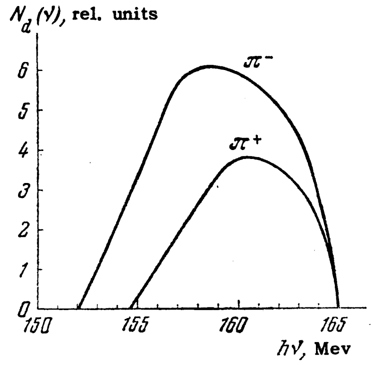

FIG. 2

$$\sigma_d^-(\nu + \Delta\nu, E_\pi)/(\sigma^-/\sigma^+),$$

where $\Delta\nu$ is the difference between the threshold energies for π^- and π^+ production on deuterium. The curves of Fig. 2 were normalized such that the ratio of the yields $N_d^\pm = \int^{\nu_m} N_d^\pm(\nu)\,d\nu$ equals the experimental value $N_d^-/N_d^+ = 2.10 \pm 0.17$.[3] Taking further into account the Coulomb correction (at a photon energy of ~ 160 Mev this amounts to 1.065) we find from the normalization the value $\sigma^-/\sigma^+ = 1.30 \pm 0.11$. Only the statistical error has been indicated. The use of the same momentum distribution for the π^+ mesons as for the π^- mesons can introduce a further error of a few percent. However, a much larger error can be present due to the uncertainty in the determination of the high energy limit of the bremsstrahlung spectrum. Thus, for example, if in the work of Carlson-Lee[3] the high energy limit were 167 instead of 165 Mev, our value of N_d^-/N_d^+ would correspond to $\sigma^-/\sigma^+ = 1.42 \pm 0.12$. The data of Carlson-Lee as reported in the abstract do not allow a more accurate determination of the quantity σ^-/σ^+ for the photon energy $\nu \approx 159$ Mev.

As can be seen from the table the ratio σ^-/σ^+ is approximately constant in the photon energy interval 159 to 200 Mev. Its value agrees well with the results of reference 1 and with the theoretical value.[5]

It should be mentioned that the influence of the sharp drop off of the bremsstrahlung spectrum near the tip has a still larger effect if one compares the π^- and π^+ yields from reactions with large difference in the thresholds. A particularly extreme example is the case of meson photoproduction on beryllium. Since π^- mesons can be created in the reaction $\gamma + Be^9 \rightarrow \pi^- + p + Be^8$ the threshold for production is roughly 17.9 Mev lower than for π^+ production. This way can be explained the anomalous behavior of the value N^-/N^+ with decreasing bremsstrahlung energy or with increasing meson energy or emission angle. This was earlier interpreted as being due to the nuclear structure or due to the special position of the weakly bound neutron. The above described effect has to be taken into account also in the study of the ratio N^-/N^+ in other complex nuclei.

*These corrections amount to around 1.5% on the values N_d^-/N_d^+ given in the table for the case with $\nu_m = 300$ Mev.

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ON THE SCATTERING OF PIONS BY DEUTERONS

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SEVERAL authors[1-5] have compared the results of theoretical calculations with experimental data on the elastic scattering of π mesons by deuterons.[2,6,7] Green[1] and Rockmore[3] have used the impulse approximation[8] and note the disagreement between calculated and experimental cross-sections at small angles.[2,7] Differential cross sections calculated by Bruekner's method[4] give better agreement with experiment. This is attributed to taking multiple scattering into account.[9] Bransden and Moorhouse[5] calculated π-d scattering using a variational method. They also obtained agreement with experiment, but in this method of calculation the contributions from multiple scattering are small.

We note in the following, however, that the calculation of the differential cross-section in the impulse approximation is based on different assumptions than those used either in Brueckner's method or the variational method: the impulse approximation partially takes into consideration the recoil of the nucleons in the deuteron, while in references 4, 5, and 9 the nucleons are considered to be infinitely heavy. It is possible to show that if the nucleon mass is considered infinite in the impulse approximation, then the differential cross-section calculated under such an assumption is not very different from the calculations of Bransden and Moorhouse[5] and Brueckner.[9]

Actually, in the impulse approximation, the differential cross section for the elastic scattering of π mesons by deuterons has the form[3,4]

$$(d\sigma(\theta)/d\Omega)_{\rm lab}=C(q_0, \theta_{\rm lab})\{(d\sigma(\theta)_p/d\Omega)_{\rm cm} + (d\sigma(\theta)_n/d\Omega)_{\rm cm}+ \text{i.t.}\}, \quad (1)$$

where q_0 is the momentum of the impinging meson, $(d\sigma(\theta)_p/d\Omega)_{cm}$ and $(d\sigma(\theta)_n/d\Omega)_{cm}$ are the differential cross sections for π mesons scattered by protons and neutrons, respectively, in the center of mass frame, and i.t. are the interference terms. The expression inside the braces in Eq. (1) is independent of the nucleon mass. For small angles:

$$C \approx 1 + 2\omega_0/M, \quad (2)$$

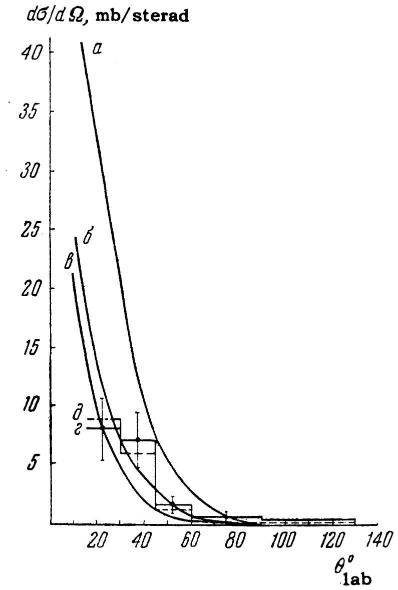
where w_0 is the total energy of the meson in the laboratory frame, while M is the nucleon mass.

From an analysis of the experimental data[2,6,7] one deduces that, for small angles, the experimental cross section is less than that calculated by Eq. (1) approximately by the amount:

$$(2\omega_0/M)\{(d\sigma(\theta)_p/d\Omega)_{\rm cm}+ (d\sigma(\theta)_n/d\Omega)_{\rm cm}+ \text{i.t.}\}.$$

At low energies this correction is small and the impulse approximation is in agreement with experiment.[3,6] For meson energies of 140 and 300 Mev, $2w_0/M \approx 0.6$ and ≈ 0.9, respectively, hence the disagreement.[1,2] If M is set equal to infinity in Eq. (1), then C = 1 for small angles and the usual impulse approximation is in agreement with experiment. In reference 9, M is also set equal to infinity, i.e., C = 1 for small angles in the impulse approximation, and consequently, the correction due to multiple scattering decreases the matrix elements in Brueckner's method insignificantly compared with the matrix elements in the impulse approximation for C = 1.

The above explanation can be illustrated by calculating elastic π-d scattering for 300-Mev mesons. The calculations are made in three ways: (a) in the impulse approximation using Eq. (1), see curve "a" in the diagram; (b) using Eq. (1), with C = 1, see curve "b," and (c) taking into account double scattering at angles $\leq 90°$ for S and P waves in the scattering of π mesons on nucleons, see curve "c." Here C is again set equal to one. The π^--nucleon phase shifts obtained by Dul'kova et al.[2] were used in the calculation. The difference between curves "b" and "c," 15% for $\theta \leq 10°$, lies beyond the limit of experimental precision.



Differential cross-sections for the elastic scattering of 300 Mev π mesons by deuterons. a, b and c – calculated curves; d – experimental data for π^+ mesons; e – experimental data for π^- mesons.[3]

Thus, if we always make the assumption that $M \rightarrow \infty$, then the different theoretical methods give the same result, which does not disagree with experiment. Clearly, the assumption that $M \rightarrow \infty$ is not justified at higher energies. On the other hand, the disagreement of the impulse approximation $(C \neq 1)$ with experiment at small angles is evidence of the incorrectness of the initial assumptions. For example, to derive Eq. (1) it is assumed that the matrix element for the scattering of π mesons by nucleons is independent of the internal motion of the nucleons in the deuteron. Taking this motion into account, a factor of ½ is introduced into Eq. (1) at small angles.

In conclusion, the authors express their thanks to V. Ya. Feĭnberg for useful discussions.

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