RADIATIVE CAPTURE OF POLARIZED μ^- MESONS BY NUCLEI

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Submitted to JETP editor November 25, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 1513-1516 (May, 1959)

The correlation between the direction of the μ-meson spin and the direction of emission of the photon in the radiative capture of μ mesons by nuclei is considered, taking into account the interaction between the μ meson and the nuclear spin (hyperfine splitting). The analysis is carried out for nuclei of arbitrary spin J.

It is known[1] that, in the case of radiation capture of a polarized μ^- meson (internal bremsstrahlung), there is correlation between the emission of the photon and the direction of the spin of the μ^- meson when parity is not conserved. An experimental observation of the asymmetry of the emission of the radiative photon makes it possible to establish the type of the interaction between the μ meson and the nucleus (A-V or T-S), if it is known whether the neutrino is right-handed or left-handed.

In the paper by Huang, Yang, and Lee,[1] the result of a calculation of the radiation capture of polarized μ^- mesons by protons is presented for the case of a two-component neutrino. As has been shown by Gershteĭn and Zel'dovich[2] μ^- mesons are fully depolarized in hydrogen because of the transition of μ^- mesons from one proton to another. It is therefore necessary to consider the capture of μ^- mesons by heavy nuclei. In addition, for nucleus with spin different from zero, it is necessary to take into account the depolarization of μ mesons due to interaction of the spins of the μ meson and the nucleus, and to consider separately two possible states with $F = J^{\pm 1/2}$ (hyperfine structure). This has first been shown by Bernstein, Lee, Yang, and Primakoff[3] in an analysis of the normal capture of μ^- mesons by nuclei. In view of the conservation of spin of the system nucleus—μ meson, this depolarization of mesons leads to a partial polarization of the nuclei. In view of this fact, taking into account the superfine structure will, apart from changing the numerical results, lead to a dependence of the studied correlation on the product of the interaction constants $C_A C_V$ and $C_S C_T$: i.e., it becomes possible to study the contribution of interference terms in the μ-meson capture.

CORRELATION IN THE PRESENCE OF HYPERFINE STRUCTURE

Let a μ meson polarized along the z axis be captured by the K orbit of the nucleus producing a mesic atom. Let the projection of the nuclear spin on the z axis be M, and the absolute value of the spin be J.

Since the frequency corresponding to the hyperfine splitting of the mesic atom levels is, in all cases, much larger than the inverse lifetime of the μ meson, one can assume that the mesic atom is in a state described by the wave function

$$\Psi = \sum_F C(F, F_z)\,\Phi_{F,\,F_z} e^{iE_F t}. \tag{1}$$

where F is the total moment of the mesic atom which assumes the two values $F = J \pm 1/2$; $F_z = M \pm 1/2$ is the projection of the total moment on the z axis; ΦF, F_z is the wave function of the system μ^-meson-nucleus in a state with determined F and F_z; E_F is the energy of the hyperfine structure level; and $C(F, F_z) = \begin{pmatrix} J\frac{1}{2}F \\ M\frac{1}{2}F_z \end{pmatrix}$ is the Clebsch-Gordan coefficient (Wigner's notation) corresponding to the condtion that, for $t = 0$, the wave function Ψ is a product of the wave functions of the nucleus with a given M and of the μ^- meson with a given projection s_z ($\psi_M \psi_{1/2}$).

From Eq. (1) we obtain the probability of μ meson capture averaged over time:*

$$W = \sum_F C^2(F, F_z)\,W_{F,\,F_z}, \tag{2}$$

where $W_{F,\,F_z}$ is the probability of capture in the $\Phi_{F,\,F_z}$ state of the mesic atom. Since

*In the above, we do not take into account the fact that the lifetime of the mesic atom with respect to the simple capture of μ^- mesons is different for states with $F = J + 1/2$ and $F = J - 1/2$, which leads to a change of the statistical weight of $W_{F,\,F_z}$. In addition, we do not consider the possibility of transition from states with $F = J + 1/2$ to those with $F = -1/2$.

$$\Phi_{F,F_z}=\begin{pmatrix}J & 1/2 & F\\ M & 1/2 & F_z\end{pmatrix}\left|\psi_M\varphi_{1/2}\right\rangle+\begin{pmatrix}J & 1/2 & F\\ M+1 & -1/2 & F_z\end{pmatrix}\left|\psi_{M+1}\varphi_{-1/2}\right\rangle,$$

$$\begin{aligned}W_{F,F_z}&\sim|(\chi_f|\Omega|\Phi_{F,F_z})|^2\\&=\begin{pmatrix}J & 1/2 & F\\ M & 1/2 & F_z\end{pmatrix}^2|(\chi_f|\Omega|\psi_M\varphi_{1/2})|^2\\&+\begin{pmatrix}J & 1/2 & F\\ M+1 & -1/2 & F_z\end{pmatrix}^2|(\chi_f|\Omega|\psi_{M+1}\varphi_{-1/2})|^2\\&+\begin{pmatrix}J & 1/2 & F\\ M & 1/2 & F_z\end{pmatrix}\begin{pmatrix}J & 1/2 & F\\ M+1 & -1/2 & F_z\end{pmatrix}\\&\times[(\chi_f|\Omega|\psi_M\varphi_{1/2})^*(\chi_f|\Omega|\psi_{M+1}\varphi_{-1/2})\\&+(\chi_f|\Omega|\psi_M\varphi_{1/2})(\chi_f|\Omega|\psi_{M+1}\varphi_{-1/2})^*],\end{aligned}\qquad(3)$$

where χ_f is the wave function of the final state, and Ω is the Hamiltonian of the considered interaction. The probabilities $\omega_{M,\,s_z}\sim|\chi_f|\Omega|\psi_{M s_z})|^2$ are calculated by the usual method and are given in the Appendix (A1). It will be shown that if $\omega_{M,\,s_z}$ is known for any values of M and s_z, then the probability $W_{F,\,F_z}$ can be found without calculating the superposition terms in the expression (3).

In calculating $W_{F,\,F_z}$, we are not interested in the spin and momentum of the recoil nucleus, the direction and polarization of the neutrino, and the polarization of the photon,* i.e. we carry out a summation and integration over the corresponding values. After this, the final state will be characterized by one vector only: the momentum of the photon **k**. The μ-meson capture probability in a mesic atom with a moment **F**, can therefore depend only on the modulus of $|\mathbf{k}|$ and $|\mathbf{F}|$ and on the different powers of the product **kF**.

Consequently, the probability $W_{F,\,F_z}$ is of the form

$$\begin{aligned}W_{F,F_z}&\sim|(\Phi_{F,F_z}|a_F+b_F(\mathbf{Fk})+c_F(\mathbf{Fk})^2|\Phi_{F,F_z})|^2\\&=a_F+b_FF_zk_z+{}^1/_2c_F\{[F(F+1)\\&-F_z^2]k^2+[3F_z^2-F(F+1)]k_z^2\}.\end{aligned}\qquad(4)$$

where a_F, b_F and c_F are coefficients depending only on F and k.

In the first approximation of perturbation theory the probability $W_{F,\,F_z}$ is proportional to k^2, i.e. $a_F\sim k^2$, $b_F\sim|k|$, c_F is independent of k. A power of (**kF**) not higher than two should therefore be taken in Eq. (4).

Equation (4) determines the dependence of the probability on F_z. To find the coefficients a_F, b_F, and c_F, a particular case $F_z=F=J+\frac{1}{2}$ has been considered. Since

$$\begin{pmatrix}J & 1/2 & J+1/2\\ M+1 & -1/2 & J+1/2\end{pmatrix}=0,$$

it follows from Eq. (3) that

$$W_{J+1/2,\,J+1/2}=\omega_{J,\,1/2},\qquad(5)$$

and $\omega_{J,\,\frac{1}{2}}$ is the known probability (see Appendix).

Equation (5) determines a_F, and c_F for $F=J+\frac{1}{2}$. If $W_{J+\frac{1}{2},\,F_z}$ is known, we find $W_{F,\,F_z}$ for the case $F=J-\frac{1}{2}$ from the same expression (3), excluding the superposition term,

$$W_{J-1/2,\,F_z}=\omega_{M,\,1/2}+\omega_{M+1,\,-1/2}-W_{J+1/2,F_z}.$$

The final expression for $W_{F,\,F_z}$ is given in the Appendix.

Taking into account Eqs. (2) and (4), and averaging over F_z, we obtain the correlation function in the form

$$\overline{W}\sim 1+\beta\cos\theta,\qquad(6)$$

where θ is the angle between the direction of the original spin of the μ meson and the momentum of the photon; the value of β is given in the Appendix.

For the case of a two-component neutrino ($C'=\pm C$), we have

$$\begin{aligned}\xi\beta=\pm\frac{1}{(2J+1)^2}\Big\{&|M_F|^2|C_V|^2\\&+\Big[1+\frac{4}{3}J(J+1)\Big]|M_{GT}|^2|C_A|^2\\&+f(J)\frac{4}{3}J(J+1)\left|M_FC_V-\frac{M_{GT}C_A}{\sqrt{J(J+1)}}\right|^2\Big\}\\\mp\frac{1}{(2J+1)^2}\Big\{&|M_F|^2|C_S|^2\\&+\Big[1+\frac{4}{3}J(J+1)\Big]|M_{GT}|^2|C_T|^2\\&+f(J)\cdot\frac{4}{3}J(J+1)\left|M_FC_S-\frac{M_{GT}}{\sqrt{J(J+1)}}C_T\right|^2\Big\},\end{aligned}\qquad(7)$$

where the upper sign should be taken for $C'=C$ and the lower for $C'=-C$.

$$\xi=|M_F|^2(|C_S|^2+|C_V|^2)+|M_{GT}|^2(|C_A|^2+|C_T|^2).$$

and f(J) equals $J_f=J$, $J+1$, J for $J_f=J$, $J-1$, $J+1$ respectively, where J_f is the spin of the recoil nucleus.

For $J=0$, we have the results which can be obtained without taking the superfine structures into account.

$$\beta=\pm(|C_A|^2-|C_T|^2)/(|C_A|^2+|C_T|^2),\quad\text{for}\quad J_f=1$$
$$\beta=\pm(|C_V|^2-|C_S|^2)/(|C_V|^2+|C_S|^2),\quad\text{for}\quad J_f=0.$$

*As is well known,[4] in radiation capture, if the neutrino is a two-component one, the photon is totally circularly polarized.

The experiments of Goldhaber, Grodzins, and Sunyar[7] indicate that, for electronic capture, the neutrino is left-handed. From the conservation of the lepton charge and from the decay mode $\mu^- = e^- + \nu + \bar{\nu}$ it follows that the leptic charge of the μ^- meson is the same as of the electron (in contrast to the assumption of Zel'dovich[5]). One should therefore expect that a left-handed neutrino is emitted in μ^- capture also. The change of the sign of β even in the case where recoil nuclei with various J_f are produced, makes it then possible to determine without ambiguity whether we are dealing with the V-A or S-T type of interaction.

A direct determination of the helicity of the neutrino in μ decay can be carried out by studying the correlation between the direction of emission of the recoil nucleus and the spin direction of the polarized μ meson, as has been recently shown by Treiman.[6]

In conclusion, the author would like to express his gratitude to Ya.B. Zel'dovich for his interest in the work and for valuable remarks.

APPENDIX

We present here certain intermediate and final formulae:

$$\omega_{M, s_z} \sim \xi + (Es_z + GM)\cos\theta + Hs_z M\cos^2\theta,$$

where

$$\xi = {}^1/_2 |M_F|^2 (|C_S|^2 + |C'_S|^2 + |C_V|^2 + |C'_V|^2) + {}^1/_2 |M_{GT}|^2 (|C_A|^2 + |C'_A|^2 + |C_T|^2 + |C'_T|^2),$$

$$E = 2|M_F|^2 \operatorname{Re}(C_V^* C'_V - C_S^* C'_S) + 2|M_{GT}|^2 \operatorname{Re}(C_A^* C'_A - C_T^* C'_T),$$

$$G = \frac{|M_{GT}|^2}{J(J+1)} \operatorname{Re}(C_A^* C'_A - C_T^* C'_T) - \frac{M_F M_{GT}^*}{\sqrt{J(J+1)}} \operatorname{Re}(C_V^* C'_A + C_A^* C'_V - C_S^* C'_T - C_T^* C'_S),$$

$$H = \frac{|M_{GT}|^2}{J(J+1)} (|C_A|^2 + |C'_A|^2 - |C_T|^2 - |C'_T|^2) - \frac{2M_F M_{GT}^*}{\sqrt{J(J+1)}} \operatorname{Re}(C_V^* C_A + C_V'^* C'_A - C_S^* C_T - C_S'^* C'_T),$$

$$W_{F, F_z} \sim \xi - \frac{H}{4} + \frac{(2J+1)G + 2(F-J)(E-G)}{2J+1} F_z \cos\theta + \frac{(F-J)H}{2J+1} \{F(F+1) - F_z^2 + [3F_z^2 - F(F+1)]\cos^2\theta\},$$

$$\xi\beta = [{}^4/_3 (E + 2G) J(J+1) + E]/(2J+1)^2.$$

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Translated by H. Kasha
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