Thus for sufficiently large $\tau\omega_{max}$ the wave front of an electromagnetic shock wave consists of a circularly polarized oscillation with a variable frequency.

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*The case of a nonlinear relation between the electric displacement D and field E can be treated similarly, as well as the case of nonlinearity with respect to both the electric and the magnetic fields.

†This circumstance (for electromagnetic waves) was first pointed out and utilized by I. G. Kataev.

‡The anisotropy field will not be considered. In the following it will be assumed that $H_{z0} = H_0 - 4\pi M > 0$ since only this leads to stability in the initial conditions in the medium.

**In a stationary wave the field components (which, in general, are not transverse) have the form $f(z - vt)$ where the velocity v = const.

***We note that the value for the velocity of the shock wave determined from (2) and (4) coincides with that from (7).

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ON THE HEAT CONDUCTIVITY AND ATTENUATION OF SOUND IN SUPERCONDUCTORS

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We have previously calculated the electronic heat conductivity[1] κ_e, of superconductors and the phonon conductivity,[2] κ_p, determined by the scattering of phonons by electrons. It will be shown here that from the theoretical temperature dependence of κ_e and κ_p found, we can explain, to a considerable extent, all the relationships in the existing experimental data on the heat conductivity of superconductors.

According to our earlier paper[2] κ_p can be expressed as:*

$$\varkappa_p^s = \varkappa_p^n F(T)/F(T_k),$$

$$F(T) = -8(b^4 + b^3)(e^b - 1)^{-1} + 6\zeta(3)(e^b + 1) - 3(e^b + 1)\sum_s s^{-3}\exp(-2bs) \times (4b^2s^2 + 4bs + 2) + 6\zeta(4)(e^b - 1) - (e^b - 1)\sum_s s^{-4}\exp(-2bs)(8b^3s^3 + 12b^2s^2 + 12bs + 6) + 32b^3(e^{2b} - 1)^{-1} - a^4\sum_s\{s\exp(-2bs)\,\mathrm{Ei}[-s(2b - a)]\} + 6\sum_s s^{-3}\exp(-2bs),$$

$$a = 2b - 0{,}16, \quad \zeta(s) = \sum_{n=1}^{\infty} n^{-s}. \qquad (1)$$

In the normal state $\kappa_p^n = \text{const}\cdot T^2$; $b = \Delta(T)/kT$, where $\Delta(T)$ is the energy gap, and κ_s/κ_n depends only on T and T/T_k. For comparison with experiment one must use a specimen with sufficient impurity concentration for κ_e to be small. In Fig. 1 the theoretical curve is drawn according to Eq. (1) and the experimental points are for an In-Tl alloy measured by Sladek.[3]

If $(T_k - T)/T_k$ is not very small, κ_e is not appreciably affected by the electron-phonon interaction.

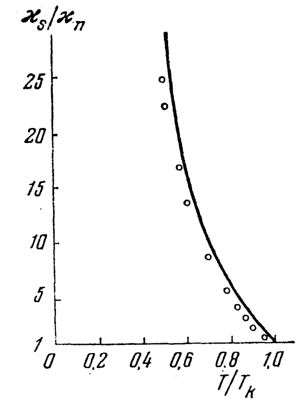

FIG. 1. Points – experimental data[3] for Tl concentration 38%. Solid curve – theoretical

As can be seen from Fig. 1, the conductivity κ_p increases exponentially as $T \to 0$, owing to the increase in phonon mean free path with decreasing scattering by electrons. At sufficiently low temperatures the lattice thermal resistance due to electron scattering, $1/\kappa_{pe}$, becomes less than the resistance due to scattering by lattice defects and crystal boundaries, $1/\kappa_{pd}$ (κ_{pd} is the same as κ_{pd} in a normal metal). Since the resulting lattice conductivity is $\kappa_p = \kappa_{pe}\kappa_{pd}/(\kappa_{pe} + \kappa_{pd})$, we get $\kappa_p \approx \kappa_{pd}$ at still lower temperatures. κ_{pd} usually decreases according to a power law[4] ($\sim T^3$) at low temperatures. For temperatures such that $\kappa_{pd} \sim \kappa_{pe}$, the lattice conductivity should then have a maximum (see curve 1 of Fig. 2). Such a maximum was found in experiments on Pb + 10% Bi.[5]

The electronic heat conductivity varies in quite a different way, because of the reduction in the number of electronic excitations, as was shown by Geĭlikman.[1] κ_e first decreases slowly and then exponentially with decreasing temperature (see curve 2 of Fig. 2).

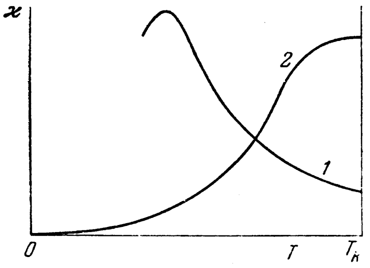

FIG. 2

The total heat conductivity κ is the sum of κ_e and κ_p. In pure specimens $\kappa_p \ll \kappa_e$ at almost all temperatures, and $\kappa \approx \kappa_e$. This is confirmed by measurements on Al and Zn,[6] Sn,[7,9] In,[7] and Pb.[8]

Only at very low temperatures do we have $\kappa_e < \kappa_p$ and $\kappa \approx \kappa_{pd}$. In very impure specimens $\kappa_p \gg \kappa_e$ and $\kappa \simeq \kappa_p$ at all temperatures.[3,5] For intermediate cases of not very pure superconductors, κ_e is the main component near T_k, so that κ falls with decreasing temperature. At sufficiently low temperatures κ_p becomes larger than κ_e, and κ is then determined by curve 1 of Fig. 2. Such a temperature dependence was found in experiments on Sn, Hg and Pb,[9-11] while de Haas and Rademakers[11] and Mendelssohn and Olsen[5] found a maximum in κ, related to the maximum in κ_p (the collected experimental data are contained in Shoenberg's book[12]).

Let us now examine the coefficient γ of absorption of sound in superconductors, due to electronic excitations, when the frequency is $\omega \gg 1/\tau$, where τ is the relaxation time. The absorption due to phonons is, under these conditions, the same as in a normal metal.

From a consideration of the probabilities of absorption of a sound quantum and of the reverse process, we obtain for the ratio

$$\frac{\gamma_s}{\gamma_n} = \frac{x - \ln[(e^{b+x}+1)(e^b+1)^{-1}] + D(x)(2b - x + 2\ln[(e^{x-b}+1)(e^b-1)^{-1}])}{\ln[(e^x+1)/2]},$$

$$x = \hbar\omega/kT;\quad D(x) = \begin{cases} 1, & x \geqslant 2b \\ 0 & x < 2b \end{cases}$$

For $x \ll 1$ this gives $\gamma_s/\gamma_n = 2/(e^b + 1)$, which agrees with the expression previously obtained by Bardeen, Cooper, and Schrieffer.[13]

We thank Academician L. D. Landau for valuable advice.

*There is a misprint in the final formula of reference 2.

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