POLARIZATION IN PAIR PRODUCTION BY CIRCULARLY POLARIZED QUANTA

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An analysis is given of the cross section formulas for bremsstrahlung and pair production by photons on a Coulomb center; the formulas are derived in the Born approximation without taking into account screening and recoil and for fixed polarization of all the particles. Transformations have been found which allow one to obtain some of the formulas from the others, and by means of them formulas have been derived for pair production by circularly-polarized quanta.

1. For the case of elliptically polarized light the polarization vector may be written in the following form

$$\mathbf{e} = (\mathbf{e}_1 + i\delta\,\mathbf{e}_2)(1+\delta^2)^{-1/2}, \qquad (1)$$

where $|\delta|$ differs from unity provided that the polarization is not circular. In this formula $\mathbf{e}_1$ and $\mathbf{e}_2$ are unit polarization vectors along two mutually perpendicular directions in a plane which is perpendicular to the momentum of the quantum, δ is a real number. On expanding (1) into a plane-polarized and a circularly-polarized wave we obtain

$$\mathbf{e} = |\delta/\sqrt{1+\delta^2}\,|\,[\mathbf{e}_1 + i(\delta/|\delta|)\,\mathbf{e}_2] + (|\delta|-1)\,\mathbf{e}_1/\sqrt{\delta^2+1}. \qquad (2)$$

The cross section for the production of a pair with prescribed polarization must be of the form

$$\sigma = \sigma_A \frac{1}{4}\frac{2\delta^2}{1+\delta^2} + \sigma_B \frac{1}{4}\frac{2(2\delta^2-3|\delta|+1)}{1+\delta^2} + \sigma_C \frac{1}{2}\frac{2\delta^2}{1+\delta^2} + \sigma_{C'}\frac{1}{2}\frac{2\delta^2}{1+\delta^2} + \sigma_D \frac{2\delta^2}{1+\delta^2} + \sigma_E \frac{2(2\delta^2-3|\delta|+1)}{1+\delta^2}. \qquad (3)$$

In the foregoing the following notation has been adopted: σ_A is the cross section given by the Bethe-Heitler formula[1] for the production of pairs of unpolarized particles by unpolarized γ quanta; σ_B is the cross section for the production of pairs of unpolarized particles by γ quanta which are plane-polarized in the $\mathbf{e}_1$ direction;[2,5] the ratios σ_C/σ_A and $\sigma_{C'}/\sigma_A$ give respectively the degree of polarization of electrons and positrons (or of μ^-- and μ^+-mesons) in the case of pair production by circularly-polarized γ quanta. The ratios σ_D/σ_A and σ_E/σ_A give the "degree of correlation of polarization" of positive and negative particles in the cases of unpolarized and plane-polarized γ quanta respectively.

The bremsstrahlung formula is of a form similar to formula (3):

$$\sigma' = \frac{\sigma'_A}{4}\frac{2\delta^2}{1+\delta^2} + \sigma'_B \frac{2(2\delta^2-3|\delta|+1)}{1+\delta^2} + \sigma'_C \frac{2\delta^2}{1+\delta^2}\frac{1}{2} + \sigma'_{C'}\frac{2\delta^2}{1+\delta^2} + \sigma'_D \frac{2\delta^2}{1+\delta^2}\frac{1}{2} + \sigma'_E \frac{2(2\delta^2-3|\delta|+1)}{1+\delta^2}. \qquad (4)$$

In this case a quantum is radiated with polarization given by formula (1), while σ'_i have the following meaning: σ'_A is the bremsstrahlung cross section for an unpolarized electron summed over the polarizations of the γ quanta (Bethe-Heitler formula);[1] σ'_B is the bremsstrahlung cross section for radiation of a given linear polarization from an unpolarized electron,[2-5] the ratios $\sigma'_{C'}/\sigma'_A$ [5] and σ'_C/σ'_A give respectively the degree of circular polarization in the case when the final or the initial electron is polarized; the ratios σ'_D/σ'_A and σ'_E/σ'_A give the "degree of correlation of polarizations" in the case of emission of an unpolarized or a linearly-polarized quantum respectively.

The difference in the coefficients in formulas (4) and (3) arises because it is necessary to take into account the fact that in the first case an averaging was carried out in the calculations, while in the other case a summation over the spins and the polarizations of the quanta was performed.

All the subsequent investigation is carried out in the Born approximation.

The formulas for σ_B, σ'_B, σ_A, σ'_A, σ'_C have been obtained and investigated a long time ago;[1-5] the formulas for $\sigma'_{C'}$, σ_C, $\sigma_{C'}$ may be easily obtained by introducing new variables (and making use of symmetry properties) from the formula for the circular polarization of bremsstrahlung by polarized electrons obtained in the paper by Vysotskiĭ et al.[5] The formulas for σ'_D, σ_D, σ_C, σ'_C

have been obtained earlier in special cases.[6,7] In the general case the expressions for σ'_D, σ_D, σ_E and σ'_E were obtained in Claesson's paper,[8] but since in deriving them McVoy's method[6,7] was used, the expressions obtained are of such an awkward form that it is possible to utilize them only by using an electronic computer.

Let us first of all investigate the question of the possibility of a direct derivation of the formulas for bremsstrahlung from the formulas for pair production, and conversely, in the case of fixed polarizations. In order to do this we shall investigate the expressions for the square of the modulus of the matrix element for both these processes.

In the case of pair production

$$|M|^2 = \frac{\varepsilon^2}{32\omega E_- E_+}\frac{1}{(1+\delta^2)}\mathrm{Sp}\left\{\left[\frac{\hat{a}_q(-\hat{p}_+ + \hat{k} + m)\hat{e}}{2(p_+k)}\right.\right.$$
$$\left. + \frac{\hat{e}(\hat{p}_- - \hat{k} + m)\hat{a}_q}{2(p_-k)}\right](-\hat{p}_+ + m)(1 - i\gamma_5\hat{S}^+)\left[\frac{\hat{a}_q(\hat{p}_- - \hat{k} + m)\hat{e}}{2(p_-k)}\right.$$
$$\left.\left. + \frac{\hat{e}(-\hat{p}_+ + \hat{k} + m)\hat{a}_q}{2(p_+k)}\right](1 - i\gamma_5\hat{S}^-)(\hat{p}_- + m)\right\}. \quad (5)$$

In the foregoing and also in subsequent material the following notation has been used: the index "+" refers to the positron (μ^+ meson), the index "−" refers to the electron (μ^- meson), $E_{(\pm)}$, ω denote the energy and $p_{(\pm)}$, k denote the 4-momentum of a positively or a negatively charged particle and of a γ quantum. The projection operators for the electron and positron polarization are of the form

$$S^{\pm} = \{(\mathbf{p}_{\pm}, \mathbf{J}_{\pm})/m;\ \mathbf{J}_{\pm} + (\mathbf{p}_{\pm}, \mathbf{J}_{\pm})\,\mathbf{p}_{\pm}/m(E_{\pm} + m)\}, \quad (6)$$

J is the spin direction.

The square of the modulus of the matrix element for pair production (5) goes over into the square of the modulus for bremsstrahlung on making the following substitution:

$$p_+ \to -p_1; \quad p_- \to p_2; \quad k \to -k, \quad (7a)$$
$$S^+ \to S^1; \quad S^- \to S^2. \quad (7b)$$

Thus from the four-dimensional expressions for the cross sections of the one process we can obtain the four-dimensional expressions for the cross sections of the other process. However, it may be seen from expression (6) for the projection operators that if one makes the substitution (7a) in (6) then in this case we have $S^- \to S^2$, but S^+ does not go over into S^1. In order to find the transformation under which it is possible to obtain from the three-dimensional expressions for the one process the three-dimensional expressions for the other process, we note that expression (5) for the square of the modulus of the matrix element for pair production goes over into itself under the transformation

$$k \to -k; \quad p_+ \rightleftarrows -p_-; \quad S_+ \rightleftarrows S_-; \quad \mathbf{e} \rightleftarrows \mathbf{e}^*. \quad (8)$$

A similar transformation for the square of the modulus of the matrix element for bremsstrahlung may, however, also be carried out in three-dimensional form, since when the transformation:

$$k \to -k; \quad p_1 \rightleftarrows p_2; \quad S^1 \rightleftarrows S^2; \quad \mathbf{e} \rightleftarrows \mathbf{e}^* \quad (9a)$$

is carried out then automatically the transformation

$$\mathbf{J}_1 \rightleftarrows \mathbf{J}_2, \quad (9b)$$

occurs, and therefore the following transformation is at once realized

$$\mathbf{e} \rightleftarrows \mathbf{e}^*; \quad k \to -k; \quad p_1 \rightleftarrows p_2; \quad \mathbf{J}_1 \rightleftarrows \mathbf{J}_2. \quad (10)$$

However, in expression (5) one may carry out another replacement in addition to (8), in which the spin directions of the electron and the positron are directly interchanged, viz. the following replacement takes place

$$k \to k; \quad p_- \rightleftarrows p_+; \quad S^- \rightleftarrows S^+, \quad (11)$$

which differs in the signs of p_+, p_-, k from the replacement (8). Indeed, only such terms in (5) differ from zero which contain an even number of factors: $\hat{p}_+$; $\hat{p}_-$; $\hat{S}^-$; $\hat{S}^+$; $\hat{k}$; therefore the following two cases are possible:

1) The term contains one of the spin projection operators ($\hat{S}^-$ or $\hat{S}^+$). Then under the replacement (11) it will change sign. But such terms may appear only in expressions corresponding to circular polarization, and moreover one must also take into account the fact that in the replacement $\mathbf{e} \rightleftarrows \mathbf{e}^*$ these (and only these) expressions have an additional change of sign, and therefore the term will not change its sign.

2) The term contains both projection operators. In this case the requirement that the square of the absolute value of the matrix element should be real leads to the result that the expressions which contain the first power of δ (cf (1)) cannot appear in such terms, and therefore in the replacement $\mathbf{e} \rightleftarrows \mathbf{e}^*$ the sign does not undergo an additional change, and since in this case there is a change of sign in an even number of factors (four-momenta), then the term does not change sign in this case also. But under the transformation (11) a replacement of spins

$$\mathbf{J}_+ \rightleftarrows \mathbf{J}_-,$$

also occurs, and therefore expression (5) turns out to be symmetric with respect to the replacement

$$k \to k; \quad p_+ \rightleftarrows p_-; \quad \mathbf{J}_- \rightleftarrows \mathbf{J}_+, \tag{12}$$

since it is equivalent to the replacement (8).

We emphasize that if in the expression for bremsstrahlung we introduce a replacement similar to (10),

$$k \to -k; \quad p_1 \rightleftarrows p_2; \quad \mathbf{J}_1 \rightleftarrows \mathbf{J}_2 \tag{13}$$

(without replacing **e** by **e***), then the expressions corresponding to circular polarization will change sign (the other expressions will not change sign).

We also note another property of the projection operators: if **J** is the transverse polarization, then any arbitrary transformations of p in S will not change S. But if **J** is a longitudinal polarization, then when the (four-dimensional) momentum p changes sign the longitudinal polarization will also change sign. We can make use of this last fact in order to obtain special cases of formulas for the correlation of polarizations.

We emphasize that although all the considerations presented below apply to the case of the absence of screening, the symmetry properties of the formulas are retained also for the case of complete screening at ultra-relativistic energies.

2. The expression for σ'_C, containing circular polarization was obtained in the general case in the paper by Vysotskiĭ, Kresnin, and Rozentsveig:[5*]

$$\sigma'_C = \frac{m}{2E_1 |\mathbf{q}'|^4}$$

$$\times\left\{\left(\frac{\mathbf{p}_1}{\varkappa_2} + \frac{\mathbf{p}_2}{\varkappa_2}\ [\mathbf{k}\times[\mathbf{k}\times(\mathbf{p}_1 - \mathbf{p}_2)]]\right)\left(\mathbf{J}_1,\ \frac{\omega\,\mathbf{p}_1 + E_1\mathbf{k}}{\varkappa_1} + \frac{\omega\,\mathbf{p}_1 - E_1\mathbf{k}}{\varkappa_2}\right)\right.$$

$$+ \frac{1}{2} m^2\omega\left[\frac{\varkappa_1}{\varkappa_2} - \frac{\varkappa_2}{\varkappa_1} + \frac{4\omega}{m^2}\left(\frac{E_1}{\varkappa_1} + \frac{F_2}{\varkappa_2}\right)\right]$$

$$\left.\times\left(\mathbf{J}_1, \frac{E_2}{\varkappa_2}(\omega\,\mathbf{p}_1 - E_1\mathbf{k}) + \frac{E_1}{\varkappa_1}(\omega\,\mathbf{p}_2 - E_2\mathbf{k})\right)\right\} N_\mathrm{b}\frac{\delta}{|\delta|}; \tag{14}$$

where ϑ_1, ϑ_2 are the angles between **k** and $\mathbf{p}_1$ or $\mathbf{p}_2$; N_b is a factor which takes into account the number of states:

$$N_\mathrm{b} = \frac{4Z^2\alpha^3\, d\omega\, do_1\, do_2}{(2\pi)^2\,\omega}\,\frac{|\mathbf{p}_2|}{|\mathbf{p}_1|}.$$

We carry out the replacement (13) in expression (14). In doing this both now and everywhere later naturally the replacement is not carried out in N. In making this replacement it is necessary to change the sign in front of the whole expression as was mentioned earlier. Then for the quantity $\sigma'_{C'}$ corresponding to circular polarization and for a fixed polarization of the final electron we obtain the following expression

$$\sigma'_{C'} = -\frac{1}{2} N_\mathrm{b}\frac{\delta}{|\delta|}\frac{m}{2E_2|\mathbf{q}'|^4}\left\{\left(\frac{\mathbf{p}_2}{\varkappa_1} + \frac{\mathbf{p}_1}{\varkappa_2},\ [\mathbf{k}\,[\mathbf{k},\,\mathbf{p}_2 - \mathbf{p}_1]]\right)\right.$$

$$\times\left(\mathbf{J}_2,\ \frac{\omega\,\mathbf{p}_2 + E_2\mathbf{k}}{\varkappa_2} + \frac{\omega\,\mathbf{p}_2 - E_2\mathbf{k}}{\varkappa_1}\right)$$

$$+ \frac{1}{2} m^2\omega\left[\frac{\varkappa_2}{\varkappa_1} - \frac{\varkappa_1}{\varkappa_2} + \frac{4\omega}{m^2}\left(\frac{E_2}{\varkappa_2} + \frac{E_1}{\varkappa_1}\right)\right]$$

$$\left.\times\left(\mathbf{J}_2,\ \frac{E_1}{\varkappa_1}(\omega\,\mathbf{p}_2 - E_2\mathbf{k}) + \frac{E_2}{\varkappa_2}(\omega\,\mathbf{p}_1 - E_1\mathbf{k})\right\}. \tag{15}$$

To obtain the expression for σ_C we shall carry out in formula (15) the replacement $\mathbf{p}_+ \to -\mathbf{p}_-$; $\mathbf{p}_- \to \mathbf{p}_2$; $\mathbf{k} \to -\mathbf{k}$; $\mathbf{J}_2 \to \mathbf{J}_-$ [which in the present case is equivalent to the replacement (7a), (7b)]

$$\sigma_C = \frac{N_\mathrm{p}\delta(-1)\,m}{|\delta|\,2E_-|\mathbf{q}|^4}\left\{\left(\frac{\mathbf{p}_-}{\varkappa_-} - \frac{\mathbf{p}_+}{\varkappa_+},\ [\mathbf{k}\,[\mathbf{k},\,\mathbf{p}_- + \mathbf{p}_+]]\right)\right.$$

$$\times\left(\mathbf{J}_-,\ \frac{-\omega\,\mathbf{p}_- - E_-\mathbf{k}}{\varkappa_+} - \frac{\omega\,\mathbf{p}_- - E_-\mathbf{k}}{\varkappa_-}\right)$$

$$-\frac{1}{2} m^2\omega\left[\frac{\varkappa_+}{\varkappa_-} - \frac{\varkappa_-}{\varkappa_+} + \frac{4\omega}{m^2}\left(\frac{E_-}{\varkappa_+} + \frac{E_+}{\varkappa_-}\right)\right]\left(\mathbf{J}_-, \frac{E_+}{\varkappa_-}(\omega\,\mathbf{p}_- - E_-\mathbf{k})\right.$$

$$\left.\left. + \frac{E_-}{\varkappa_+}(\omega\,\mathbf{p}_+ - E_+\mathbf{k})\right)\right\}. \tag{16}$$

Here

$$\varkappa_- = -2E_-\omega(1 - V_-\cos\vartheta_-);$$

$$\varkappa_+ = -2E_+\omega(1 - V_+\cos\vartheta_+); \quad \mathbf{q} = \mathbf{p}_- + \mathbf{p}_+ - \mathbf{k};$$

$$N_\mathrm{p} = 4Z^2\alpha^3|\mathbf{p}_+||\mathbf{p}_-|\,dE_+\,do_+\,do_-/(2\pi)^2\,\omega^3.$$

We shall obtain the expression containing positron polarization from (15) by making use of the replacement (12)

$$\sigma'_C = \frac{N_\mathrm{p}\delta(-1)\,m}{|\delta|\,2E_+|\mathbf{q}|^4}\left\{\left(\frac{\mathbf{p}_+}{\varkappa_+} - \frac{\mathbf{p}_-}{\varkappa_-},\ [\mathbf{k}\,[\mathbf{k},\,\mathbf{p}_+ + \mathbf{p}_-]]\right)\right.$$

$$\times\left(\mathbf{J}_+,\ \frac{-\omega\,\mathbf{p}_+ - E_+\mathbf{k}}{\varkappa_-} - \frac{\omega\,\mathbf{p}_+ - E_+\mathbf{k}}{\varkappa_+}\right)$$

$$-\frac{1}{2} m^2\omega\left[\frac{\varkappa_-}{\varkappa_+} - \frac{\varkappa_+}{\varkappa_-} + \frac{4\omega}{m^2}\left(-\frac{E_+}{\varkappa_-} + \frac{E_-}{\varkappa_+}\right)\right]$$

$$\left.\times\left(\mathbf{J}_+, \frac{E_-}{\varkappa_+}(\omega\,\mathbf{p}_+ - E_+\mathbf{k}) + \frac{E_+}{\varkappa_-}(\omega\,\mathbf{p}_- - E_-\mathbf{k})\right)\right\}. \tag{17}$$

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*Quite recently the paper by Fronsdal and Uberall was published[10] which contains a part of the results already obtained previously[5] and in agreement with them.

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