

WALL PROBE IN A MAGNETIC FIELD

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The current flowing to a wall probe in a strong magnetic field is computed in the case when the motion of the electrons can be considered free along the magnetic field and diffuse across the field.

SPIVAK and Reikhrudel',¹ in a series of articles, have extended the Langmuir theory of probe measurements to include the case of a weak magnetic field (~ 10 oe), when the condition $\rho_e \gg a$ is satisfied (ρ_e is the Larmor radius of the electron, and a is the dimension of the probe).

In a magnetic field H , the electron moves along a circle of radius $\rho_e = mv/eH$ (e , m , and v are the charge, mass, and velocity of the electron, respectively). The center of the circle moves in space only because of the collisions between the electron and other particles, or under the influence of the plasma fields. After each collision the center of the circle is displaced on the average by ρ_e . As a result, ρ plays the role of effective mean free path in a direction perpendicular to the magnetic field. In fields of ~ 1000 oe, the value of ρ is $\sim 10^{-3}$ cm and it is impossible to satisfy the condition $\rho_e \gg a$, since the dimension of the probe is usually ~ 1 mm. For this reason, the Spivak-Reikhrudel' theory cannot be used to interpret probe data on discharges in strong magnetic field. Nevertheless, the Larmor radius of positive ions is usually noticeably greater than the dimension of the probe, and the magnetic field can be neglected in calculating the ion current.

Bohm, Barhop, and Massey² examined in detail the question of the ion current flowing in the presence of a strong magnetic field into a negatively-charged probe. According to reference 2, the ion flow is independent, within 20%, of the ion temperature, as long as the latter is lower than the electron temperature. The following formula is given for the total current:

$$J = 0.4n_0S\sqrt{2kT/M},$$

where S is the probe area, M the ion mass, and n_0 the electron concentration outside the layer. The criterion of the validity are the inequalities $\rho_+ \gg a$ and $\lambda_+ \gg a$, which are readily satisfied in low-pressure discharges. This formula should therefore give the correct order of magnitude of the ion current.

As to the electron portion of the probe characteristic, Bohm, Barhop, and Massey,² in view of the difficulty in its interpretation, restricted themselves only to a positively-charged probe, which repels the ions. Their premises are equivalent to the assumption of a plasma of infinite extent along the direction of the magnetic field.

The case most frequently encountered in practice is the reverse, when the length of the plasma along the magnetic field is $l \lesssim \lambda_e$, where λ_e is the range of the electron along the magnetic field. It is absolutely impossible to neglect in this case the finite extent of the plasma in this direction.

A compensated ion beam is used in many physical setups. One of the methods used to investigate the properties of the ion beam is the probe method. The theory of probe measurements in compensated ion beams is therefore of definite practical interest. In the first part of this paper we calculate the current into the probe in the limiting case, when the concentration of the slow ions is negligibly small compared with the concentration of the fast ions of the beam. In the second part we calculate the current into the probe for the opposite limiting case, when the concentration of the slow ions is high. This case corresponds to a discharge plasma. The motion of the electrons along the magnetic field is assumed free, $\lambda_e > 1$, and the transverse motion is assumed diffuse with a diffusion coefficient D .

I. PROBE MEASUREMENTS IN A COMPENSATED ION BEAM

1. Statement of the Problem

A quasi-neutral plasma, produced by ionization of the residual gas by fast ions from the beam, is located between conducting planes AB and CD, (Fig. 1). The density of the current of fast ions is constant along z . The magnetic field H is directed perpendicular to the planes that bound the plasma. The plasma is infinite in the direction perpendicular to the magnetic field.

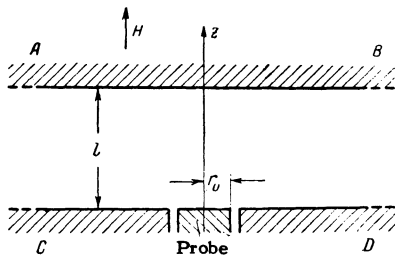


FIG. 1. Geometry of the problem.

Located in one of the planes (CD, Fig. 1) is a probe, which is a disk of radius r_0 , the surface of which is perpendicular to the magnetic field. The potential of the probe is measured relative to the potential of plane CD. To be specific, we consider a case when the plane AB is negative relative to CD. In this case the plasma electrons cannot fall on AB. The case in which both planes are of the same potential can be considered in an analogous manner.

The electron current \mathbf{j} satisfies the equation

$$\operatorname{div} \mathbf{j} = Q, \quad (1)$$

where Q is the ionization density. In a cylindrical coordinate system with the z axis passing through the center of the probe and directed along the magnetic field, we can rewrite (1) as

$$\nabla_{\perp} j_{\perp} + \partial j_z / \partial z = Q. \quad (2)$$

The properties of the plasma change little in the z direction, except for the thin layer at the walls, of thickness on the order of the Debye radius. Integration of Eq. (2) with respect to z yields

$$l \nabla_{\perp} j_{\perp} = -j_z|_{z=0} + Ql. \quad (3)$$

It is obvious that a change in the probe potential produces an axially symmetrical change in the current \mathbf{j} ; Eq. (3) can therefore be averaged over the azimuth angle. This yields

$$\frac{l}{r} \frac{dr j_r}{dr} = -j_z|_{z=0} + Ql. \quad (4)$$

The density of the electron current on the wall, in the presence of a potential barrier, is: outside the probe ($r > r_0$)

$$j_z|_{z=0} = (nv/4) e^{-U_{pl}/T}, \quad (5)$$

and inside the probe ($r \leq r_0$)

$$j_z|_{z=0} = (nv/4) e^{-(U_{pl} - U_s)/T}. \quad (6)$$

Here n is the concentration of the electrons, equal to the concentration of the fast ions, T is the electron temperature in volts, U_{pl} is the potential of the plasma relative to the wall, $U_{pl} > 0$, and v is a certain velocity, which coincides with the mean velocity of random motion of the electrons in the case of Maxwellian distribution.

We denote the current corresponding to $U_{pr} = 0$ by j_z^0 :

$$j_z^0 = (nv/4) e^{-U_0/T}, \quad (7)$$

where U_0 is the potential of the unperturbed plasma.

Using (5), (6), and (7), we can rewrite (4) as

$$\frac{l}{r} \frac{dr j_r}{dr} = -j_z^0 - \frac{nv}{4} e^{-U_0/T} (e^{-(\varphi - U_{pr})/T} - 1) + Ql, \quad (8)$$

where $\varphi = U_{pl} - U_0$ is the perturbation of the plasma potential when a potential U_{pr} is applied to the probe.

Inserting into (8) an expression for j_r in terms of the coefficient of diffusion of the electrons across the magnetic field,

$$j_r = -D \frac{\partial n}{\partial r} + \frac{D}{T} n \frac{\partial U_0}{\partial r} + \frac{D}{T} n \frac{\partial \varphi}{\partial r}, \quad (9)$$

and attributing the terms that do not contain φ to j_z^0 , i.e., putting

$$j_z^0 = \frac{lD}{r} \frac{\partial}{\partial r} r \frac{\partial n}{\partial r} - \frac{lD}{r} \frac{\partial}{\partial r} r \frac{n}{T} \frac{\partial U_0}{\partial r} + Ql, \quad (10)$$

we obtain for $r \leq r_0$

$$\frac{d^2 \varphi}{dr^2} + \frac{1}{r} \frac{d\varphi}{dr} = Tk^2 (1 - e^{-(\varphi - U_{pr})/T}) \quad (11)$$

and for $r > r_0$

$$\frac{d^2 \varphi}{dr^2} + \frac{1}{r} \frac{d\varphi}{dr} = Tk^2 (1 - e^{-\varphi/T}), \quad (12)$$

where

$$k^2 = (v/4Dl) e^{-U_0/T}. \quad (13)$$

Let us estimate the value of k^2 . Comparing (13) and (7), we have

$$k^2 = j_z^0 / nDl.$$

If a is the characteristic length over which the drop in n , U_0 , and T takes place, then, taking (10) into account,

$$k^2 \sim \frac{1}{a^2} + \frac{U_0}{Ta^2} + \frac{Q}{Dn} \sim \frac{U_0}{Ta^2} + \frac{Q}{Dn}. \quad (14)$$

The probe current is

$$J = \int_0^{r_0} 2\pi r j_z|_{z=0} dr = \int_0^{r_0} 2\pi r \frac{nv}{4} e^{-U_0/T} (e^{-(\varphi - U_{pr})/T} - 1) dr + J_0. \quad (15)$$

Here J_0 is the probe current when $U_{pr} = 0$. Using (11), we get

$$J - J_0 = -2\pi n D l \frac{x_0}{T} \left. \frac{d\varphi}{dx} \right|_{x=x_0}, \quad (16)$$

where $x = kr$ and $x_0 = kr_0$. To find $d\varphi/dx$, it is necessary to solve Eqs. (11) and (12) with suitable boundary conditions. These equations can be generally solved only numerically. In particular cases, however, results can be obtained without resorting to numerical methods.

2. Low Probe Potentials

The most simple case for which an approximate solution of (11) and (12) is possible when the probe potential is low. Since $|\varphi| \leq U_{pr}$ always, we have also $|\varphi - U_{pr}|/T \ll 1$ when $|U_{pr}| \ll T$, and $\exp\{-\frac{(\varphi - U_{pr})}{T}\}$ can therefore be expanded into a series, with accuracy to first-order quantities. Solving the resultant equations and substituting the expression obtained for φ in (16), we obtain for the probe current

$$J - J_0 = 2\pi n D l \frac{K_1(x_0) I_1(x_0)}{I_0(x_0) K_1(x_0) + I_1(x_0) K_0(x_0)} x_0 \frac{U_{pr}}{T}, \quad (17)$$

where $I_n(x)$ is the Bessel function of imaginary argument and $K_n(x)$ is the MacDonald function.

The probe thus has a linear characteristic at low probe potentials. It must be noted that for low probe potentials the solution is valid for all values of $x_0 = kr_0$. At large values of x_0 we obtain, using the asymptotic values of the functions $I_0, I_1, K_0,$ and $K_1,$

$$J - J_0 = \pi n D l k r_0 \frac{U_{pr}}{T}. \quad (18)$$

It follows from (18) and (17) that the probe current is proportional to the probe radius at large kr_0 and to the square of the radius at small kr_0 . This is due to the fact that the potential barrier decreases essentially at the edge of the probe, and the electrons therefore spill out over the ring near the edge of the probe, without being able to reach its inner parts. If the probe dimension kr_0 is sufficiently small, the electrons spill out over the entire probe area.

3. The General Case

Equations (11) and (12) were solved approximately for the limiting cases of small ($kr_0 \ll 1$) and large ($kr_0 \gg 1$) probes. The physical meaning of the concepts "small" and "large" probe is as follows. If there is no ionization near the probe ($Q = 0$) or if it is very small, and if U_0/T is relatively small, then, considering (14) and the fact that $r_0/a \ll 1$, we get $kr_0 \ll 1$. This is the case of a small probe. But if Q is sufficiently large, and the coefficient of transverse diffusion, D , is small, corresponding to a strong magnetic field, the second term will be large and $kr_0 \sim r_0 \sqrt{Q/Dn} \gg 1$, which is the condition of a large probe. Approximate solutions of Eqs. (11) and (12), and consequently the probe currents, were found for these two cases. For intermediate probe dimensions, these solutions were interpolated over the probe dimensions for each U_{pr}/T .

Figure 2a shows a plot of j vs. the probe dimension x_0 for various probe potentials, with the probe current determined from the following formula:

$$J - J_0 = 0.4\pi n D l j. \quad (19)$$

The dotted curves delimit the interpolation region. For comparison, Fig. 2b shows the dependence of the current on the probe dimension x_0 (in the same units) for the Langmuir case.

It follows from these results that at small probe dimensions, no matter what the probe potential, the probe current, as in the Langmuir case, is proportional to the square of the probe radius, i.e., the density of the electron current is more or less uniform over the probe. If the probe is large, when $x_0 \gg 1$, the probe current, unlike the Langmuir case, is proportional to first power of x_0 , i.e., the density of the electron current is large essentially on the periphery of the probe, and is small near its center.

At sufficiently high negative probe potentials the dependence of the probe current on the probe potential is nearly exponential. This permits de-

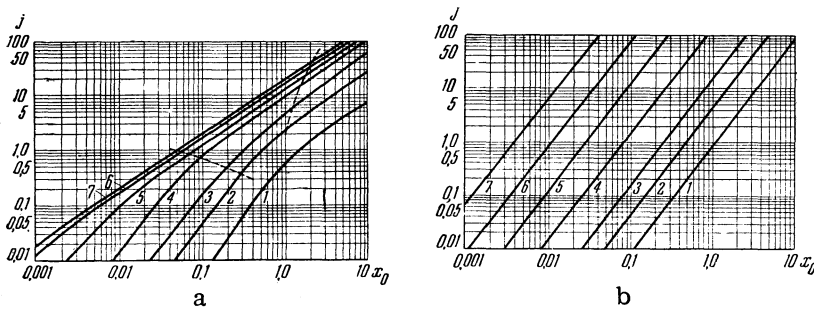


FIG. 2. Dependence of probe current on probe dimensions: a - in the presence of a magnetic field, b - in the absence of a magnetic field. Curve 1) $U_{pr}/T = 0.3$; 2) $U_{pr}/T = 1$; 3) $U_{pr}/T = 2$; 4) $U_{pr}/T = 4$; 5) $U_{pr}/T = 6$; 6) $U_{pr}/T = 8$; 7) $U_{pr}/T = 10$.

termination of the electron temperature by the Langmuir method.

Using the curves of Fig. 2 and Eq. (19), and knowing T , we can determine x_0 . If x_0 is known, the coefficient of transverse electron diffusion, D , can be determined from (19). Using Eq. (13) for k , we can determine U_0 from x_0 .

II. PROBE MEASUREMENTS IN GAS-DISCHARGE PLASMA

4. Statement of the Problem

We consider the limiting case, when the concentration of the slow ions is large compared with the concentration of the fast ions in the plasma discharge.

The plasma consists of electrons with a temperature T and of slow ions with a temperature T_+ , with $T_+ \ll T$. The slow ions are formed by ionization of the residual gas by external sources, by fast ions, and by electrons; they also arise through ion charge exchange. In this case the change produced in the plasma potential by a change in the probe potential is limited to a small quantity, on the order of T_+ . The potential of the plasma, U_0 , can therefore be considered constant with sufficient degree of accuracy, inasmuch as $T_+ \ll T$ and $U_0 > T$. A change in the probe potential will influence the ion and electron concentration. The geometry of the problem remains the same.

Under these assumptions, Eq. (3) holds for the electron current. On the other hand

$$j_{\perp} = -D\nabla_{\perp} n + \frac{D}{T} n \nabla_{\perp} U_0. \quad (20)$$

We put $n = n_0 + \nu$, where n_0 is the electron concentration at $U_{pr} = 0$ and ν is the change in electron concentration due to the change in the probe potential U_{pr} .

The ionization density may be a function of the electron concentration n , provided the electron temperature is sufficiently high. We assume $Q(n) = \beta + \alpha n$, where β is the ionization density due to external ionization sources, such as the discharge radiation and α is the coefficient of ionization by the electrons.

Inserting into (3) the value of j_{\perp} from (20) and Q , and attributing the terms not containing ν to the probe current density j_z^0 at $U_{pr} = 0$, i.e., putting

$$j_z^0 = lD\nabla_{\perp}^2 n_0 - \frac{lD}{T} \nabla_{\perp} n_0 \nabla_{\perp} U_0 - \frac{lD}{T} n_0 \nabla_{\perp}^2 U_0 + \beta l + \alpha n_0 l, \quad (21)$$

we get

$$j_z|_{z=0} - j_z^0 = \alpha \nu l + lD\nabla_{\perp}^2 \nu - \frac{lD}{T} \nabla_{\perp} \nu \nabla_{\perp} U_0 - \frac{lD}{T} \nu \nabla_{\perp}^2 U_0. \quad (22)$$

Considering that the change in electron concentration ν , produced by a change in the probe potential, is axially symmetric, we average Eq. (22) over the azimuth. The third term in (22) then vanishes. It is easy to show that $\nabla_{\perp}^2 U_0 \cong \nabla_{\perp}^2 U_0(0)$, with sufficient accuracy. We shall henceforth denote this quantity by $\nabla_{\perp}^2 U_0$. Thus

$$j_z|_{z=0} - j_z^0 = \alpha \nu l + lD\nabla_{\perp}^2 \nu - \frac{lD}{T} \nu \nabla_{\perp}^2 U_0. \quad (23)$$

On the other hand,

$$j_z|_{z=0} = \frac{(n_0 + \nu)v}{4} e^{-(U_0 - U_{pr})/T}, \quad (24)$$

$$j_z^0 = \frac{n_0 v}{4} e^{-U_0/T}. \quad (25)$$

Insertion of (24) and (25) into (23) results in an equation for ν :

$$\begin{aligned} \nabla_{\perp}^2 \nu - \left(\frac{1}{T} \nabla_{\perp}^2 U_0 - \frac{\alpha}{D} + \frac{v}{4Dl} e^{-(U_0 - U_{pr})/T} \right) \nu \\ = \frac{n_0 v}{4Dl} e^{-U_0/T} (e^{U_{pr}/T} - 1). \end{aligned} \quad (26)$$

Equation (26) has been derived for the region over the probe, i.e., for $r \leq r_0$. Outside the probe ($r > r_0$) it is necessary to put in (26) $U_{pr} = 0$. This yields

$$\nabla_{\perp}^2 \nu - \left(\frac{1}{T} \nabla_{\perp}^2 U_0 - \frac{\alpha}{D} + \frac{v}{4Dl} e^{-U_0/T} \right) \nu = 0. \quad (27)$$

To find the electron component of the current into the probe by integrating $J_z|_{z=0}$, it is necessary to find ν in accordance with Eq. (24).

5. Electron Current in Probe

A. Absence of Ionization. In the region above the probe, ν changes substantially at distances $\sim r_0$, while U_0 changes substantially at distances on the order of a , where $a \gg r_0$ is the characteristic length for the potential. It is therefore possible to neglect the term $\nu \nabla_{\perp}^2 U_0 / T \sim U_0 \nu / Ta^2$ as compares with $\nabla_{\perp}^2 \nu \sim \nu / r_0^2$ for $r \leq r_0$, provided U_0 / T is not too large.

We introduce

$$\omega^2 = k^2 + \frac{1}{T} \nabla_{\perp}^2 U_0, \quad (28)$$

where k^2 is determined from (13). Taking this into account, Eqs. (26) and (27) yield for ν , in the region $r \leq r_0$:

$$\nabla_{\perp}^2 \nu - \nu k^2 e^{U_{\text{pr}}/T} = \frac{n_0 \nu}{4Dl} e^{-U_{\text{pr}}/T} (e^{U_{\text{pr}}/T} - 1) \quad (29)$$

and for $r > r_0$

$$\nabla_{\perp}^2 \nu - \omega^2 \nu = 0. \quad (30)$$

From (13) and (25) we find $j_z^0 = k^2 n_0 D l$. Making use of (21), we get

$$k^2 = \frac{\nabla_{\perp}^2 n_0}{n_0} - \frac{\nabla_{\perp} n_0}{n_0} \frac{\nabla_{\perp} U_0}{T} \sim \frac{1}{a^2}.$$

Consequently $kr_0 \sim r_0/a \ll 1$. Solving (29) and (30), we obtain the distribution of electron concentration. After inserting ν into (24) and integrating with respect to r from $r = 0$ to $r = r_0$, we obtain the final equation for the electron current in the probe:

$$J_- - J_-^0 = \frac{4J_-^0}{kr_0} \sinh \frac{U_{\text{pr}}}{2T} \times \frac{K_1(\omega r_0) I_1(ke^{U_{\text{pr}}/2T} r_0)}{I_0(ke^{U_{\text{pr}}/2T} r_0) K_1(\omega r_0) + \frac{k}{\omega} e^{U_{\text{pr}}/2T} I_1(ke^{U_{\text{pr}}/2T} r_0) K_0(\omega r_0)}, \quad (31)$$

where J_-^0 is the probe current at $U_{\text{pr}} = 0$. It follows from (28) that ω and k are of the same order of magnitude. We can therefore simplify (31) by expanding $K_0(\omega r_0)$ and $K_1(\omega r_0)$ in series.

B. Presence of Ionization. In this case $\alpha \neq 0$ and $\beta \neq 0$. An estimate of the value of k^2 yields for this case

$$k^2 \approx \beta/Dn_0 + \alpha/D + 1/a^2 \approx \beta/Dn_0 + \alpha/D,$$

since $1/a^2 \ll \beta/Dn_0 + \alpha/D$ for sufficiently small D . We can therefore disregard terms of order $1/a^2$. Equations (26) and (27) become for this case

$$\nabla_{\perp}^2 \nu - \omega_1^2 \nu = n_0 k^2 (e^{U_{\text{pr}}/T} - 1) \quad (32)$$

for $r \leq r_0$, and

$$\nabla_{\perp}^2 \nu - \omega_2^2 \nu = 0 \quad (33)$$

for $r > r_0$. Here

$$\omega_1^2 = k^2 e^{U_{\text{pr}}/T} - \alpha/D; \quad \omega_2^2 = k^2 - \alpha/D. \quad (34)$$

After finding ν and inserting it into the current equation, we obtain

$$J_- - J_-^0 = J_-^0 (e^{U_{\text{pr}}/T} - 1) \left[1 - \frac{k^2}{\omega_1^2} e^{U_{\text{pr}}/T} + \frac{2e^{U_{\text{pr}}/T} K_1(\omega_2 r_0) I_1(\omega_1 r_0) k^2 / \omega_1^2}{\omega_1 r_0 [I_0(\omega_1 r_0) K_1(\omega_2 r_0) + (\omega_1 / \omega_2) K_0(\omega_2 r_0) I_1(\omega_1 r_0)]} \right]. \quad (35)$$

It must be noted that this result differs from the expression for the electron current in the probe in the Langmuir case by the factor in the square brackets. The value of this factor depends on the

actual mechanism of electron production and on the value of the coefficient of transverse diffusion.

If only external ionization is present, i.e., $\alpha = 0$, $\beta \neq 0$, we have $\omega_1^2 = k^2 \exp(U_{\text{pr}}/T)$ and $\omega_2^2 = k^2$. Equation (35) then coincides with (31), but it must be recognized that in this case $k^2 \approx \beta/Dn_0$ is not small when D is small.

If there is no external ionization, we can no longer neglect terms on the order of $1/a^2$ and $\omega_2^2 \approx 1/a^2$, and therefore $\omega_2 r_0 \approx r_0/a$ is always small.

6. Ion Current in Probe

In the preceding computations (Secs. 4 and 5) it is assumed that the probe is negative relative to the plasma, and there is no potential barrier for the ions. Therefore, a change in the probe potential causes a change in the probe ion current through a change in the electron concentration and the associated quasi-neutral ion concentration. The ion current in the probe is given by³

$$J_+ = 2\pi v_+ \int_0^{r_0} n_+ r dr, \quad (36)$$

where $v_+ = 0.345 \sqrt{2T/M}$, M is the ion mass, and T is the electron temperature. But we have by Poisson's equation

$$n_+ = n_- - (1/4\pi e^2) \nabla^2 \varphi.$$

Therefore, considering the axial symmetry of φ , we get

$$J_+ = J_+^0 + 2\pi v_+ \int_0^{r_0} v r dr - \frac{v_+}{2e^2} r_0 \left. \frac{d\varphi}{dr} \right|_{r=r_0}. \quad (37)$$

Here φ is the change in the plasma potential produced by the change in the probe potential due to the violation of quasi-neutrality.

The correction to the ion current, necessitated by the violation of quasi-neutrality of the plasma, can be found approximately in the following manner. We can write for the ion concentration $n_+ = n_0 \exp(-\varphi/T_+)$ and obtain from the quasi-neutrality condition $n_0 + \nu = n_+ = n_0 \exp(-\varphi/T_+)$. Therefore $\varphi = -T_+ \ln(1 + \nu/n_0)$.

Making use of this result, we get

$$J_+ - J_+^0 = v_+ \left[2\pi \int_0^{r_0} v r dr + \frac{T_+}{T} r_0 \lambda^2 \frac{2\pi dv/dr}{(1 + \nu/n_0)_{r=r_0}} \right], \quad (38)$$

where $\lambda^2 = T/4\pi e^2 n_0$ is the Debye length. The second term in (38) is the correction to the ion current, necessitated by the fact that n_+ and $n_- = n_0 + \nu$ are not exactly equal, i.e., necessitated by the violation of quasi-neutrality of the plasma upon change in the probe potential. The correc-

tion should be small compared with the principal term. If the probe potential is very high, then $\nu|_{r=r_0} \rightarrow -n_0$ and the correction may prove to be not small. This indicates that one cannot calculate the concentration $n_0 + \nu$, as was done above, and the change in the plasma potential must be taken into account.

When calculating the correction to the current, we inserted into the expression for φ a value of ν calculated without allowance for the change in the plasma potential. It is easy to show that the correction computed in this manner is greater than the true correction.

7. Probe Characteristic

Under experimental conditions, one measures the total probe current, $i_{pr} = J_- - J_+$. The expressions for J_- and J_+ were given above. It is of interest to examine several limiting cases:

1. Only external ionization exists, i.e., $\alpha = 0$ and $\beta \neq 0$. In this case $k^2 = \beta/Dn_0$ can be large when D is small. Using the asymptotic expression for the Bessel function, we obtain for sufficiently high probe potentials

$$i_s \rightarrow J_-^0 + (2/kr_0)J_-^0. \quad (39)$$

The second term is the electron current due to diffusion. In the case of a probe with large kr_0 , we see from the results obtained that the electron current, $J_- - J_-^0$, is proportional to the probe radius at high probe potential. An explanation for this was given above.

2. There is no external ionization, the ionization is proportional to the electron concentration, $\alpha \neq 0$, and $\beta = 0$. In this case $k^2 = \alpha/D > 1$ at sufficiently small D . If $kr_0 > 1$ it turns out that i_{pr} , at sufficiently large U_{pr}/T , is less than J_-^0 . This becomes understandable if it is taken into account that $kr_0 > 1$ corresponds to small diffusion, and that in this case the ionization is proportional to the electron concentration. It sufficiently high probe potentials, the electron concentration over the probe drops to zero, and the external diffusion,

which places the electrons over the probe, is small. The ion current into the probe is therefore zero, and the electron current starts dropping.

3. If ionization is present in the case of greatly hindered diffusion, i.e., at small D and correspondingly large kr_0 , the electron concentration over the probe increases considerably when the probe potential is negative, and this leads to violation of the quasi-neutrality condition and to a substantial change in the plasma potential. Under these conditions, the results obtained no longer hold, for it has been assumed in the derivation that the plasma potential changes by an amount proportional to T_+ and that $T_+ \ll T$. This case should be investigated separately.

On the other hand, if the diffusion is not very small, i.e., kr_0 is small, the diffusion will again not let the electron concentration rise over the probe. The quasi-neutrality will therefore not be violated. The results obtained above will remain in force.

The expressions obtained for the probe current make it possible to determine, from the experimental probe characteristic, the principal parameters of the plasma and the coefficient of diffusion D .

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