

Thus all the statistical characteristics of the system are determined by the mass operator of the "one-particle" Green's function. This fact makes the application of field-theory methods to statistics very fruitful. It is not hard to show from Eq. (8) that for the case  $\beta = \infty$  ( $T = 0$ ) one gets the following well known relation for the energy of the ground state

$$\bar{E} = E(g=0) + \int_0^g \frac{dg'}{g'} \int M(x\beta, x't') G(x't', x\beta) d^3x d^3x' dt'. \quad (9)$$

One can verify this by choosing as the complete orthogonal set of functions for the summation the eigenfunctions of the total Hamiltonian and noting that for  $\beta = \infty$  the ground-state term is the only one contributing to the sum.

\*The integration with respect to  $t$  is everywhere taken from 0 to  $\beta$ .

<sup>1</sup>T. Matsubara, Prog. Theor. Phys. **14**, 351 (1955).

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### POLARIZATION OF THE NUCLEUS BY CAPTURE OF POLARIZED NEGATIVE MUONS INTO THE MESONIC K SHELL

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THE formation of  $\mu^-$ -mesonic atoms with transition of  $\mu^-$  mesons to the K shell is accompanied by considerable depolarization of the originally polarized  $\mu^-$  mesons.<sup>1-3</sup> If the nucleus has a spin  $I$  a polarization of the nucleus is also produced, owing to the magnetic interaction of the spins.

The hyperfine splitting in the ground state of the mesonic atom is much larger than  $\hbar/\tau$ , where  $\tau$  is the lifetime of the  $\mu^-$  meson. Therefore the states with  $F = I + \frac{1}{2}$  and  $F = I - \frac{1}{2}$  form an incoherent mixture.<sup>4</sup> Accordingly the spin state of the mesonic atom is best described by the corresponding density matrix  $\rho$ . Since the nucleus is supposed unpolarized before the capture, the only direction

singled out if that of the original polarization of the  $\mu^-$  mesons. Let  $\mathbf{j}$  be the unit vector in that direction. For a given  $F$ , from the requirements of invariance and the Hermitian character of  $\rho$  and its being linear in  $\mathbf{j}$ , we have

$$\rho_F = \frac{1}{2F+1} \left( 1 + \frac{3\lambda_F}{F+1} \mathbf{jF} \right) P_F, \quad (1)$$

$P_F$  is the projection operator onto states with the given value of  $F$ . From the condition  $\mathbf{j}\bar{\mathbf{F}} = \text{Sp}(\rho_F \mathbf{jF}) \leq F$  it follows that  $|\lambda_F| \leq 1$ .

$$\text{For } F = I + \frac{1}{2} \quad P_F = \frac{I+1+2\mathbf{Is}}{2I+1};$$

$$\text{for } F = I - \frac{1}{2} \quad P_F = \frac{I-2\mathbf{Is}}{2I+1},$$

$\mathbf{s}$  and  $\mathbf{I}$  are the spin operators of the  $\mu^-$  meson and the nucleus. Substituting Eq. (2) in Eq. (1), we get:

for  $F = I + \frac{1}{2}$ :

$$\rho_+ = \frac{1}{2(2I+1)} \left\{ 1 + \frac{2\mathbf{Is}}{I+1} + \lambda_+ \mathbf{j} \left( \frac{3\mathbf{I}}{I+1} + 2\mathbf{s} \right) + \frac{3\lambda_+}{(I+1)(I+\frac{3}{2})} \left[ (\mathbf{jI})(\mathbf{sI}) + (\mathbf{sI})(\mathbf{jI}) - \frac{2I(I+1)}{3} \mathbf{js} \right] \right\}, \quad (3)$$

from which we have

$$\frac{\bar{\mathbf{s}}}{s} = \frac{1}{s} \text{Sp}(\rho_+ \mathbf{s}) = \lambda_+ \mathbf{j}, \quad \frac{\bar{\mathbf{I}}}{I} = \frac{1}{I} \text{Sp}(\rho_+ \mathbf{I}) = \lambda_+ \mathbf{j}; \quad (4)$$

for  $F = I - \frac{1}{2}$ :

$$\rho_- = \frac{1}{2(2I+1)} \left\{ 1 - \frac{2\mathbf{Is}}{I} + \lambda_- \frac{I-\frac{1}{2}}{I+\frac{1}{2}} \mathbf{j} \left( \frac{3\mathbf{I}}{I} - 2\mathbf{s} \right) - \frac{3\lambda_-}{I(I+\frac{1}{2})} \left[ (\mathbf{jI})(\mathbf{sI}) + (\mathbf{sI})(\mathbf{jI}) - \frac{2I(I+1)}{3} \mathbf{js} \right] \right\}; \quad (5)$$

$$\frac{\bar{\mathbf{s}}}{s} = \frac{1}{s} \text{Sp}(\rho_- \mathbf{s}) = -\lambda_- \frac{I-\frac{1}{2}}{I+\frac{1}{2}} \mathbf{j},$$

$$\frac{\bar{\mathbf{I}}}{I} = \frac{1}{I} \text{Sp}(\rho_- \mathbf{I}) = \frac{(I-\frac{1}{2})(I+1)}{(I+\frac{1}{2})I} \lambda_- \mathbf{j}. \quad (6)$$

For nuclei with  $I = \frac{1}{2}$ , for example for  $\mu^-$ -mesonic hydrogen,

$$\rho_+ = \frac{1}{4} [1 + \lambda_+ \mathbf{j}(\sigma_p + \sigma_\mu) + \frac{1}{3} \sigma_p \sigma_\mu],$$

$$\rho_- = \frac{1}{4} (1 - \sigma_p \sigma_\mu). \quad (7)$$

One must take this effect (polarization of the nucleus) into account in the analysis of experiments relating to the capture of  $\mu^-$  mesons by nuclei, in particular in the determination of the angular distribution of the neutrons produced. Formulas for the asymmetry parameter of the angular distribution of the neutrons produced in the reaction  $\mu^- + p \rightarrow n + \nu$  have been obtained in references 5-7. In these papers it was assumed that before the capture the proton is unpolarized, despite the fact that the  $\mu^-$  meson located in the K shell is polarized. It thus appears that the results in question are unreliable.

<sup>1</sup>Garwin, Lederman, and Weinrich, Phys. Rev. **105**, 1415 (1957).

<sup>2</sup>Ignatenko, Egorov, Khalupa, and Chultém, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 1131 (1958), Soviet Phys. JETP **8**, 792 (1959).

<sup>3</sup>I. M. Shmushkevich, J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 645 (1959), Soviet Phys. JETP **9**, 000 (1959); Nucl. Phys. (in press).

<sup>4</sup>Bernstein, Lee, Yang, and Primakoff, Phys. Rev. **111**, 313 (1958).

<sup>5</sup>Shapiro, Dolinskiĭ, and Blokhintsev, Dokl. Akad. Nauk SSSR **116**, 946 (1957), Soviet Phys. "Doklady" **2**, 475 (1957).

<sup>6</sup>Huang, Yang, and Lee, Phys. Rev. **108**, 1340 (1957).

<sup>7</sup>B. L. Ioffe, J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 308 (1957), Soviet Phys. JETP **6**, 239 (1958).

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### GAMMA RAYS ACCOMPANYING THE FISSION OF $U^{238}$ BY 2.8 AND 14.7 Mev NEUTRONS

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MOST recent investigations of the prompt fission gammas pertain either to fission of  $U^{235}$ , by neutrons or to spontaneous fission of  $Cf^{252}$ . The latest determinations of the average total energy of the gamma-quanta emitted in one fission yield approximately 7.5 Mev for fission of  $U^{235}$  by thermal neutrons<sup>1</sup> and 8.2 Mev for spontaneous fission of  $Cf^{252}$  (reference 2). It was found experimentally that when  $U^{235}$  is fissioned by thermal neutrons and by neutrons of energies 2.8 and 14.7 Mev, the total energy of the gamma quanta is the same in all cases, within a range of  $\pm 15\%$  (reference 3).

The purpose of this work was to obtain data on the energy liberated in the form of gamma rays during fission of  $U^{238}$  by fast neutrons. Using a procedure and apparatus previously employed,<sup>3</sup> we compared the gamma-ray spectra obtained in the fission of  $U^{238}$  by fast neutrons, with the gamma-ray spectrum obtained in fission of  $U^{235}$  by thermal neutrons. The gamma quanta were registered with a scintillation counter with a

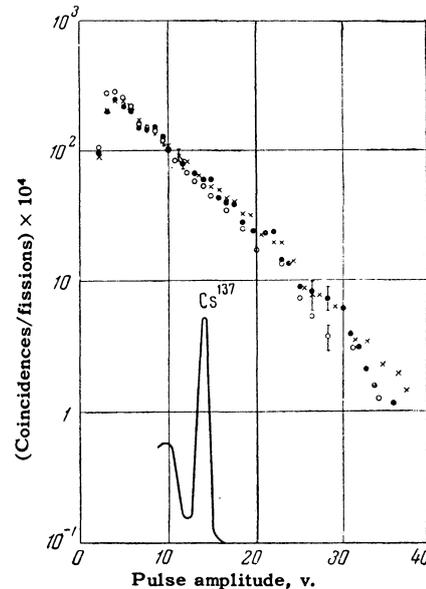


FIG. 1. Distribution of  $\gamma$ -quanta pulses by amplitudes. ● – fission of  $U^{235}$  by thermal neutrons. ○ – fission of  $U^{238}$  by 2.8-Mev neutrons. × – fission of  $U^{238}$  by 14.7-Mev neutrons.

NaI(Tl) crystal, connected for coincidence with a single-layer fission chamber that registered the fissions. Placed in the chamber were targets of equal diameter made of  $U^{235}$  and  $U^{238}$  with an average density of 1.8 and 2.2 mg/cm<sup>2</sup> respectively. The relative placement of the crystal and of the layers of the fissioning substance was the same for thermal and fast neutrons. The spectra of the pulses obtained from the scintillation counter, after eliminating the random coincidences, are shown in the diagram. To estimate the values of the energy, the abscissa of the diagram shows the amplitude distribution of the momenta for gamma rays of 661 keV energy ( $Cs^{137}$ ). The ratio of the number of coincidences between gamma quanta and fragments obtained by fission of  $U^{238}$  by 2.8 and 14.7 Mev neutrons to the number of coincidences for fission of  $U^{235}$  by thermal neutrons, are respectively  $1.03 \pm 0.03$  and  $1.00 \pm 0.02$ . For the measured range of amplitudes, as can be seen from the diagram, the spectra have the same appearance for all cases. Taking account, however, of the fact that the indeterminacy in the final results is greater, owing to the possible divergence of the spectra at large energies, one can conclude that in the fission of  $U^{238}$  by 2.8 and 14.7 Mev neutrons the average total energies of the gamma rays are the same, within 15%, as in the case of fission of  $U^{235}$  by thermal neutrons.

In comparing the data obtained with the results of other investigations we see firstly, that the average total gamma-quanta energies per fission are nearly equal for all the investigated nuclei ( $U^{235}$ ,