THE KINEMATICS OF ELEMENTARY INTERACTIONS

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Submitted to JETP editor October 4, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 1159-1167 (April, 1959)

Kinematic methods for the analysis of nuclear reactions of fast particles are considered. Application of such a method of analysis of the interactions observed in cloud chambers and photographic emulsions permits us to obtain angular and energy characteristics of the interaction in the center-of-mass system of the colliding particles.

STUDY of the interaction of particles of high energy ($> 10^9$ ev) with nucleons and nuclei are made difficult by two fundamental causes:

a) The absence of any sort of complete theory for the description of the processes of interaction of particles leads to the result that it is not possible to analyze the experimental data from a single point of view. In the majority of cases there are no theoretical representations at all which could be verified or refuted by experiment.*

b) Experimental setups currently used for the study of the interaction of high-energy particles, both in cosmic rays[1-4] and in accelerators,[5-7] as a rule do not yield exhaustive information on the observed processes.

Such a situation leads to the fact that at present partial methods are used for the analysis of the experimental data, which permit us to find individual phenomenological characteristics of the processes.

The purpose of the present research is the consideration of methods of analysis of the nuclear interactions that make use of kinematic relations. Such an analysis permits one to obtain separate particular characteristics of processes that are free from specific model representations.

1. APPLICATION OF CONSERVATION LAWS

In what follows we shall denote the total energy, momentum, angle of flight of the particle relative to the primary, and mass by E, p, θ, and M, respectively.

In the laboratory system of coordinates (l-system) we shall denote the first particle by the index 0, the more rapid of the two nucleons after the nucleon-nucleon collision by the index n, and the much slower nucleon by the index δ. The same system of characterization is preserved in the center-of-mass system of two colliding particles (c-system), but in this case all of the parameters are identified by an additional index c. In what follows we use a system of units in which the velocity of light $c = 1$.

Considering the kinematics of the interaction of fast particles, especially when the colliding particles possess the same mass, it is convenient in a number of cases to transform to the coordinate system of the center-of-mass of the colliding particles, inasmuch as in this system, in the mean, there should be symmetry relative to the plane that is perpendicular to the direction of motion of the primary particle. In a determination in a c-system, $\sum p = 0$, whence is easy to obtain γ_c — the Lorentz factor of the c-system for the case in which one of the interacting particles is at rest in the l-system:

$$\gamma_c = \frac{\gamma_0 + M_t/M_0}{\sqrt{2\gamma_0 M_t/M_0 + (M_t/M_0)^2 + 1}}; \quad \beta_c = \frac{\beta_0}{1 + M_t/M_0\gamma_0}, \qquad (1)$$

where M_t is the mass of the target nucleus. In the particular case in which two nucleons interact,

$$\gamma_c = \sqrt{(\gamma_0 + 1)/2}; \qquad \beta_c = \beta_0/(1 + 1/\gamma_0). \qquad (1')$$

the angles of flight of the secondary particles in the c-system are determined by the formula

$$\tan\theta_c = \frac{1}{\gamma_c}\frac{\sin\theta}{\cos\theta - \beta_c/\beta}. \qquad (2)$$

For the determination of the angles θ_c, it is convenient to use a graphical representation of Eq. (2) put forth by Bradt et al.[9] Figure 1 shows a nomogram, with the help of which we can quickly carry out a transformation of the angles. The

*An exception is the statistical theory, the development of which has permitted a satisfactory description of the multiplicity of generation of particles in the region of energy of primary particles $10^9 - 10^{10}$ ev.[8]

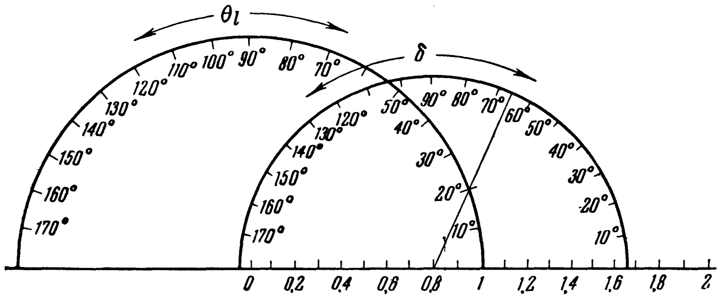

FIG. 1

values of the ratio β_c/β are marked along the horizontal diameter in units of the radius of the larger semicircle. The divisions on the larger semicircle correspond to the values of the angles θ_l. Setting the center of the smaller semicircle at the point corresponding to the given value of β_c/β, we draw a ray from its center to the point on the larger semicircle corresponding to the angle θ_l. The point of intersection of the ray with the smaller semicircle gives the value of the angle $\delta = \tan^{-1}(\gamma_c \tan \theta_c)$. To accelerate the transformation, it is best to draw the small semicircle on a separate sheet of transparent material and to mark the corresponding values of $\ln \tan \delta$ against the values of the angle δ.

Hereinafter, the only general laws that will be used in the analysis of the interaction of high-energy particles will be the laws of conservation of energy and momentum:

$$E_0 + M_t = \sum E_i,$$

$$p_0 = \sum p_i \cos\theta_i, \quad \sum p_i \sin\theta_i = 0. \qquad (3)$$

The index i denotes all particles after the collision, independent of their nature.

Inasmuch as we are interested in interactions due to particles, whose energy is large in comparison with their rest mass, it is convenient to write (3) in the form

$$E_0 - p_0 + M_t = \sum (E_i - p_i \cos\theta). \qquad (3')$$

We write $E_i - p_i \cos\theta = \Delta_i$, while Δ is written in turn as the sum of two quantities ϵ and κp:

$$\varepsilon = E - p = p\left(\sqrt{1-(M/p)^2} - 1\right),$$

$$\varkappa = 1 - \cos\theta. \qquad (4)$$

Then Eq. (3′) takes the form*

$$\sum \varepsilon_i + \sum \varkappa_i p_i = M_t + \varepsilon_0. \qquad (5)$$

In Fig. 2 we have plotted the curves $\epsilon = \epsilon(p)$ for pions and nucleons. It is seen from the curve that when the primary particles are nucleons, we

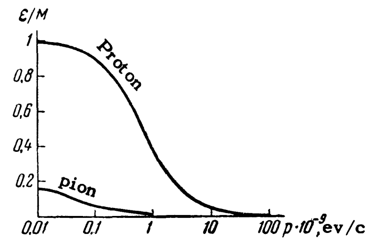

FIG. 2

can neglect ϵ_0 in comparison with M_n in Eq. (5) for all energies $E_0 > 5 \times 10^9$ ev (here $E_0 = 5 \times 10^9$, $\epsilon_0 = 0.1\, M_n$) and, consequently,

$$\sum \varepsilon_i + \sum \varkappa_i p_i = M_t\,. \qquad (6)$$

In the case in which the interaction takes place on one of the nucleons of the nucleus, one must take account of the Fermi momentum of these nucleons. This inaccuracy in Eq. (6) is less than $0.1\, M_n$.

The laws of conservation of energy and longitudinal momentum, written in this fashion, can serve as an excellent criterion for discovering the lack of a nucleon-nucleon interaction. Actually, in a certain interaction produced by a primary nucleon, let the sum of all quantities Δ_i for all observable secondary particles exceed the value of the rest mass of the nucleon, $\sum \Delta_i > M_n$. In this case we can state unambiguously that the target is not a free nucleon and that the interaction takes place either simultaneously with several nucleons of the nucleus or that more than one consecutive reaction takes place inside the nucleus.* The opposite conclusion, that a nucleon-nucleon reaction apparently occurs if $\sum \Delta_i < M_n$, is not possible, since the summation is carried out only over the observed charged particles, and the presence of secondary neutral particles (neutron, π^0

*The conservation laws were first written in this form by S. N. Vernov.

*It should be noted that a strict application of the criterion (6) pertains to the nucleon-nucleus class of interactions and to the case in which evaporation of the nucleus takes place owing to the small energy transmitted to the nucleus in the interaction of the primary nucleon with one of the peripheral nucleons of the nucleus. In the analysis of similar case, it is possible to form the sum Δ_i, without considering the heavy nuclear fragments that are formed in the evaporation.

meson) can reverse the inequality sign. However, if it is known that there is a nucleon-nucleon interaction, and $\sum \Delta_i^{ch} < M_n$, then it can be verified that there are neutral particles among the secondary particles.

If it is assumed that only a single neutral particle is not found, then we have for it $\Delta_x = M_n - \sum \Delta_i$. The value of the quantity Δ_x can serve as an indication of the nature of the neutral particle.

In the case in which there are π^0 mesons as well as a slow neutron among the secondary neutral particles, the neutron makes the principal contribution to the total value of Δ. Actually, the value of ϵ for relativistic pions is small, as is also $\kappa p = p \sin\theta \tan(\theta/2)$, since $p \sin\theta \sim M_\pi$,[10] while the angular distribution of secondary pions is rather narrow for large values of E_0. Neglecting $\sum \Delta_i$ for neutral mesons and a fast nucleon, we can estimate Δ_δ for the δ neutron: $\Delta_\delta \lesssim M_n - \sum \Delta_i^{ch}$.

2. KINEMATICS OF A SLOW NUCLEON IN A NUCLEON-NUCLEON INTERACTION

A series of important conclusions on the nucleon-nucleon interactions can be made by considering the data applying to a nucleon emitted, in the center-of-mass system, in the direction opposite to the motion of the primary nucleon (δ nucleon). In the laboratory system, such a nucleon will be comparatively slow, whence it is easily shown that the range of permitted angles of emission is bounded from above. For the case in which only the number of secondary pions is known, the value for the limiting angle of emission of the δ nucleon in the laboratory system was obtained by Sternheimer.[11] The dependence of θ_{max} on E_0 is shown in Fig. 3 for various values of n_π calculated according to his formulas. Actually, the value of θ_{max} depends on the angular and momentum distribution of secondary pions in the laboratory system.

Let us look at the case in which the angles of emission and the values of the momentum of the relativistic particles are known, or at least if their lower boundary in the laboratory system is

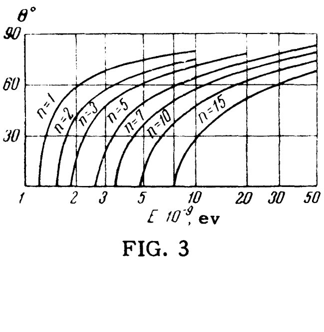

FIG. 3

known. In this case, as was shown above, one can estimate the value of Δ for the slow neutron, $\Delta_\delta \leq M_n - \sum \Delta_i^{ch}$. The connection between the angle of emission θ_δ and the momentum p_δ in the laboratory system for a given value of Δ_δ is given by the relation

$$p_\delta = [\Delta_\delta \cos\theta_\delta \pm \sqrt{\Delta_\delta^2 \cos^2\theta_\delta - (1 - \Delta_\delta^2)\sin^2\theta_\delta}] / \sin^2\theta_\delta. \quad (7)$$

This dependence is shown graphically in Fig. 4. It is easy to see from the drawing that to each value of Δ_δ (the numbers on the curves given the value of Δ_δ, measured in nucleon masses) correspond a maximum possible angle of emission $(\theta_\delta)_{max}$ and a minimum momentum $(p_\delta)_{min}$. The analytic expression for these quantities can be represented by the formulas

$$\tan(\theta_\delta)_{max} = (\Delta_\delta^{-2} - 1)^{-1/2}, \quad (8)$$

$$(p_\delta)_{min} = (1 - \Delta_\delta^2)/2\Delta_\delta, \quad (9)$$

where Δ_δ is measured in nucleon masses. In the case in which Δ_δ is known precisely, Eqs. (7) — (9) give precise values for $p_\delta(\theta)$, $|p_\delta|_{min}$ and $|\theta_\delta|_{max}$.

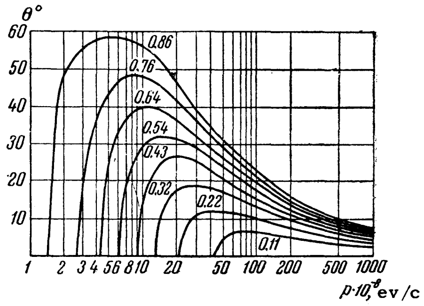

FIG. 4

However, only the highest value of Δ_δ is always found experimentally, inasmuch as the neutral particles are not included in $\sum \Delta_i$, while in certain cases, only the lower boundary is known for the values of Δ of observable particles in certain cases. This leads to the result that the value $|p_\delta|_{min}$, determined from the curves of Fig. 4, may prove to be only underestimated and the values of $|\theta_\delta|_{max}$ only overestimated in comparison with actual values. Particles emitted at angles exceeding $|\theta_\delta|_{max}$, determined by the formulas developed above, certainly cannot be nucleons.

For nucleon-nucleon collisions, it can be considered that, on average, the scattering of nucleons after the collision takes place symmetrically in the center-of-mass system. In this case, knowledge of the value of Δ_δ allows us to determine the character of the distribution of energy between the secondary particles of different types.

We denote by α the fraction of the energy associated with the faster of the two nucleons after the interaction.

Let us consider a system of coordinates bound to the incoming nucleon (all quantities in this system are identified by the index s). Then the energy of a fast nucleon after the collision will be, in the laboratory system,

$$E_n = \gamma_0 (E_s + \beta_0 p_s \cos\theta_s)$$

and, if E_s and p_s are measured in nucleon masses,

$$\alpha = E_n/E_0 = E_s + \beta_0 p_s \cos\theta_s.$$

For all interactions in which multiple meson production takes place, we can set $\beta_0 = 1$. Then

$$\alpha = E_s + p_s \cos\theta_s. \tag{10}$$

As a consequency of symmetry, the distribution of α in the s system will coincide with the distribution of the values of $\Delta_\delta = E_\delta - p_\delta \cos\theta_\delta$ in the laboratory system, while the mean value is

$$\bar{\alpha} = \bar{\Delta}_\delta. \tag{11}$$

At the same time, as shown above, the quantity Δ_δ determines the maximum value of the angle of emission of the δ nucleon in the laboratory system. It then follows that the existence of δ nucleons, emitted at large angles, demonstrates the presence of interactions for which a significant fraction of the energy is connected with the fast nucleon. Thus, to interactions in which the δ nucleons are emitted at angles $\theta_\delta > 32°$, there should correspond interactions for which $\alpha \geq 0.5$ (see Fig. 4).* It must be emphasized that in separately chosen interactions there is no direct connection between the quantities Δ_δ and α, inasmuch as in the individual interaction the scattering of the nucleons after collision can also take place unsymmetrically.

We now find a connection between the energy spectrum of the δ nucleons in the laboratory system and the angular distribution of the nucleons in the center-of-mass system.

For simplicity, we assume that in the center-of-mass system the scattering of nucleons takes place isotropically. In this case it is easy to find the relative number of interactions in which δ nucleons emerge in the laboratory system with energy E_δ less than a certain value $E_{\delta a}$. Let the energy of the nucleons in the center-of-mass system be equal to $E_{\delta c}$; then all the nucleons lying within the cone of generating angle θ_{ac}, defined by the relation

$$\cos\theta_{ac} = (E_{\delta a}/\gamma_c - E_{\delta c}) / \beta_c \sqrt{E_{\delta c}^2 - M_n^2},$$

will possess energies $\leq E_{\delta a}$ in the laboratory system. For the condition of isotropy, the relative number of such nucleons k will be determined by the fraction of the solid angle Ω $(\theta_{\delta c} \geq \theta_{ca})$ 4π.

Taking it into account that two nucleons take part in the interaction, we get

$$k = \frac{2}{4\pi} \int_{\theta_{ca}}^{180} d\Omega = 1 + (E_{\delta a}/\gamma_c - E_{\delta c}) / \beta_c \sqrt{E_{\delta c}^2 - M_n^2}; \tag{12}$$

k reaches a maximum value for $E_{\delta c} = \gamma_c / E_{\delta a}$:

$$k_{max} = 1 - \sqrt{\gamma_c^2 - E_{\delta a}^2} / \gamma_c \beta_c. \tag{13}$$

The values of k_{max} for $E_0 = 3.5$ and 10 nucleon masses are given at the end of the article. We used here a value of 1.08 Bev for $E_{\delta a}$, which corresponds to a value $J/J_{min} = 2$, at which the proton still differs from the pions under experimental conditions.

3. APPLICATION OF THE RELATIONS OBTAINED TO EXPERIMENTAL DATA

a) We apply the criterion of non nucleon-nucleon collisions [Eq. (6)] to the interaction of nucleons with various nuclei: the light nucleus Be_4^9 and the nuclei in the composition of photoemulsions. Data on the interaction of protons of cosmic rays with nuclei of beryllium were taken by us from references 1 — 3, where Wilson cloud chambers located in a magnetic field were used.* For analysis of the interactions observed in photoemulsions,† sixty seven interactions were selected which were brought about by charged particles with $E_0 \sim 10^{10}$ ev, in which the secondary particles with ionizations exceeding seven fold were lacking and in which the number of "gray" tracks with ionizations with one half up to 7-fold did not exceed 1. The result of the analysis is shown in Table I.

It is seen from the table that in the interaction

*A. E. Chudakov first pointed out this relation between θ_δ and α.

*It should be observed that kinematic analysis can be rigorously applied in those cases in which the charged products of the interaction are observed. In Wilson chambers with plates there is a possibility of distortion of the results because of secondary processes in the plates.

†A survey and measurement of the angles and ionization of tracks of secondary particles were carried out in the Cosmic Ray Laboratory of the Physics Institute, Academy of Sciences, U.S.S.R. under the direction of G. B. Zhdanov, to whom the authors express their gratitude for making the experimental results available for the present analysis (prior to publication).

TABLE I

Target and reference	Energy of the primary nucleon E_0, 10^9 ev	Number of relativistic particles n_s	$N_g = 1$		$N_g = 0$	
			Total number of interactions	Number of non N–N interactions	Total number of interactions	Number of non N–N interactions
Be[1,2]	~5	2–3	14	0	34	0
		⩾4	0	0	6	0
Be[3,4]	5–50	⩾4	3	1	15	1
Nuclei of photo-emulsions, G. B. Zhdanov et al.	9	$1 \leqslant n_s < 4$	17	12	25	—*
		>4	5	5	20	—*

*Lack of data on the momenta of relativistic particles prevents an application of criterion (6) to the interactions with $N_g = 0$.

TABLE II

Number of particles in the shower, n_s	$\sum \Delta_i^{ch}/M_m$	$(\Delta_\delta)_{max}/M_m$	$(\theta_\delta)^0_{max}$	Experimental date for particles whose nature has not been established from the momenta and ionizations			
				θ	J/J_{min}	$p \cdot 10^8$ ev/c	Sign of charge
8	0.8	0.2	10	48	1	>13	?
				11	1	—	?
				25	1	>9	?
4	0.4	0.6	34	48	1	>9	?
7	0.5	0.5	30	32	1	12^{+6}_{-3}	+
7	0.4	0.6	34	45	1	—	
6	0.5	0.5	30	55	1	>3	?
				37	1	>3	?

of protons with nuclei of beryllium, the only insignificant fraction of the total number of interactions is not nucleon-nucleon. For interactions of protons with nuclei of the photoemulsion, there is a significantly different result: among the interactions which in most researches on photoemulsions are interpreted as nucleon-nucleon interactions, a significant fraction (not less than 0.5) are interactions of more than with a single nucleon of the nucleus.

b) In Sec. 1 it was shown that in the case in which the difference $M_n - \sum \Delta_i^{ch}$ for the interaction of nucleons with nucleons $\sum \Delta_i < M_n$ (the summation is taken over all observed charged particles) determines the maximum value of Δ for the δ nucleon.

Knowing $(\Delta_\delta)_{max}$, it is possible to find the value of $(\theta_\delta)_{max}$ from the curves of Fig. 4. Comparing the value of $(\theta_\delta)_{max}$ thus obtained with the experimental value, we can establish the nature of the particles emitted at large angles in the laboratory system.

The result of such an analysis carried out by us according to the data of reference 3, under the assumption that nucleon-nucleon interactions are generally absent in Be, is given in Table II. All the particles listed in the table were pions.

In the data of Fowler et al.,[5] the nature of the particles emitted at large angles in the interactions of neutrons of $\overline{E}_0 = 2.7$ Bev with protons was identified by means of the curves of Sternheimer (see Fig. 3). Application of the method that we have advanced would in this case permit a substantial reduction of the value of $(\theta_\delta)_{max}$ and at the same time determine the nature of the particles in most cases.

c) It was shown above that the distribution of values of Δ_δ should coincide with the distribution of values of α — the coefficient of elasticity for the interaction in which the pions are generated. The distribution of values of Δ_δ is plotted in Fig. 5 from the data of Smorodin.*

Out of a total number of 54 cases of production of one or more pions, in 6 cases, $n_s \geq 4$. In 14 of 48 cases with $n_s < 4$, showers were observed with a slow δ proton, the momentum of which was less than 700 Mev/c. The distribution of Δ_s for these cases is shown in Fig. 5 by the solid lines. In 10 cases, analysis based on the law of

*Only the research of Smorodin is used for such an analysis inasmuch as the operation of the Wilson chamber in this case was achieved by means of a system of counters which did not introduce any appreciable discrimination as to the character of the interaction.

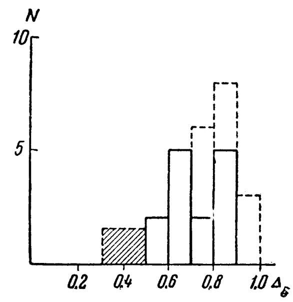

FIG. 5

conservation of electrical charge and of the quantity $M_n - \sum \Delta_i$ showed that in the interactions slow protons ought to be released, which are absorbed in the plate of Be. The value of Δ_δ in these cases exceeds 0.7. These cases are shown by the dashed lines. Thus, among showers with $n_s < 4$, in half the cases a proton is emitted with $p < 700$ Mev/c. It is natural to assume that in the remaining half of the cases with $n_s < 4$ the slow particle is a neutron. In showers with $n_s \geq 4$, not in a single case was a proton observed with increased ionization. This shows directly that even in these cases the momentum of the nucleon must be greater than 700 Mev/c, and consequently, $\Delta_\delta > 0.5$. Assuming that in these cases also, a δ proton is emitted in half of the interactions, we complete the histogram in Fig. 5 by three cases with $\Delta_\delta > 0.5$, distributing them conditionally in the interval from 0.3 to 0.5 (the shaded rectangle), since at $E_0 \sim 5 \times 10^9$ ev, α cannot be less than 0.3.

The value $\overline{\alpha} = 0.7$, which is obtained from the histogram, is in excellent agreement with the average value of the elastic coefficient obtained by Grigorov[12] from the analysis of the energy losses of the nucleon component of cosmic rays with energies $\sim 10^{10}$ ev in their passage through the atmosphere.

The demonstration obtained in reference 2 of the small value of the coefficient of elasticity in those showers in which a large number of pions is generated is confirmed by the research of reference 3, in which the interactions of rather high energy $(5 \times 10^9 - 5 \times 10^{10}$ ev) were separated. In this work, among 18 recorded showers with $n_s \geq 4$, only a single case was observed of a proton with $J/J_{min} > 2.0$ (Δ_δ in this case is equal to 0.64) and at the same time relativistic particles emitted at large angles (which could be identified with δ protons) were not observed. This circumstance suggests that the recorded cases are characterized by a small value of Δ_δ.*

*In reference 3, the coefficient of elasticity is determined in a different manner; the result obtained there agrees well with our conclusion.

The determination of the coefficient of elasticity in a fashion not connected with the determination of the energy of the primary particle at high energies of primary particles is of fundamental interest. We have considered the value of $\sum \Delta_i$ for showers described in the research of De Benedetti et al.[13] The shower, recorded in a photoemulsion, contained 39 relativistic particles whose momenta were measured. The authors of reference 13 estimated the energy of the primary particle to be $E_0 \sim 5 \times 10^{12}$ ev. The value of $\sum \Delta_i^{ch}$ was determined to be $0.7\,M_n$, while in this sum there were not taken into account two pions emitted at a large angle with respect to the axis of the shower. The momenta of these pions are not found in reference 13. We can consider this case to be interaction of two nucleons only if the number of emitted mesons is significantly smaller than one half the number of charged mesons. If this shower is the result of the collision of two nucleons, then $\Delta_\delta \leq 0.3\,M_n$, and consequently the coefficient of elasticity is small.

d) Kinematic analysis permits us to establish the fact that in the center-of-mass system, the nucleons are emitted anisotropically after the interaction, being concentrated in directions close to the direction of motion of the nucleons before the collision.

Let us write out the data taken from reference 2 for the values of k_{max}:

	Calculated			Experimental	
$E_0/M_n =$	3	5	10	3	5
k_{max} (%) $=$	35	15	5	60 [4]	80

(13a)

In the calculation, the value of $E_{\delta a}$ was taken equal to 1.08 Bev. On the right we have given the values of k found by experiment (the data of reference 4 applied to the interaction of neutrons with mean energy ~ 2.7 Bev with hydrogen). From the data set forth it is evident that the observed number of slow nucleons does not agree with the assumption on the isotropic scattering of nucleons in the center-of-mass system. A similar conclusion was obtained earlier in the work of Grigorov.[12]

The anisotropy of the scattering of nucleons in the center-of-mass system after the interaction takes place is in accord with the results of reference 3, evidently in those cases also in which the number of pions generated is large.

In conclusion, the authors consider it their duty to note that individual aspects of the work were discussed with many co-workers in the Cosmic Ray Laboratories of the Physical Institute, Academy of Sciences, U.S.S.R. and the Scientific Research Institute for Nuclear Physics, to whom they express their acknowledgment.

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Translated by R. T. Beyer
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