b} |^{-5} \\
&+ (2 / m_b) I^{(b)} [-6 \omega_{i_j}^{(b)} (a_j - b_j) (a_s - b_s) (\dot{a}_s - \dot{b}_s) \\
&+ 3 \omega_{j_s}^{(b)} (2 \dot{a}_s - \dot{b}_s) (a_j - b_j) (a_i - b_i)] | \mathbf{a} - \mathbf{b} |^{-5} \}. \quad (6)
\end{aligned}$$

As is easily verified, formulas (4) and (6) under conditions (5) only coincide if  $\sigma_a = 0$ ,  $b_s = 0$  (a case considered by Lense and Thirring, Das, and the author, giving the motion of a satellite of small mass around a rotating central body). In the general case (4) and (6) do not coincide. Consequently, Ryabushko's assertion in reference 1 that for spherically symmetric bodies the first members of (1) coincide with the results of Fock<sup>3</sup> is therefore incorrect.

Formula (1) cannot be correct even under the following considerations:  $D_a^i$  from (1) and (6) can be got from Lagrange's equation

$$D_a^i = \partial L'_\omega / \partial a_i - (d / dt) \partial L'_\omega / \partial \dot{a}_i \quad (7)$$

with a correction to the Lagrangian because of the rotation of the bodies

$$\begin{aligned}
L'_\omega &= f c^{-2} [m_a \omega_{s_i}^{(b)} I_{s_j}^{(b)} (2 \dot{b}_i - 4 \dot{a}_i) \\
&+ m_b \omega_{s_i}^{(a)} I_{s_j}^{(a)} (2 \dot{a}_i - 4 \dot{b}_i)] (a_j - b_j) / | \mathbf{a} - \mathbf{b} |^3 \quad (8)
\end{aligned}$$

using condition (5). It is easily verified that, under the interchange of a and b,  $L'_\omega$  goes over to  $-L'_\omega$ . This fact contradicts the requirement that the correction to the Lagrangian, just like the full Lagrangian, must be invariant under the interchange of the two bodies; Fock's correction to the Lagrangian obviously satisfies this requirement.

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## ON THE DEPTH OF THE POTENTIAL WELL FOR $\Lambda$ PARTICLES IN HEAVY HYPERNUCLEI

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THE energy of the  $\Lambda$  particles in heavy hypernuclei, together with the binding energies of the light hypernuclei, imposes certain restrictions on the  $\Lambda$ -nucleon potential. The  $\Lambda$  particles in heavy hypernuclei can be regarded as moving in a square potential well whose depth is determined by the interaction of the  $\Lambda$  particle with the nucleons.<sup>1,2</sup> The author and Lyul'ka<sup>3,4</sup> considered  $\Lambda$ -nucleon potentials derived from meson theory. In order to avoid singularities at small distances, the momenta of the virtual mesons had to be cut off. These potentials yield the correct dependence of the binding energy  $B_\Lambda$  on the number of particles in light hypernuclei. They lead to a stronger interaction of the  $\Lambda$ -nucleon pair in the singlet state, which is in agreement with the value zero for the spin of  $\Lambda H^4$ , as estimated from the ratio of the number of mesonic and non-mesonic decays. In the present note we make an estimate of the potential energy of the interaction of  $\Lambda$  particles in nuclear matter on the basis of the potentials obtained in references 3 and 4. In estimating the potential energy the nucleons in the nuclear matter were regarded as an incompressible degenerate Fermi gas. We carried out calculations for two values of the nuclear matter density, given by the radii  $R = 1.2 A^{1/3} \times 10^{-13}$  and  $R = 1.4 A^{1/3} \times 10^{-13}$ . The actual density lies apparently somewhere between these limits.<sup>5</sup>

In the table we given the values of the potential energy of  $\Lambda$  particles in nuclear matter,  $U^1K$ ,  $U^2\pi$ ,  $UK\pi$ , and  $U^2K$ , for  $\Lambda$ -nucleon potentials corresponding to the exchange of a single K, two  $\pi$ , a K and a  $\pi$ , and two K mesons, respectively. We also list the total potential energy  $U$  (all values are in Mev). In computing these values we assumed two types of coupling between the K mesons and the baryons: the scalar and the pseudoscalar coupling. The coupling between the baryons and the  $\pi$  mesons was assumed to be pseudovector with the coupling constant  $f^2 = 0.08$ . We used a rectangular cut-off with  $k_m = 6\mu_\pi$ . The resulting potential energy  $U$

| K-Baryon coupling                             | $R = 1.4 A^{1/3} \cdot 10^{-12}$ |            |            |          |     | $R = 1.2 A^{1/3} \cdot 10^{-12}$ |            |            |          |     |
|---|----------------------------------|------------|------------|----------|-----|----------------------------------|------------|------------|----------|-----|
|   | $U^{1K}$                         | $U^{2\pi}$ | $U^{K\pi}$ | $U^{2K}$ | $U$ | $U^{1K}$                         | $U^{2\pi}$ | $U^{K\pi}$ | $U^{2K}$ | $U$ |
| Pseudoscalar,<br>$f^2=0,08$                   | +1                               | -50        | +22        | -20      | -47 | +3                               | -78        | +34        | -32      | -73 |
| Scalar, $g_{\Lambda}^2 = g_{\Sigma}^2 = 1.1$  | +17                              | -50        | +25        | -21      | -29 | +25                              | -78        | +39        | -33      | -47 |
| Scalar, $g_{\Lambda}^2 = 3g_{\Sigma}^2 = 1.1$ | +17                              | -50        | +17        | -10      | -26 | +25                              | -78        | +26        | -16      | -43 |

is negative, guaranteeing a bound state for the  $\Lambda$  particle. The data in the table give information on the size of the contributions from the various processes to the potential energy of the  $\Lambda$  particle. In view of the exchange character of the  $1K$  and  $K\pi$  mesonic potentials, these make the interaction of the  $\Lambda$  particles with the nuclear matter nonlocal. However, in the approximation of "zero interaction range," we can also in this case introduce an effective potential for the  $\Lambda$  particles in the nucleus. The data of the table were obtained in this approximation. Owing to the effects of the Pauli principle in the system of nucleons, the contributions to the potential energy from the exchange-type  $1K$  and  $K\pi$  forces are strongly suppressed. They are positive and have about the same absolute value as the contributions from the  $2K$  forces.

The uncertainties in the experimental binding energies for  $\Lambda$  particles in heavy hypernuclei are quite large, and it is therefore impossible to obtain from them any accurate information about the well depth. Apparently, the depth should be 20 to 30 Mev for  $10 < A < 20$ . The depth of the square potential well in hyperfragments with  $A < 10$  is about 20 Mev. The estimates of Dalitz and Downs<sup>6</sup> indicate that the well depth in heavy hypernuclei can be 29 to 38 Mev. The comparison of these data with those of the table shows that the scalar variant is not in disagreement with these data, while the pseudoscalar variant gives too high values for the depth, using our assumed values for the coupling constants.

Since there are at present no accurate experimental data on the binding energies of heavy hypernuclei, it is not possible to make a detailed comparison.

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### ON THE AXIAL ASYMMETRY OF ATOMIC NUCLEI

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To investigate the equilibrium shape of atomic nuclei, we considered previously<sup>1</sup> the behavior of nucleons in an infinite ellipsoidal square well with semi-axes  $a_x r_0$ ,  $a_y r_0$ , and  $a_z r_0$ . This was done by means of a coordinate transformation which transforms the ellipsoid into a sphere of radius  $r_0$ . In the new coordinates the kinetic-energy operator of the nucleons has two parts,  $-\hbar^2/2M\Delta$  and  $\hat{V}$ . The second part,  $\hat{V}$ , can be expanded into a series in powers of the deformation with a linear leading term. It is convenient to choose as the deformation parameters  $\rho$  and  $\gamma$  which are connected with the semi-axes by the relations

$$a_x^{-1} + a_y^{-1} - 2 = \rho \cos \gamma, \quad a_y^{-1} - a_x^{-1} = \sqrt{3} \rho \sin \gamma. \quad (1)$$

It is easy to see that  $\gamma$  coincides with the parameter  $\gamma$  introduced by Bohr,<sup>2</sup> and  $\rho$  is in first order proportional<sup>2</sup> to  $\beta$ :  $\rho \approx (5/4\pi)^{1/2} \beta$ . The parameters  $\alpha$  and  $\delta$  of reference 1 are connected with  $\rho$  and  $\gamma$  by the relations

$$\alpha = \rho \cos \gamma, \quad \delta = -\sqrt{3} \rho \sin \gamma. \quad (2)$$

Considering  $\rho$ , and consequently  $\hat{V}$ , to be small